FUZZY FAULT TREE ANALYSIS FOR SOME HEALTHCARE RELATED PROBLEMS USING LEVEL (λ, ρ) INTERVAL VALUED TRAPEZOIDAL FUZZY NUMBERS.

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ABSTRACT:

In Clinical process there exist some interaction between patients, providers, and technologies. Therefore there are some chances exist for medical errors due to the involvement of human beings and machines. In general, available data is not precise and sufficient to assess a clinical process up to a desired degree of accuracy due to various practical and economical reason. Thus, collected data may have some sort of uncertainties. In this paper, a new fuzzy fault tree approach has been presented for patient safety risk modelling in healthcare. This approach applies fault-tree, inter-valued trapezoidal fuzzy numbers, and the weakest-t-norm (T_w) based approximate arithmetic operation to obtain fuzzy failure probability of the system. The effectiveness of the developed approach is illustrated with two different kinds of problems taken from literature related to healthcare. Also, Tanaka et al.' approach has been used to rank the critical basic events of the considered problems. Computed results have been compared with results obtained from other existing techniques.

KeyWords: healthcare, Fuzzy sets, inter-valued trapezoidal fuzzy numbers, Fault tree analysis, weakest t norm(T_w)

1. INTRODUCTION

The real world is complex. Complexity generally arises from uncertainty which in turn is responsible for some kind of ambiguity. Zadeh's principle of incompatibility suggests that complexity and ambiguity are correlated. The closer one looks at a real world problem, the fuzzier becomes its solutions. As we learn more about a system its complexity decreases and our understanding increases. Yet in many cases, the precision afforded by crisp methods cannot be made useful in modelling the system. Most of our traditional tools for formal modelling, reasoning and computing are crisp, deterministic and precise in character. By crisp, we mean dichotomous, that is yes-or-no type rather than more-or –less, In conventional dual logic, for instance ,a statement can be true or false and nothing in between. In set theory, an element can either belong to a set or not and in optimization, a solution is either feasible or not.

Fuzzy set theory provides a means for representing uncertainties. Historically, probability theory has been the primary tool for representing uncertainty in mathematical models. Because of this, all uncertainties were assumed to follow the characteristics of the random uncertainty. However, all uncertainties are not random. These may be due to imprecision/approximation in measurement, and vagueness in expressions due to the use of linguistic terminology. Such uncertainties are not suited to treatment or modelling by probability theory. In fact it could be argued that the overwhelming amount of uncertainty associated with complex system and issues, which humans address on a daily basis, is non random in nature. Fuzzy set theory is marvellous tool for modelling the kind of uncertainty associated with vagueness and imprecision. We thus have a clear distinction between fuzziness and randomness. Fuzziness describes the ambiguity of an event, Where as randomness describes the uncertainty in the occurrences of the event.

As the living standard of human beings increases due to the increases in their economical and social levels, the need in health-care is also increasing. Here, the term healthcare can be defined as a series of processes with a number of interrelated interventions leading to a particular outcome. For example, for a patient to receive the correct medication, there is a process in which a drug is first prescribed, then dispensed and then administered as shown in fig 1(1).

In order to execute the safe medication treatment process, each of steps shown in fig.1 must be completed correctly by giving full consideration to patient safety. However, a Joint Health Commission report indicates that medical errors result in the death of between 44,000 and 98,000 patients every year and concludes that healthcare is a high risk, error prone industry [2]. The healthcare institutions are exceedingly complicated systems where the likelihood of happening accidents, errors, close calls, sentinel events, failures, and adverse events are always exist. One possibility to have a safer healthcare system is to support the healthcare processes such as prescribing, medication administration by information technology (IT). Research showed that IT applications can have a potential to reduce clinical errors (e.g. medication errors, diagnostic errors), to support healthcare professionals (e.g. availability of timely, up-to-date patient information), to increase the efficiency of care (e.g. less waiting for patients) and to improve the quality of care [3]. In recent years, healthcare systems have been involved in a number of different changes, ranging from technological to normative ones, all asking for increased efficiency [4]. Since patient safety related problems are major concern for healthcare institutions around the world, so the healthcare institutions have to pointed out the main reasons of different kinds of medical errors and to find out then ways for reducing their frequency. In healthcare, more proactive risk analysis techniques should be applied for better and safe medication processes [5]. There are several examples where reliability analysis methods such as root cause analysis (RCA), failure mode and effect analysis (FMEA), fault tree analysis (FTA) and event tree analysis (ETA) have been applied for patient safety risk modelling in healthcare [3,6–9]. Fault tree analysis has been extensively used as a powerful technique in health related risk analysis from both qualitative and quantitative perspectives [8–10]. Hyman and Johnson [7] present a FTA of the patient harm-related clinical alarms failures. They have also

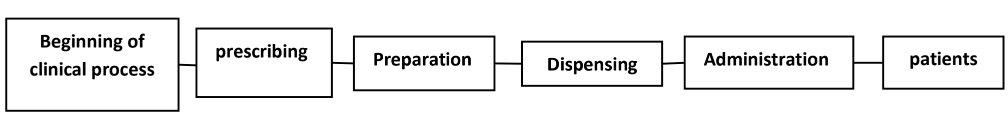


Fig .1.Execution of clinical processes between providers and patients

addressed human factor issues associated with setting, observing, and responding to alarms. Park and Lee [8] constructed a FTA of hand washing process to investigate the causes for faults in hygiene management. They suggested FTA as a good alternative approach to hazard analysis in hazard analysis and critical control point (HACCP) system implementation. Abe cassis et al. [9] used FTA for preventing wrong-site surgery. They also suggested that FTA can be adapted by institutions or specialties to lead to more targeted interventions to increase redundancy and reliability within the preoperative process. Raheja and Escano [10] pointed out some medical error prone are as where FTA can be implemented. Some of the suggested healthcare areas where FTA can be used are equipment failures and malfunctions, material faults, human errors, environment-related risks, management deficiencies, communication and measurement errors, etc

2. FUZZY SETS:

Fuzzy set theory is a paradigm shift of classical set theory. Actually the bivalent nature of classical set theory constraints the decisions to lie between yes and no. The data for most of the real life problems, particularly related to humans judgement and decision cannot be confined to only two options. Fuzzy set theory given by Zadeh is best suited for dealing with the type of uncertainties caused due to vagueness and lack of precisions in the available data. It is an extension of classical set theoretic approach that provides a cover to the uncertainty between yes and no and opens scope for linguistic terms and qualifier in input data. A fuzzy set is defined by a membership function from the universal set to the interval [0,1], as given below;

$$\mu_A(x) : X \to [0,1] \tag{1}$$

where $\mu_A(x)$ gives the degree of belongingness of x in the set A. A fuzzy set A can be expressed as follows:

$$\tilde{A} = \{(x, \mu_A(x)) : x \in X\} \tag{2}$$

Fuzziness can be found in many areas of daily life such as in engineering, medicine, manufacturing and others. In all areas in which human judgements, evaluation and decision are important. These are the areas of decision making reasoning, learning and so on.

