Comparison in between the Homotopy Perturbation and Variational Iteration Transform method to Finding Estimation Error with Burger's Fisher Equation

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ABSTRACT

In this article, we consider Cauchy problem for the nonlinear parabolic-hyperbolic partial differential equations is considered. Numerical solutions of the generalized Burgers-Fisher equation are obtained by using an Efficient Variational Iteration Transform method and Homotopy perturbation method (HPM).We compare the results with approximate solution for this equation ,Our numerical results show that the HPM is more efficient and more accurate than VITM[14] .

KEYWORDS

Homotopy perturbation method, Variational Iteration Transform method, Generalized Burger's-Fisher equation, Partial differential equations.

I. INTRODUCTION

Most of problems and phenomena in different fields of science occur nonlinearly, Laplace transform and Variational Iteration Method (VIM) is to construct an iteration method based on a correction functional that includes a generalized Lagrange multiplier

Many methods have been developed for providing approximate solutions of nonlinear partial differential equations (NPDEs). Some of these methods are Variational Iteration Method (VIM) [3] differential transform method (RDTM)[4,5]. The Laplace decomposition method have been used to solve nonlinear differential equations [6 ,7 ,8 ,9]. Generalized Burgers-Fisher equation was investigated by Satsuma in 1986.Non-linear Burger's-Fisher equation is of high importance used for finding analytic or / and approximate solution VIM [10], ADM [11], homotopy analysis method (HAM) [12], are some of the analytical methods. In this article, we will use the Homotopy perturbation method (HPM)[13]) and an Efficient Variational Iteration Transform method[14] to solve the Burger's-Fisher equation and some of the nonlinear mixed parabolic hyperbolic differential equations..We compare the results with approximate solution for this equation Variational Iteration Transform Method show that the used method is exact and feasible for solving such problems. This paper contains basic idea of homotopy perturbation in section 2, The generalized Burger's- Fisher equation in 3, **.**Homotopy-Perturbation Method in 4, .5 is Numerical Result, conclusion is in 6.

II. BASIC IDEA OF HOMOTOPY PERTURBATION METHOD

To explain this method, let us consider the following function

$$A(p) - f(q) = 0, \qquad q \in \Omega \tag{2.1}$$

With boundary conditions

$$\text{of } B(p, p_u) = 0, q \in \Gamma \tag{2.2}$$

Where A, B is a common differential operator and boundary operator respectively, u is known analytical function, and Γ is the boundary of the domain Ω. The operator A can be separated into two parts L and N, where L is linear, while N is nonlinear. So (2.1) can be rewritten as

$$L(p) + N(p) - f(q) = 0. \tag{2.3}$$

By Liao [16] we can construct a homotopy

$w(q, S) : \Omega \times [0,1] \to R$ which satisfies

$$H(w, S) = (1 - S)[L(w) - L(p_0)] + S[A(w) - f(q)] = 0, S \in [0,1] \in \Omega \tag{2.4}$$

$$H(w,S) = L(w) - L(p_0) + SL(p_0) + S[N(w) - f(q)] = 0 \qquad (2.5)$$

Where **q ∈ Γ** and **S∈ [0, 1]** is an embed parameter ,p_0 is an initial approximation of (2.1), which satisfies the boundary conditions. Audibly from Equations (2.4) and (2.5) we will have:

$$H(w,0) = L(w) - L(p_0) = 0 \qquad (2.6)$$

$$H(w,1) = A(w) - f(q) = 0, \qquad (2.7)$$

Altering process of **S** from zero to unity is just that of **H (w , S)** from $L(w) - L(p_0)$ to $A(w) - f(q)$ In topology, this is called deformation, $L(w) - L(p_0)$ and $A(w) - f(q)$ is called homotopic. The embed parameter **S** is introduced a great deal more logically, unaltered by mock factors. In addition, it can be considered as a small parameter for **0 < s ≤ 1.** So it is very usual to assume that the solution of (2.4), (2. 5) can be written as

$$w = w_0 + sw_1 + s^2 w_2 + \ldots. \qquad (2.8)$$

When **s=1** then (2.8) become

$$p = \lim_{s\to 1} . w_0 + w_1 + w_2 + \ldots$$

III. THE GENERALIZED BURGER'S- FISHER EQUATION

Consider the generalized burgers-fisher equation [14].

$$u_r + au^\delta u_y - u_{yy} = bu(1-u^\delta) \qquad 0 \le y \le 1, \quad r \ge 0 \qquad (2.9)$$

With the initial condition **u(y, 0) =f(y),** and exact solution is

$$u\,(y,r) = \left(\frac{1}{2} + \frac{1}{2}\tanh\left[\frac{-a\delta}{2(\delta+1)}\left(y - \left(\frac{a}{\delta+1}\right) + \frac{b(\delta+1)}{a}\right)\right]y\right)^{\frac{1}{\delta}} \qquad (2.10)$$

Where $a, b \ge 0$ and $\delta > 0$. Are given constants if δ **=1**,(2.9) is called Burger's –Fisher Equation.

