

Reliability Analysis of a System consisting Main Units and Subunits

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Abstract

A standby redundant system is one in which one operating unit is followed by spare units called standbys. On failure of operating unit, a standby unit is in working mode. In the present paper we investigate the probabilistic analysis of a two-main unit and four subunit system. The system remain operative if one main unit and two subunit are in working mode. System is failed when the main unit fails. A failed system is replaced by new one. The failed sub-unit are repairable.

Key Words: *Availability, Markov renewal process, Mean time to system failure Reliability.*

Introduction

Two/three unit standby systems with two stages, that is, working or failed, have been widely discussed by a large number of researchers including (Mokaddis et. Al. 1997, Taneja, 2005, Goel et. Al. 2010). Introduction of redundancy is one of the well-known methods by which the reliability of a system can be improved. A standby redundant system is one in which one operating unit is followed by spare units called standbys. On failure of operating unit, a standby unit is in working mode. Ritu Mittal (2006) Analyzed stochastic analysis of a compound redundant system consisting three subsystems.

System Description and Assumption.

Initially, the system starts operation with two main units and four subunits. If one main unit and two subunits work then system is in operative mode. After failure of both the main units, it is replaced by new – one. Failure time distributions of main unit and sub-units are arbitrary functions of time. A single repair facility is available to repair the sub-unit. The repair rate of the sub-unit is constant. The repair of sub-unit is as good as new. Using regenerative point technique with the Markov-renewal process, the following reliability characteristics have been obtained:

- (i) Reliability of the system.
- (ii) Steady state availability of the system.

Notation and States of the System.

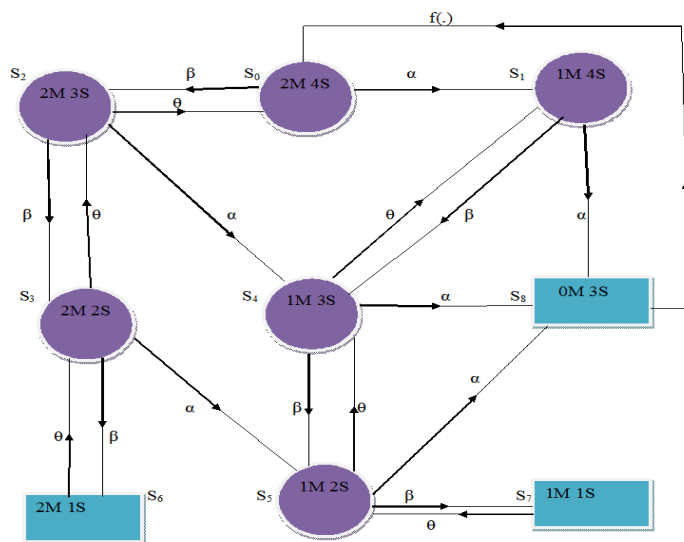
Constant failure rate of main unit

Constant failure rate of sub-unit

repair rate of sub-unit.

f(.) = replacement time pdf of main unit.

- $S_0 (M_2 S_4)$ 2 main units and 4 sub-units are in normal mode.
- $S_1 (M_1 S_4)$ 1 main unit and 4 sub-units are in working mode.
- $S_2 (M_2 S_3)$ 2 main units and 3 sub-units are in working mode.
- $S_3 (M_2 S_2)$ 2 main units and 4 sub-units are in working mode.
- $S_4 (M_1 S_3)$ 1 main unit and 3 sub-units are in working mode.
- $S_5 (M_1 S_2)$ 1 main unit and 2 sub-units are in working mode.
- $S_6 (M_2 S_1)$ System is in failed mode.
- $S_7 (M_1 S_1)$ System is in failed mode.



$S_8 (M_1 S_4)$ System is in failed mode

Analysis of Reliability and Mean Time to system Failure

Assuming that the failed states S_7 and S_8 to be the absorbing states and employing the arguments used in the theory of regenerative process, the following relation among $R_i(t)$ be obtained :

$$\begin{aligned}
 R_0(t) &= Z_0(t) + q_{01}(t) \odot R_1(t) + q_{02}(t) \odot R_2(t) \\
 R_1(t) &= Z_1(t) + q_{14}(t) \odot R_4(t) \\
 R_2(t) &= Z_2(t) + q_{20}(t) \odot R_0(t) + q_{24}(t) \odot R_4(t) + q_{23}(t) \odot R_3(t) \\
 R_3(t) &= Z_3(t) + q_{35}(t) \odot A_5(t) + q_{32}(t) \odot A_2(t) \\
 R_4(t) &= Z_4(t) + q_{41}(t) \odot A_1(t) \\
 R_5(t) &= Z_5(t) + q_{54}(t) \odot A_4(t)
 \end{aligned}
 \tag{1-5}$$

where,

$$\begin{aligned}
 Z_0(t) &= e^{-(\alpha+\beta)t}, & Z_1(t) &= e^{-(\alpha+\beta)t}, \\
 Z_2(t) &= e^{-(\theta+\delta+\beta)t}, & Z_3(t) &= e^{-(\theta+\delta+\beta)t},
 \end{aligned}$$