3. FAULT TREE ANALYSIS BY FUZZY PROBABILITY

Fault-tree analysis (FTA) is a logical and diagrammatic method to evaluate the probability of an accident resulting from sequences and combinations of faults and failure events. A fault tree describes an accident-model and interprets the relations between malfunctions of the components and observed symptoms. Thus, the fault tree is useful for understanding logically the mode of occurrence of an accident. Furthermore, given the failure probabilities of system components, the probability of the top event can be calculated.

In conventional fault-tree analysis, the failure probabilities of system components are treated as exact values, For many systems, however, it is often difficult to evaluate the failure probabilities of components from past occurrences, because the environments of the systems change. Moreover, we often need to consider failures of components which have never failed before.

Instead of the probability of failure, we propose the possibility of failure, viz. a fuzzy set [1] defined in probability space. By resorting to this concept, we can allocate a degree of uncertainty to each value of the probability of failure; in this manner, different aspects of uncertainty probability and possibility can be simultaneously treated. For example, if information that "the probability of failure is between 0.01 and 0.1, and is perhaps around 0.07" is given, it can be represented as a fuzzy set, ie. possibility of failure. This possibility of failure includes the probability of failure as a limiting case, and thus, the present approach might be more predictive and useful than the conventional uncertainty analysis.

In the present approach based on fuzzy fault-tree model [2], the possibility of failure of the top event is calculated from the possibilities of failure of its components according to the extension principle [3, 4]. In this paper, the possibilities of failure are limited to the trapezoid shape for simplicity; this assumption leads to a reasonable approximation to the mode by which we assess the possibilities of failure.

3.1. FUZZIFICATION AND TRAPEZOIDAL FUZZY NUMBER

In practice, the accurate values of any model parameters reflecting a real system are not known precisely due to unavailability of sufficient amount of data and complete knowledge about the system, and thus the issue is a concern of uncertainty. In order to quantify uncertainty, fuzzifications of model parameters' values or collected data are done by system experts. In the process of fuzzification, crisp data is transformed into fuzzy data with the help of fuzzy membership functions. In literature, a variety of fuzzy membership functions exist for performing fuzzification including triangular, trapezoidal, Cauchy and Gaussian, etc. [25]. For analysing safety and healthcare related problems, trapezoidal fuzzy membership functions or more simply trapezoidal fuzzy number

(TPFNs) are often utilized to provide more precise descriptions and to obtain more accurate solutions [27]. In this paper, TPFNs are used for quantifying data uncertainty associated with basic events. Mathematically, a TPFN ˜A is expressed as ˜A = (a1, a2, a3, a4) and defined by the following membership function

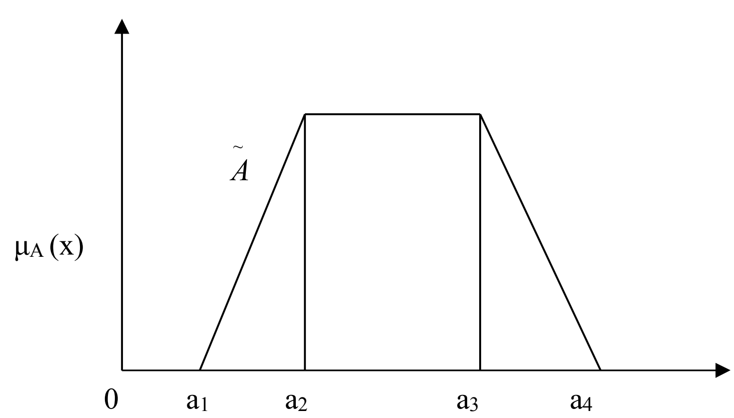


Fig.2 trapezoidal fuzzy number $\tilde{A}$

$$\mu_A(x)=\begin{cases}0 & x\le a_1\\ \dfrac{(x-a_1)}{(a_2-a_1)} & a_1\le x\le a_2\\ 1 & a_2\le x\le a_3\\ \dfrac{(a_4-x)}{(a_4-a_3)} & a_3\le x\le a_4\\ 0 & x\ge a_4\end{cases} \quad (3)$$

3.2.FORMULATION OF FUZZY FAULT-TREE ANALYSIS

This section generalizes evaluation of a fault tree to fuzzy sets. Fault-tree analysis consists of two major parts: construction and evaluation. Here, we are mainly concerned with the fuzzy evaluation of failure probability of the top event of a fault-tree.

A fault-tree is a logic model that represents the combinations of events which lead to the top (undesirable) event. Figure 2 is an example that uses two types of event symbols and two types of gates. The rectangle defines an intermediate or top event that is the output of a logic gate. The circle indicates a fundamental event, viz, a primary failure of a system element. The symbol "+ " stands for an OR gate and the symbol ". " for an AND gate.

In figure 2, the top event can be expanded as:

$$T = A_1 \cup A_2$$

$=(x_1 \cap x_2)\ U\ (x_3\ UA_3)$

$=(x_1 \cap x_2)\ U\ X_3\ U\ (x_4 \cap X_5)$

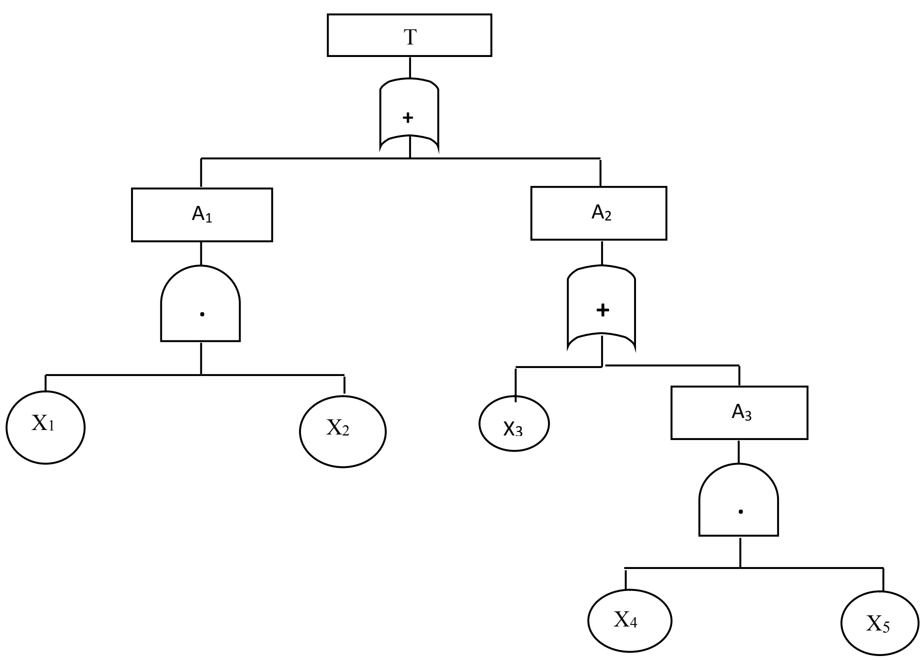


Fig.3 Fuzzy fault tree

4. Procedural Steps of the methodology:

Step 1. Construction of fault-tree diagram

Construct fault-tree diagram for some complex healthcare related problems(e.g. a medication pump failing to deliver medicine to a patient, execution of redundant processes during inpatient transfers to radiology, wrong site surgery, etc.) by using fault-tree logical symbols and trace back the entire process from top to bottom events.