IV. BURGER'S FISHER EQUATION BY HOMOTOPY-PERTURBATION METHOD

For the solution of (2.9) with initial condition, according to homotopy perturbation Method, we construct the following homotopy: Let

δ=1 $(1-s)(w_r - v_r) + s(w_r + aww_x - w_{xx} - bw(1-w)) = 0$ (2.11)

$$w_x - p_{0_t} = s(-p_{0_t} - aww_x + w_{xx} + bw(1-w)) \quad (2.12)$$

Therefore the solution is written as:

$$w = w_0 + \mathrm{s}w_1 + \mathrm{s}^2 w_2 + \ldots. \quad (2.13)$$

Put (2.13) in (2.12) and then compare coefficient with the same powers of **s**, we get:

$$s^0 : w_{0_t} - p_{0_t} = 0 \quad (2.14)$$

$$s^1 : w_{1_t} + p_{0_t} + aw_0 w_{0_x} - w_{0_{xx}} = bw_0(1-w_0) \quad (2.15)$$

$$\vdots$$

$$s^k : w_{k_t} + \sum_{j=0}^{k-1} aw_k w_{(k-j-1)_x} - w_{(k-1)_{xx}} = bw_{k-1}(1-kw_0) \quad (2.16)$$

$$w_0 = p_0 = f(t) = \frac{1}{2} - \frac{1}{2}\tanh(\frac{1}{4ax}) \quad (2.17)$$

We get

$$w_0 = \frac{1}{2} - \frac{1}{2}\tanh(\frac{1}{4ax}) \quad (2.18)$$

$$w_1 = [(\frac{a^2}{16} - \frac{a^2}{16}\tanh(\frac{1}{4ax})^2 + bw_0(1-w_0)]t \quad (2.19)$$

$$w_2 = [(\frac{a^4}{256} - \frac{a^4}{256}\tanh(\frac{1}{4at})^2)\tanh(\frac{1}{4at}) + bw_1(1-2w_0)]t^2 \quad (2.20)$$

$$\vdots$$

$$w_k = \int_0^t (w_{(k-1)_{xx}} - \sum_{j=0}^{k-1} w_j w_{(k-j-1)_x} + bw_{k-1}(1-kw_0))dt \quad (2.21)$$

By HPM calculate the approximate solution

$$p(x,t)=\lim_{s\to 1} w(x,t)=w_0(x,t)+w_1(x,t)+w_2(x,t)+\cdots \quad (2.22)$$

V. NUMERICAL RESULT

To solve (2.1) by using HPM we take **δ=1** for various value of **a,b** and comparing with results of VITM [14] for this equation [1], our results show that HPM is more efficient and more accurate than the VITM [14], it can be conclude that HPM is very powerful and efficient Method in finding the solution for wide set of problems.

In table 1 explains the comparison of approximate solution by HPM and VITM with exact solution, taken **a=0.001and b=0.001** and δ **=1.** Where $p_{Approximation}=w_0(x,t)+w_1(x,t)+w_2(x,t)+\cdots$

Table 1: (Error Estimate) Error=Exact solution-Numerical solution

X	T	**Exact solution**	**Approx. HPM**	**Approx. VITM [14]**	**Error HPM**	**Error VITM**
0.01	0.02	0.5000038125	0.500004045	0.5000037500	-2.325E-07	6.25E-08
	0.04	0.5000088124	0.500009786	0.5000087500	-9.736E-07	6.24E-08
	0.06	0.5000138124	0.500014034	0.5000137500	-2.216E-07	6.24E-08
	0.08	0.5000188124	0.500017987	0.5000187500	8.254E-07	6.24E-08
0.04	0.02	0.500000625	0.500000001	0.5000000000	6.239E-07	5.75E-07
	0.04	0.5000050625	0.500005001	0.5000050000	6.15E-08	6.25E-08
	0.06	0.5000100624	0.500010001	0.5000100000	6.14E-08	6.24E-08
	0.08	0.5000150624	0.500015077	0.5000150000	-1.46E-08	6.24E-08