System sojourns in state S_0 up to time t . Th $Z_4(t) = e^{-(\theta+\alpha+\beta)t}$, $Z_5(t) = e^{-(\theta+\alpha+\beta)t}$.

For an illustration, the equation for $R_0(t)$ is the sum of the following mutually exclusive contingencies :

- (i) probability of this contingency is

$$e^{-(\alpha+\beta)t} = Z_0(t), \text{ (Say).}$$

- (ii) System transits from state S_0 to S_1 during time $(u, u+du)$; $u \leq t$ and then starting from S_1 , it survives for the remaining time duration $(t - u)$. The probability of this event is

$$\int_0^t q_{01}(u) du R_1(t - u) = q_{01}(t) \odot R_1(t).$$

- (iii) System transits from state S_0 to S_2 during time $(u, u+du)$; $u \leq t$ and then starting from state S_2 , it survives for the remaining time duration $(t - u)$. The probability of this event is

$$\int_0^t q_{02}(u) du R_2(t - u) = q_{02}(t) \odot R_2(t).$$

Taking Laplace Transform of relations (1-5) and writing the solution of resulting set of algebraic equations in the matrix form as follows

$$\begin{bmatrix} R_0^* \\ R_1^* \\ R_2^* \\ R_3^* \\ R_4^* \\ R_5^* \end{bmatrix} = \begin{bmatrix} 1 & -q_{01}^* & -q_{02}^* & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -q_{14}^* & 0 \\ -q_{20}^* & 0 & 1 & -q_{23}^* & -q_{24}^* & 0 \\ 0 & 0 & -q_{32}^* & 1 & 0 & -q_{35}^* \\ 0 & -q_{41}^* & 0 & 0 & 1 & -q_{45}^* \\ 0 & 0 & 0 & 0 & -q_{54}^* & 1 \end{bmatrix}^{-1} \begin{bmatrix} Z_0^* \\ Z_1^* \\ Z_2^* \\ Z_3^* \\ Z_4^* \\ Z_5^* \end{bmatrix} \quad (6)$$

The argument 's' has been omitted from $R_i^*(s)$, $q_{ij}^*(s)$ and $Z_i^*(s)$ for brevity. Computing the above matrix equation for $R_0^*(s)$, we get

$$R_0^*(s) = \frac{N_1(s)}{D_1(s)} \quad (7)$$

where

$$N_1(s) = \begin{vmatrix} Z_0^* & -q_{01}^* & -q_{02}^* & 0 & 0 & 0 \\ Z_1^* & 1 & 0 & 0 & -q_{14}^* & 0 \\ Z_2^* & 0 & 0 & -q_{23}^* & 0 & 0 \\ Z_3^* & 0 & 0 & 1 & 0 & -q_{35}^* \\ Z_4^* & -q_{41}^* & 0 & 0 & 1 & 0 \\ Z_5^* & 0 & 0 & 0 & -q_{54}^* & 1 \end{vmatrix}$$

$$D_1(s) = \begin{vmatrix} 1 & -q_{01}^* & -q_{02}^* & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -q_{14}^* & 0 \\ 0 & 0 & 0 & -q_{23}^* & 0 & 0 \\ 0 & 0 & -q_{32}^* & 1 & 0 & -q_{35}^* \\ 0 & -q_{41}^* & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -q_{54}^* & 1 \end{vmatrix}$$

Taking the inverse Laplace transforms of the expression (7), we can get the reliability of the system when system initially starts from state S_0 .

To get the MTSF, we have a well-known formula

$$E(T_0) = \int R_0(t)dt = \lim_{S \rightarrow 0} R_0^*(S)$$

So that, using $Q_{ij}^*(0) = p_{ij}$ and $Z_i^*(0) = \mu_i$ we get

$$E(T_0) = \frac{\mu_0 + p_{01}\mu_1 + p_{02}\mu_2 + p_{01}p_{23}\mu_3 + p_{02}p_{14}\mu_4}{1 - p_{01}p_{14}p_{24} - p_{02}p_{35}p_{32}p_{54}}. \quad (8)$$

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