Step 2. Obtain bottom events failure possibilities in the form of level (λ, ρ) interval-valued trapezoidal fuzzy numbers

Possible failure of each bottom event is obtained by aggregating experts knowledge and experience, and represented in terms of level (λ, ρ) interval-valued trapezoidal fuzzy numbers.

Step 3. Computation of system top event fuzzy failure probability($\tilde{q}_T$)

Using fault-tree diagram and possible failure of bottom events represented in terms of level (λ, ρ) interval-valued trapezoidal fuzzy numbers, the system top event fuzzy failure probability($\tilde{q}_T$) can be computed utilizing inter-connection between basic events connected by OR and AND gates, and fuzzy arithmetic operations defined on level (λ, ρ) interval-valued trapezoidal fuzzy numbers. Also, the defuzzified value of system top event can be easily computed using its fuzzy failure probability and Centre of Gravity method of defuzzification.

Step 4. Compute system top event fuzzy reliability

Compute system top event fuzzy reliability which is equal to one minus the fuzzy failure probability of the top event.

Step 5. Find the most and least influential bottom events of the problem.

Tanaka et. al. V −index will be extended for level (λ, ρ) interval-valued trapezoidal fuzzy numbers and then using it, the most and least influential basic events of the considered problems will be evaluated by finding $\max\{V(\tilde{q}_T, \tilde{q}_{T_i}) \forall\, i\}$ and $\min\{V(\tilde{q}_T, \tilde{q}_{T_i}) \forall\, i\}$ values respectively for the whole system, where $\tilde{q}_{T_i}$ is the system top event fuzzy failure probability after eliminated i^{th} basic event.

Step 6. Analyze the results and give suggestions based on it for improving the efficiency of considered healthcare related problems.

5. LEVEL (λ, ρ) INTER-VALUED FUZZY NUMBERS

The following definitions are proposed to use fuzzy numbers and level (λ, ρ) interval-valued fuzzy sets in the fuzzy reliability of serial systems.

Definition 1. $\widetilde{A}$ is called a level λ triangular fuzzy number $0 < \lambda \leq 1$ if its membership functionon R= $(-\infty, \infty)$ is

$$\mu_A(x) = \begin{cases} \frac{\lambda(x-a)}{b-a} & a \le x \le b \\ \frac{\lambda(c-x)}{c-b} & b \le x \le c \\ 0 & otherwise \end{cases} \quad (4)$$

where a<b<c we denote $\widetilde{A}$ = (a, b, c, λ) when $\lambda = 1$, it is called a triangular fuzzy number. The family {(a, b, c, $\lambda \forall a < b < c, a, b, c \in R$) of all level λ fuzzy number is denoted by $F_N(\lambda)$

Definition 2 A fuzzy set $[a, b, \alpha]$ where 0 $\le \alpha \le 1$ and defined on R, is called a level α fuzzy interval, if its membership function is

$$\mu_{(a,b,\alpha)}(x) = \begin{cases} \alpha, & a \le x \le b \\ 0 & otherwise \end{cases} \quad (5)$$

Definition 3.(pu and liu [1 1]) . $\widetilde{a}$ is called a fuzzy point at a if its membership function on R = $(-\infty, \infty)$ is

$$\mu_{\widetilde{a}}(x) = \begin{cases} 1 & x = a \\ 0 & x \ne a \end{cases} \quad (6)$$

Definition 4. (Gorzalczalczany [9]; Deschrijiver [8]) An interval-valued fuzzy set $\widetilde{A}(i - v\ fuzzzy\ set)$ on R is derived by $\widetilde{A} \equiv \{(x, [\mu_{\widetilde{A}^L}(x), \mu_{\widetilde{A}^u}(x)]) / x \in R\}$ 0 ≤ $\mu_{\widetilde{A}^L}(x) \le \mu_{\widetilde{A}^u}(x) \le 1 \forall x \in R$, It is denoted by $\mu_{\widetilde{A}}(x) = [\mu_{\widetilde{A}^L}(x)\, \mu_{\widetilde{A}^U}(x),]\ x \in R$ or $\widetilde{A} = [\widetilde{A}^L, \widetilde{A}^U]$

The i-v fuzzy set $\widetilde{A}$ indicates that, when the membership grade of x belongs to the interval $[\mu_{\widetilde{A}^L}(x)\, \mu_{\widetilde{A}^U}(x),]$ the largest grade is $\mu_{\widetilde{A}^U}(x)$ and the smallest grade is $\mu_{\widetilde{A}^L}(x)$

$$\mu_{\widetilde{A}^L}(x) = \begin{cases} \frac{\lambda(x-a)}{b-a} & a \le x \le b \\ \lambda & b \le x \le c \\ \frac{\lambda(d-x)}{d-c} & c \le x \le d \\ 0 & otherwise \end{cases} \quad (7)$$

Therefore, $\widetilde{A}^L = (a, b, c, d, \lambda)\ a < b < c < d$

Let

$$\mu_{\tilde{A}^U}(x) = \begin{cases} \dfrac{\rho(x-p)}{b-p} & p \le x \le b \\ \rho & b \le x \le c \\ \dfrac{\rho(r-x)}{(r-c)} & c \le x \le r \\ 0 & otherwise \end{cases} \quad (8)$$

Therefore $\tilde{A}^U = (p, b, c, r, \rho), p < b < c < r$, Consider the case in which $0 < \lambda \le \rho \le 1$ and $p < a < b < c < d < r$. From (5) and (6) we obtain $\tilde{A} = [\tilde{A}^L, \tilde{A}^U]\,[(a, b, c, d; \lambda), (p, b, c, r; \rho)]$, Which is called the level (λ, ρ) $i - \text{v}$ fuzzy number.