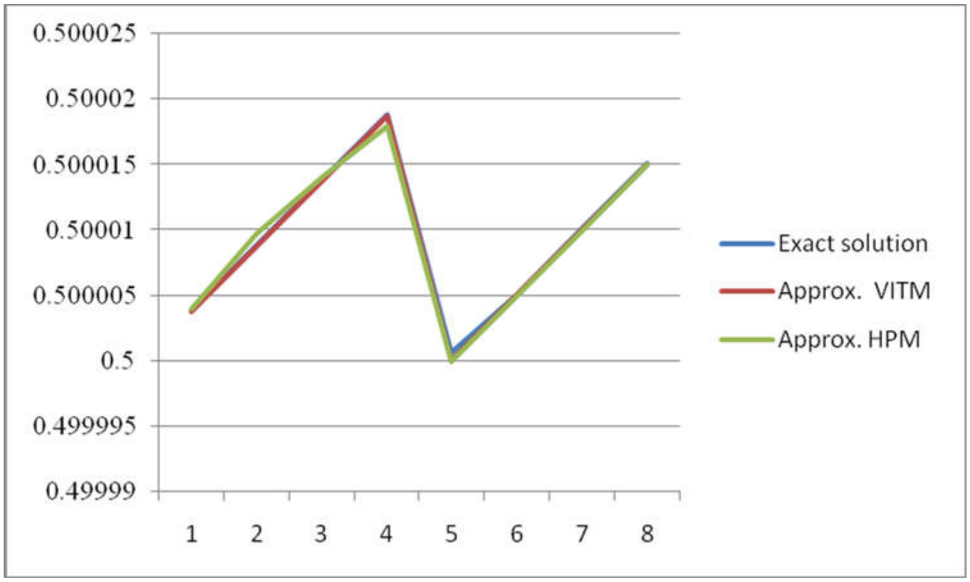

Fig 1.1 shows HPM is more effective as compared to VITM

In Table 2 explains when **a=0.001**and **b=0.001,** and δ **=2** we Can find approximate solution, and make comparison between the exact solution and HPM as well as VITM [14]

Table 2: (Error Estimate) Error=Exact solution-Numerical solution

X	T	**Exact solution**	**Approx. HPM**	**Approx. VITM[14]**	**Error HPM**	**Error VITM**
0.01	0.02	0.707107960089704	0.707108765	0.707104429356344	-8.0491E-07	3.53073E-06
	0.04	0.707105603067101	0.70710681	0.707102072321866	-1.20693E-06	3.53075E-06
	0.06	0.707103246036642	0.70710328	0.707099715279531	-3.39634E-08	3.53076E-06
	0.08	0.707100888998326	0.70710289	0.707097358229340	-2.001E-06	-8.84682E-06
0.04	0.02	0.707118567772678	0.7071185	0.707104444945331	6.77727E-08	4.12283E-06
	0.04	0.707116210785435	0.70711	0.707102087910584	6.21079E-06	1.12287E-06
	0.06	0.707113853790336	0.7071138	0.707099730867981	5.37903E-08	-5.9233E-05
	0.08	0.707111496787380	0.7071114	0.707097373817520	9.67874E-08	3.4705E-07

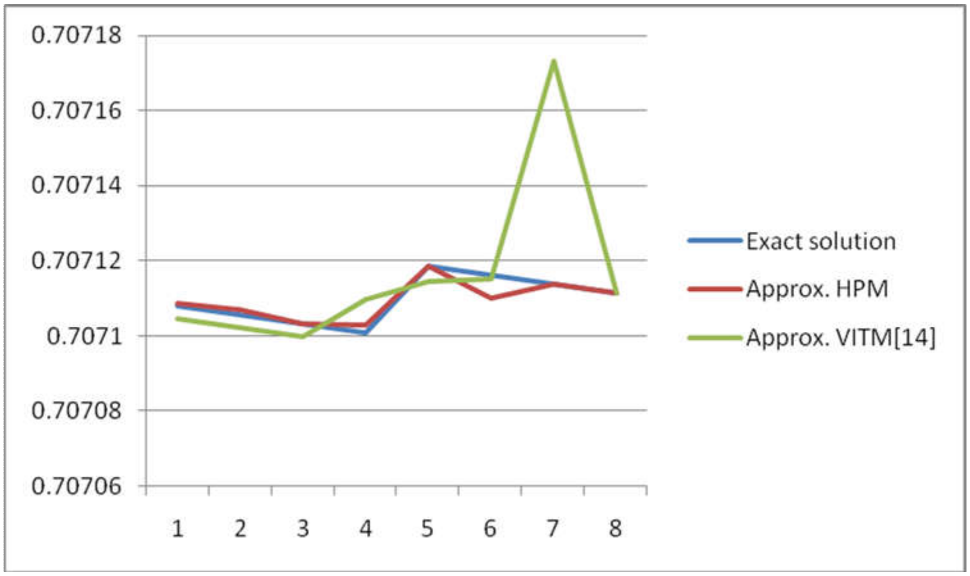

Fig: 2.1 shows HPM is more effective as compared to VITM

VI. CONCLUSION

Table (1) and (2) shows the comparison of results obtained by the HPM and Table (1.1) and (1.2) shows the comparison of results obtained by the HPM and Modified VITM[14] In this study, HPM has been successfully implemented to solve nonlinear mixed parabolic-hyperbolic differential equations. Variational Iteration Transform [14] method reveals its capability of reducing the volume of the computational work and gives high accuracy in the numerical results. . On the other hand, comparison shows that the solution of Burger's–Fisher equation by the HPM is in rather good agreement with the exact solutions and better than the existing methods such as VITM, RDTM and VIM.

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