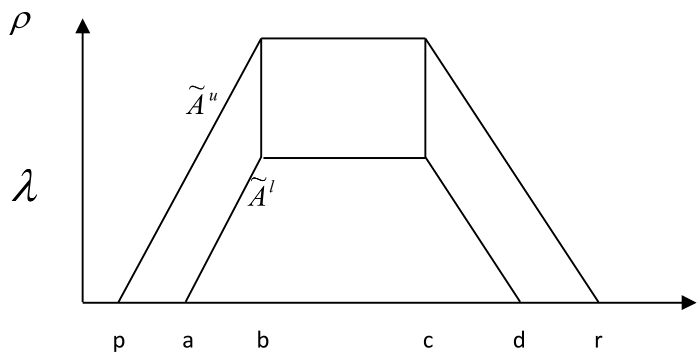


Fig 4 **LEVEL (λ, ρ) INTERVAL-VALUED FUZZY NUMBERS**

2. Table 1Fuzzy arithmetic operation on two positive TPFN's $\tilde{A}$ and $\tilde{B}$

Operation	Fuzzy Expression
(1) Addition	$\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$
(2) Multiplication	$\tilde{A} \otimes \tilde{B} = (a_1b_1, a_2b_2, a_3b_3, a_4b_4)$
(3) Subtraction	$\tilde{A} \Theta \tilde{B} = (a_1-b_1, a_2-b_2, a_3-b_3, a_4-b_4)$
(4) Compliment	$\tilde{1} \Theta \tilde{B} = (1-a_4, 1-a_3, 1-a_2, 1-a_1)$

5.1. t-norm and the weakest t-norm (T_w)

A t-norm is a binary function, t: $[0,1] \times [0, 1] \rightarrow [0, 1]$ which satisfies the axioms of (1) commutativity, (2) associativity, (3) monotonicity and (4) boundary condition [34]

In literature various kind of t –norms exist such as min {x, y}, $x \times y$, max{0, x+y-1} and T_w norm (Eq.(3)) due to its shape preserving characteristics while applying fuzzy arithmetic operations [23].

$$Tw = \begin{cases} x & if \quad y = 1 \\ y & if \quad x = 1 \\ 0 & otherwise \end{cases} \quad (9)$$

Another advantages of Tw norm based approximate fuzzy arithmetic operation is that they give smaller fuzzy accumulation within uncertain environment [34]. These advantages inspired researchers to use Tw norm for successful decreasing growing phenomena of fuzziness in reliability assessment of complex systems

\5.2. Tw based fuzzy arithmetic operation on TPFNs

Lin et al.[34] gave Tw based approximate fuzzy arithmetic operations defined on TFNs. Following same logical approach, this setion introduces Tw based approximate fuzzy arithmetic operations defined for TPFNs. Let $\widetilde{A}$ = (a_1, b_1, c_1, d_1: λ),(p_1, b_1 ,c_1, r_1: ρ) and $\widetilde{B}$ = (a_2, b_2, c_2, d_2: λ),(p_2, b_2, c_2, r_2: ρ) are two i-v TPFNs. Then Tw norm based four basic fuzzy arithmetic operations used in this study are given in table 2.

5.3 Defuzzification

Defuzzification is the process of converting a fuzzy quantity into a precise numerical quantity. In literature various methods have been developed for this purpose[36], Among the many methods available in literature for defuzzification, present study utilizes centre of gravity (COG) method. Let $\mu_{\widetilde{A}}(x)$ be the membership function of the output fuzzy set $\widetilde{A}$, then the algebraic expression of the COG method is given by

$$\widetilde{A} = \frac{\int x \, \mu_{\widetilde{A}}(x)}{\int \mu_{\widetilde{A}}(x)} \quad (10)$$

Where $\widetilde{A}$ is the defuzzified value of the output fuzzy set $\widetilde{A}$. Eq. (4) can be applied to find the defuzzified value of a TPFNs. Let $\widetilde{A}$ =(a_1, b_1, c_1, d_1: λ),(p_1, b_1 ,c_1, r_1: ρ) be a i-v trapezoidal fuzzy numbers then its defuzzified value can be computed by following expression [35].

$$\widetilde{A} = 1/2 \left[\frac{\int_{a_1}^{b_1} \frac{(x-a_1)}{(b_1-a_1)} xdx + \int_{b_1}^{c_1} xdx + \int_{c_1}^{d_1} \frac{(d_1-x)}{(d_1-c_1)} xdx}{\int_{a_1}^{b_1} \frac{(x-a_1)}{(b_1-a_1)} dx + \int_{b_1}^{c_1} dx + \int_{c_1}^{d_1} \frac{(d_1-x)}{(d_1-c_1)} dx} + \frac{\int_{p_1}^{b_1} \frac{(x-p_1)}{(b_1-p_1)} xdx + \int_{b_1}^{c_1} xdx + \int_{c_1}^{r_1} \frac{(r_1-x)}{(r_1-c_1)} xdx}{\int_{p_1}^{b_1} \frac{(x-p_1)}{(b_1-p_1)} dx + \int_{b_1}^{c_1} dx + \int_{c_1}^{r_1} \frac{(r_1-x)}{(r_1-c_1)} dx} \right] \quad (11)$$

$$\widetilde{A} = 1/2 \left[\frac{c_1^2 + d_1^2 + c_1 d_1 - a_1^2 - b_1^2 - a_1 b_1}{3(d_1 + c_1 - b_1 - a_1)} + \frac{c_1^2 + r_1^2 + c_1 r_1 - p_1^2 - b_1^2 - p_1 b_1}{3(r_1 + c_1 - b_1 - p_1)} \right] \quad (12)$$

Traditional FTA and FFTA

3.1. FTA

FTA is an extensively used technique was developed in 1962 at Bell telephone laboratories in USA for safety evaluation of the minuteman launch control system. A fault tree is a logical and graphical description of various combinations of failure events to estimate the probability of an accident from both qualitative nd quantitative perspectives

Table2

T_w based approximation fuzzy arithmetic operation defined on two positive TPFNs

$\widetilde{A} = (a_1, b_1, c_1, d_1 : \lambda), (p_1, b_1, c_1, r_1 : \rho)$ and $\widetilde{B} = (a_2, b_2, c_2, d_2 : \lambda)\ (p_{2,}, b_2, c_2, r_2 : \rho)$

Table 2

Operation	fuzzy expression Tw-norm definition
(1)Addition	$\widetilde{A}^L \oplus_{T_w} \widetilde{B}^L$ =(b_1+b_2-max(b_1-a_1, b_2-a_2),$b_1+b_2, c_1+c_2, d_1+d_2$+max($d_1-c_1, d_2-c_2$))
	$\widetilde{A}^U \oplus_{T_w} \widetilde{B}^U$ = (b_1+b_2-max(b_1-p_1, b_2-p_2),$b_1+b_2, c_1+c_2, r_1+r_2$+max($r_1-c_1, r_2-c_2$))
(2) Multiplication	$\widetilde{A}^L \otimes_{T_w} \widetilde{B}^L$ =(b_1b_2-max(($b_1-a_1)c_2, (b_2-a_2)c_1$),$b_1b_2, c_1c_2, d_1d_2$+max(($d_1-c_1)c_2, (d_2-c_2)c_1$)
	$\widetilde{A}^U \otimes_{T_w} \widetilde{B}^U$ =(b_1b_2-max(($b_1-p_1)c_2, (b_2-p_2)c_1$),$b_1b_2, c_1c_2, r_1r_2$+max(($r_1-c_1)c_2, (r_2-c_2)c_1$)
(3) Subtraction	$\widetilde{A}^L\ \Theta_{T_w} \widetilde{B}^L$ =(b_1-c_2-max(b_1-a_1, d_2-c_2),b_1-c_2, c_1-b_2, c_1-b2+max(d_1-c_1, b_2-a_2)
	$\widetilde{A}^U \Theta_{T_w} \widetilde{B}^U$ = (b_1-c_2-max(b_1-p_1, r_2-c_2),b_1-c_2, c_1-b_2, c_1-b2+max(r_1-c_1, b_2-p_2)
(4)compliment	$\widetilde{1}\Theta_{T_w} \widetilde{B}^L$ =($1-d_2, 1-c_2, 1-b_1, 1-a_2$)
	$\widetilde{1}\Theta_{T_w} \widetilde{B}^U$ =($1-r_2, 1-c_2, 1-b_2, 1-p_2$)

Table3

Row. No.	Approach	Gate	Operation	Equation
(1)	Traditional	OR	Conjunction	$P_{OR} = 1-[(1-q_1)\times(1-q_2)\times\ldots\times(1-q_n)]$
	FTA	AND	Intersection	$P_{AND} = q_1 \times q_2 \times \ldots. \times q_n$
(2)	Traditional	OR	Conjunction	$P_{OR} = \widetilde{1}\Theta[(\widetilde{1}\Theta\widetilde{q}_1)\otimes(\widetilde{1}\Theta\widetilde{q}_2)\otimes\ldots.\otimes(\widetilde{1}\Theta\widetilde{q}_n)]$
	FFTA	AND	Intersection	$P_{AND} = \widetilde{q}_1 \otimes \widetilde{q}_2 \otimes \ldots\ldots \otimes \widetilde{q}_n$
(3)	Proposed	OR	Conjunction	$P_{OR} = \widetilde{1}\Theta_{T_w}[(\widetilde{1}\Theta_{T_w}\widetilde{q}_1)\otimes_{T_w}(\widetilde{1}\Theta_{T_w}\widetilde{q}_2)\otimes_{T_w}\ldots.\otimes_{T_w}(\widetilde{1}\Theta_{T_w}\widetilde{q}_n)$
	FFTA	AND	Intersection	$P_{AND} = q_1 \otimes_{T_w} q_2 \otimes_{T_w} \ldots. \otimes_{T_w} q_n$

Qualitative analysis of a fault tree helps to find out its minimal-cut sets while its quantitative analysis computes the failure probabilities of top event using its basic events' exact failure probabilities. A convential FTA can be easily understood by the basic events representing core causes of system failure while logic gates (OR and AND) are used to represent logical relationship between the basic and top events. The AND gate is used to represent the occurrence of an event requires the happening of all its related causes simultaneously while, the OR gate means that the occurrence of an event requires the happening of any of its related cause. Boolean algebra is applied to estimate system top event failure probability. Once the FTA is constructed for the system, its quantitative analysis can be performed by following the traditional assumption and applying mathematical operation given in table(3)(first row) for n basic event failure probabilities q_i's.

5.4. FFTA

FTA is based on assumption that there are sound and clear success and failure states in a system and that failures occurs at random. So the quantitative evaluation of the top event in FTA is highly dependent on the quality of knowledge such as failure probabilities of basic events and independence between them. The estimation of precise values of basic events' failure probabilities are scarce or either difficult to acquire. In this context, fuzzy fault tree analysis(FFTA)which is an extension of classical FTA may be used[14]. A FFTA can be implemented when

- There are no such clear boundaries exist between failure and success states of the system,
- The probability of system failure cannot be calculated precisely due to lack of sufficient amount of data and due to the existence of uncertainty the data,
- There is subjective evaluation of the reliability.

To overcome the problem of inadequacy in conventional FTA, extensive research has been performed by using fuzzy sets theory. The pioneering work on this issue belongs to Tanaka et.al.[19]. Which treated probabilities of basic events as TPFNs, and applied the fuzzy extension principle to determine the fuzzy probability of top event using mathematical expression given in table3 (second row) associated with n basic events with fuzzy failure probabilities q_i's .

5.5. Development of proposed FFTA

To implement fuzzy sets in proposed FFTA, we used following definition 4.1 and 4.2 to implement OR and AND operation respectively.

Definition 5.1. Let $\widetilde{A}$ =(a_1, b_1, c_1, d_1: λ),(p_1, b_1 ,c_1, r_1: ρ) and $\widetilde{B}=(a_2,b_2,c_2,d_2:\lambda)\ (p_{2,},b_2,c_2,r_2:\rho)$ be two i-v trapezoidal fuzzy numbers then the failure possibility $F(\widetilde{A}\cap\widetilde{B})$ for $\widetilde{A}>0$ and $\widetilde{B}>0$ can be defined using AND operator [19,36] as

$$F(\widetilde{A}\cap B)=F(\widetilde{A})\otimes_{T_w}F(\widetilde{B})$$

The mathematical expressions for OR and AND gates associated with n basic events with fuzzy failure possibilities $\widetilde{q}_i{}'_s$ can be easily formulated using Eqs.(6) and (7), respectively. The formulated mathematical expressions for OR and AND gates are given in Table3 (third row)

To measure the degree of influence of every bottom event in the FFTA, this paper utilizes V measure introduced by Tanaka et.al.[19] and defined as follows:

Definition 5.2.

Let $\widetilde{q}_T$ denote the fuzzy failure probability of the system top event which depends on its components 'fuzzy failure probabilities. if system is constituted by n number of components whose fuzzy failure probabilities($\widetilde{q}_i{}'_s$) are TPFNs then the fuzzy failure probability of system top event is given by the equation,

$$\widetilde{q}_T=\widetilde{q}_T(\widetilde{q}_1,\widetilde{q}_2,.....,\widetilde{q}_i,...,\widetilde{q}_n)\equiv(\widetilde{q}^L{}_{T_1},\widetilde{q}^L{}_{T_2},\widetilde{q}^L{}_{T_3},\widetilde{q}_{T_4}{}^L:\lambda):(\widetilde{q}^U{}_{T_5},\widetilde{q}^U{}_{T_2},\widetilde{q}^U{}_{T_3},\widetilde{q}^U{}_{T_6}:\rho)$$

(8)

Where $\widetilde{q}_T=(\widetilde{q}^L{}_{T_1},\widetilde{q}^L{}_{T_2},\widetilde{q}^L{}_{T_3},\widetilde{q}_{T_4}{}^L:\lambda):(\widetilde{q}^U{}_{T_5},\widetilde{q}^U{}_{T_2},\widetilde{q}^U{}_{T_3},\widetilde{q}^U{}_{T_6}:\rho)$ is a λ,ρ i-v TPFN

Let $\tilde{q}_{T_i}$ be the fuzzy failure probability of system top event after preventing system ith component failure (i.e. $\tilde{q}_i = \tilde{0}$) then the value of $\tilde{q}_{T_i}$ is given by the equation

$$\tilde{q}_{T_i} = \tilde{q}_T(\tilde{q}_1, \tilde{q}_2, ..., \tilde{0}, ..., \tilde{q}_n) \equiv (\tilde{q}^L{}_{Ti_1}, \tilde{q}^L{}_{Ti_2}, \tilde{q}^L{}_{Ti_3}, \tilde{q}^L{}_{Ti_4} : \lambda) : (\tilde{q}^U{}_{Ti_5}, \tilde{q}^U{}_{Ti_2}, \tilde{q}^U{}_{Ti_3}, \tilde{q}^U{}_{Ti_6} : \rho) \quad (13)$$

Where $\tilde{q}_{T_i} = (\tilde{q}^L{}_{Ti_1}, \tilde{q}^L{}_{Ti_2}, \tilde{q}^L{}_{Ti_3}, \tilde{q}^L{}_{Ti_4} : \lambda) : (\tilde{q}^U{}_{Ti_5}, \tilde{q}^U{}_{Ti_2}, \tilde{q}^U{}_{Ti_3}, \tilde{q}^U{}_{Ti_6} : \rho)$ is λ, ρ i-v TPFN

Then the index V, measure the difference between $\tilde{q}_T$ and $\tilde{q}_{T_i}$, and defined as

$$V(\tilde{q}_T, \tilde{q}_{T_i}) = \begin{matrix} (\tilde{q}^L{}_{T_1} - \tilde{q}^L{}_{Ti_1}) + (\tilde{q}^L{}_{T_2} - \tilde{q}^L{}_{Ti_2}) + (\tilde{q}^L{}_{T_3} - \tilde{q}^L{}_{Ti_3}) + (\tilde{q}^L{}_{T_4} - \tilde{q}^L{}_{Ti_4}) + \\ (\tilde{q}^U{}_{T_5} - \tilde{q}^U{}_{Ti_5}) + (\tilde{q}^U{}_{T_2} - \tilde{q}^U{}_{Ti_2}) + (\tilde{q}^U{}_{T_3} - \tilde{q}^U{}_{Ti_3}) + (\tilde{q}^U{}_{T_6} - \tilde{q}^U{}_{Ti_6}) \end{matrix} > 0 \quad (14)$$

$V(\tilde{q}_T, \tilde{q}_{T_i})$ indicates the extent of improvement in eliminating the failure of the ith component.

If $V(\tilde{q}_T, \tilde{q}_{T_i}) \geq V(\tilde{q}_T, \tilde{q}_{T_j})$ then preventing failure of i-th component is more effective than the

preventing failure of jth component of the system.

Illustrative application

To illustrate the proposed method of FFTA, two different kinds of example have been taken from literature related to healthcare. The first example is a medication pump failing to deliver medication [1], while the second example of execution of redundant processes during impatient transfer to radiology is adopted from Ong and Coiera [37]

6 Example 1: FFTA of a medication pump failing to deliver medication[1]

The FTA of a medication pump failing to deliver medication to a patient is shown in Fig.3[1]. This fault tree model has four combination of failures leading to the top event i.e. medication not delivered to patient, immediately below the top event is an OR gate meaning that any individual item below the gate is sufficient by itself to cause the next higher level failure state. For example, pump failure, clamp not removed from tube, pump not activated, and tubing kinked by patient movement are each independently associated with the top event. In this example, the pump and the alarm work together. Pump failure event occurs due to two events (the pump stops and the alarm does not alert to the practitioner regarding the pump stopping) connected by an AND gate. The pump stops due to either an electrical power failure, a pump motor failure, or tubing occlusion.

The fault tree in Fig.3 is typical of how the equipment designer might see the problem. The designer has produced an outstanding product design whose output is concordant with a six sigma level of reliability (three defects per million). For the equipment manufacturer, a pump failure (undetected stopping of the pump) is the top level event of interest. In the healthcare setting, human behaviours and errors are very important contributors to failure of the system. In this fault tree, we have considered three human errors plus one patient factor. Marx and slonim[1] considered the values of failure probabilities of all the basic events as 0.001(column 3 of table 4) However, this could not be possible for real system, and so we have considered these values as different TPFNs as given in table 4(column 4).

Table 4

Basic event	Failure possibility	Crisp value	TPFNs representation
A_1	$\tilde{q}_{A_1}$	0.001	$\begin{pmatrix} 0.0006, 0.0008, 0.0012, 0.0015 : 0.8 \\ 0.0004, 0.0008, 0.0012, 0.0017 : 1.0 \end{pmatrix}$
B_1	$\tilde{q}_{B_1}$	0.001	$\begin{pmatrix} 0.0006, 0.0008, 0.0012, 0.0015 : 0.8 \\ 0.0004, 0.0008, 0.0012, 0.0017 : 1.0 \end{pmatrix}$

C_1	$\tilde{q}_{C_1}$	0.001	$\begin{pmatrix} 0.00055, 0.0007.0.0013, 0.0014 : 0.8 \\ 0.00035, 0.0007, 0.0013, 0.0016 : 1.0 \end{pmatrix}$
D_1	$\tilde{q}_{D_1}$	0.001	$\begin{pmatrix} 0.0006, 0.0007.0.0012, 0.00145 : 0.8 \\ 0.0004, 0.0007, 0.0012, 0.00145 : 1.0 \end{pmatrix}$
E_1	$\tilde{q}_{E_1}$	0.001	$\begin{pmatrix} 0.0005, 0.0007, 0.0013, 0.0016 : 0.8 \\ 0.0003, 0.0007, 0.0013, 0.0018 : 1.0 \end{pmatrix}$
F_1	$\tilde{q}_{F_1}$	0.001	$\begin{pmatrix} 0.0005, 0.0007, 0.0013, 0.0016 : 0.8 \\ 0.0003, 0.0007, 0.0013, 0.0018 : 1.0 \end{pmatrix}$
G_1	$\tilde{q}_{G_1}$	0.001	$\begin{pmatrix} 0.00055, 0.00065, 0.0013, 0.0015 : 0.8 \\ 0.00035, 0.00065, 0.0013, 0.0015 : 1.0 \end{pmatrix}$
H_1	$\tilde{q}_{H_1}$	0.001	$\begin{pmatrix} 0.0005, 0.0007, 0.0013, 0.0016 : 0.8 \\ 0.0003, 0.0007, 0.0013, 0.0018 : 1.0 \end{pmatrix}$

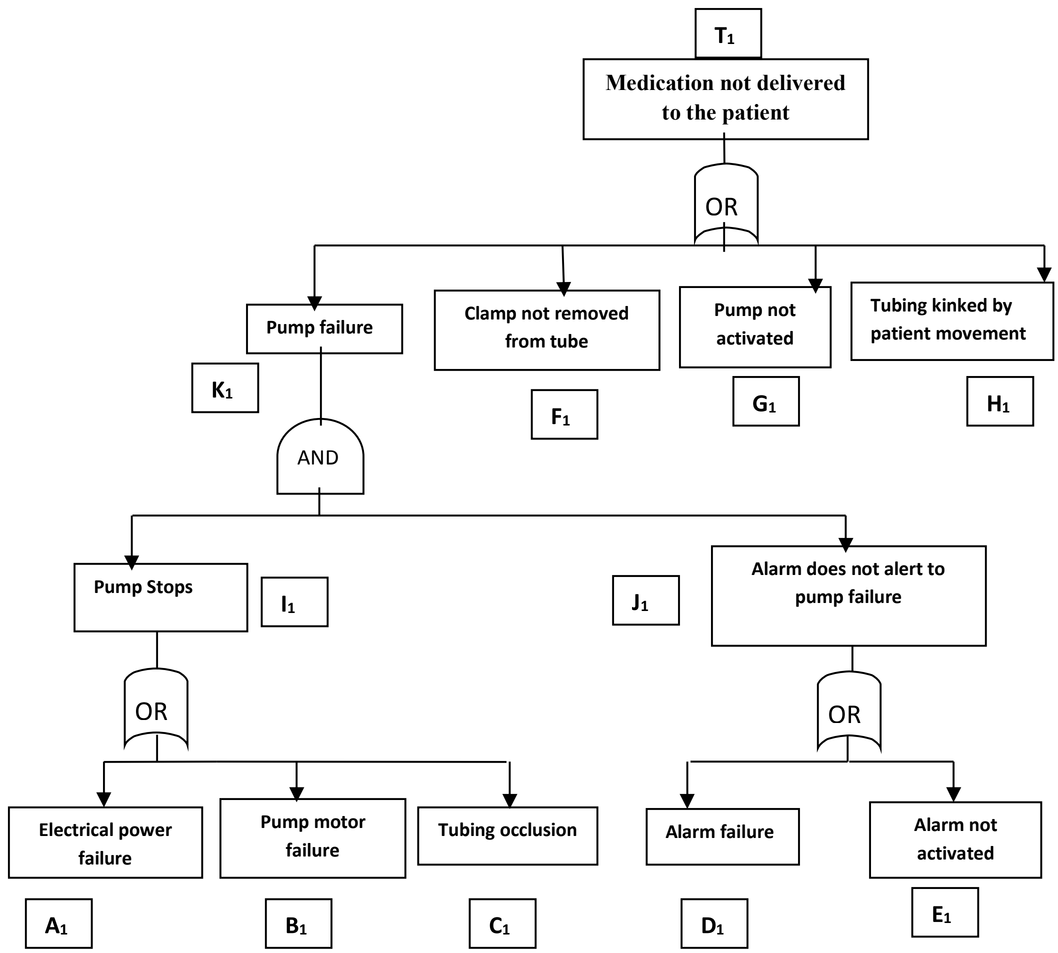


Fig.3. A medication pump fault tree with human error factor failing to deliver medication [1]

Mathematical expression of event is given by

$$T = K_1 \cup F_1 \cup G_1 \cup H_1$$

$$= (I_1 \cap J_1) \cup F_1 \cup G_1 \cup H_1$$

$$= ((A_1 \cup B_1 \cup C_1) \cap (D_1 \cup E_1)) \cup F_1 \cup G_1 \cup H_1$$

And mathematical formula of this expression is given as :

$$q_{T_1} = 1-[(1-q_{K_1})\times(1-q_{F_1})\times(1-q_{G_1})\times(1-q_{H_1})]$$

$$= 1-[(1-q_{I_1}\times q_{J_1})\times(1-q_{F_1})\times(1-q_{G_1})\times(1-q_{H_1})]$$

$$= 1-[(1-(1-(1-q_{A_1})\times(1-q_{B_1})\times(1-q_{C_1}))$$

$$\times(1-(1-q_{D_1})\times(1-q_{E_1})))\times(1-q_{F_1})\times(1-q_{G_1})\times(1-q_{H_1})] \quad (16)$$

6.1.Traditional FTA

Traditionally, probability method is adopted for dealing with the uncertainty [1,12]. However, we know that probability can only quantify the randomness of success or failure event. This research calculated the failure possibility of top event "Medication not delivered to the patient using Eq.[12] and collected data tabulated in table 4(column 3) as follows:

$$q_{T_1} = 1-[(1-(1-(1-q_{A_1})\times(1-q_{B_1})\times(1-q_{C_1}))$$

$$\times(1-(1-q_{D_1})\times(1-q_{E_1})))\times(1-q_{F_1})\times(1-q_{G_1})\times(1-q_{H_1})]$$

$$= 1-[(1-(1-(1-0.001)\times(1-0.001)\times(1-.001))$$

$$\times(1-(1-.001)\times(1-.001)))\times(1-.001)\times(1-.001)\times(1-.001)]$$

$$=0.00301 \quad (17)$$

After the above calculation, we find that the failure probability of top event "Medication not delivered to the patient" is 0.99699.

6.2. Huang et al's method

Huang et.al. [26] posbist FTA method is application is applicable when the failure probability of a system is extremely small or when essential statistical data are scarce. The method could be applied to predict and diagnose a system failure and evaluate its reliability and safety. Using Eq.(11) and crisp failure possibilities given in table 4(column 3), calculation have been done for huang et.al.[26] approach to access the failure possibility of top event "Medication not delivered to the patient" which are as followings:

$$P_{oss}(I_1) = \max(P_{oss}(A_1), P_{oss}(B_1), P_{oss}(C_1))$$

$$= \max(0.001, 0.001, 0.001)$$

$$=0.001$$

$$P_{oss}(J_1) = \max(P_{oss}(D_1), P_{oss}(E_1))$$

$$= \max(0.001, 0.001)$$

=0.001

$P_{oss}(K_1)$ $=\min(P_{oss}(I_1), P_{oss}(J_1))$

=min (0.001, 0.001)

=0.001

Then, the top event failure possibility of top event "medication not delivered to the patient" is 0.001 and the reliability of "Medication delivered to the patient" is 0.999.

6.3.Tanaka et al's approach

Sometimes it is difficult to assign a unique crisp numerical value between 0 and 1 to a failure probability to that situation, the failure probability can be defined as a closed interval [0,1]. Specifically, to quantify the uncertainty, fuzzy sets theory can be used. To compute the fuzzy failure probability of top event. Tanaka et.al [19] approach can be used. To implement this approach, fuzzy arithmetic operations given in Table 1, the definitions of OR and AND gates given in Table 3 (second row), and the fuzzy failure probabilities of the fundamental events given in Table 4(column 4) have been used. The mathematical calculations are given below.

$$q_{T_1} = \tilde{1}\Theta[(\tilde{1}\Theta(\tilde{1}\Theta(\tilde{1}\Theta\tilde{q}_{A_1})\otimes(\tilde{1}\Theta\tilde{q}_{B_1})\otimes(\tilde{1}\Theta q_{C_1}))$$

$$= \otimes(\tilde{1}\Theta(\tilde{1}\Theta\tilde{q}_{D_1})\otimes(\tilde{1}\Theta\tilde{q}_{E_1})))$$

$$= \otimes(\tilde{1}\Theta\tilde{q}_{F_1})\otimes(\tilde{1}\Theta\tilde{q}_{G_1})\otimes(\tilde{1}\Theta\tilde{q}_{H_1})]$$

$$= \begin{pmatrix} 0.001551120, 0.00205181, 0.003964129, 0.004705971 \\ 0.001150374, 0.00205181, 0.003964129, 0.005307615 \end{pmatrix} \quad (18)$$

The computed fuzzy probability or possibility of failure of top event "Medication not delivered to the patient" is $\begin{pmatrix} 0.001551120, 0.00205181, 0.003964129, 0.004705971 \\ 0.001150374, 0.00205181, 0.003964129, 0.005307615 \end{pmatrix}$,while the fuzzy reliability of "Medication not delivered to the patient" is

$$\begin{pmatrix} 0.9953073562, 0.9961050678, 0.9979513997, 0.99844507999 \\ 0.9947093545, 0.9961050678, 0.9979513997, 0.99844507999 \end{pmatrix}$$

The defuzzified values of fuzzy failure probability and fuzzy reliability are 0.0031031549 and 0.996896845

6.4. Proposed Method:

Following the same situation as discussed in previous method in section 5.1.3, the fuzzy failure probability of top event can be computed using developed FFTA which is completely discussed in section 4. To compute the fuzzy failure probability of top event using developed FFTA, T_w based fuzzy arithmetic operations given in table 3(third row), and the fuzzy failure probabilities of the fundamental events given in table 4 (column 4) have been used while the fundamental calculations are given below.

$$\tilde{q}_{T_w} = \tilde{1}\Theta_{T_w}[(\tilde{1}\Theta_{T_w}(\tilde{1}\Theta_{T_w}(\tilde{1}\Theta_{T_w}\tilde{q}_{A_1})\otimes_{T_w}(\tilde{1}\Theta_{T_w}\tilde{q}_{B_1})\otimes_{T_w}(\tilde{1}\Theta_{T_w}q_{C_1}))$$

$$= \otimes_{T_w}(\tilde{1}\Theta_{T_w}(\tilde{1}\Theta_{T_w}\tilde{q}_{D_1})\otimes_{T_w}(\tilde{1}\Theta_{T_w}\tilde{q}_{E_1})))$$

$$= \otimes_{T_w}(\tilde{1}\Theta_{T_w}\tilde{q}_{F_1})\otimes_{T_w}(\tilde{1}\Theta_{T_w}\tilde{q}_{G_1})\otimes_{T_w}(\tilde{1}\Theta_{T_w}\tilde{q}_{H_1})]$$

$$= \begin{pmatrix} 0.0018519146, 0.0020518103, 0.0039041291, 0.0042037178 \\ 0.0016529093, 0.0020518103, 0.0039041291, 0.0044027561 \end{pmatrix} \quad (19)$$

The fuzzy failure probability of top event "Medication not delivered to the patient" is $\begin{pmatrix} 0.0018519146, 0.0020518103, 0.0039041291, 0.0042037178 \\ 0.0016529093, 0.0020518103, 0.0039041291, 0.0044027561 \end{pmatrix}$ and fuzzy reliability of "Medication delivered to the patient is $\begin{pmatrix} 0.9957962822, 0.9960958709, 0.9979451897, 0.9981480854 \\ 0.9955972439, 0.9960958709, 0.9979451897, 0.9983470907 \end{pmatrix}$. The defuzzified values of fuzzy failure probability and reliability of the problem are 0.00330559 and 0.99669441

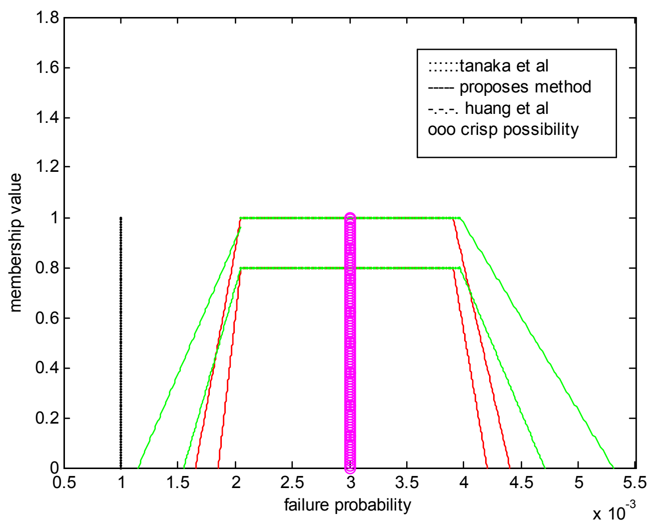


Table 5 Ranking of basic event of Example 1 using failure difference

Eliminated event	$\tilde{q}_{Ti_1}$	$V(\tilde{q}_{T_1}, \tilde{q}_{T_{1i}})$	Rank
$A_1(i=1)$	$\begin{pmatrix} 0.0018503965, 0.0020506945, 0.0059011501, 0.042007453 \\ 0.0016512353, 0.0020506945, 0.0059011501, 0.004400475 \end{pmatrix}$	0.0000166353	5
$B_1(i=2)$	$\begin{pmatrix} 0.0018503965, 0.0020506945, 0.0059011501, 0.042007453 \\ 0.0016512353, 0.0020506945, 0.0059011501, 0.004400475 \end{pmatrix}$	0.0000166353	5
$C_1(i=3)$	$\begin{pmatrix} 0.0018511048, 0.0020408343, 0.0039009015, 0.004200497 \\ 0.0016513751, 0.0020408343, 0.0039009015, 0.004400227 \end{pmatrix}$	0.0000165011	6
$D_1(i=4)$	$\begin{pmatrix} 0.0018504762, 0.0020502058, 0.0038997175, 0.004199313 \\ 0.0016507464, 0.0020502058, 0.0038997175, 0.004399043 \end{pmatrix}$	0.0000233772	4

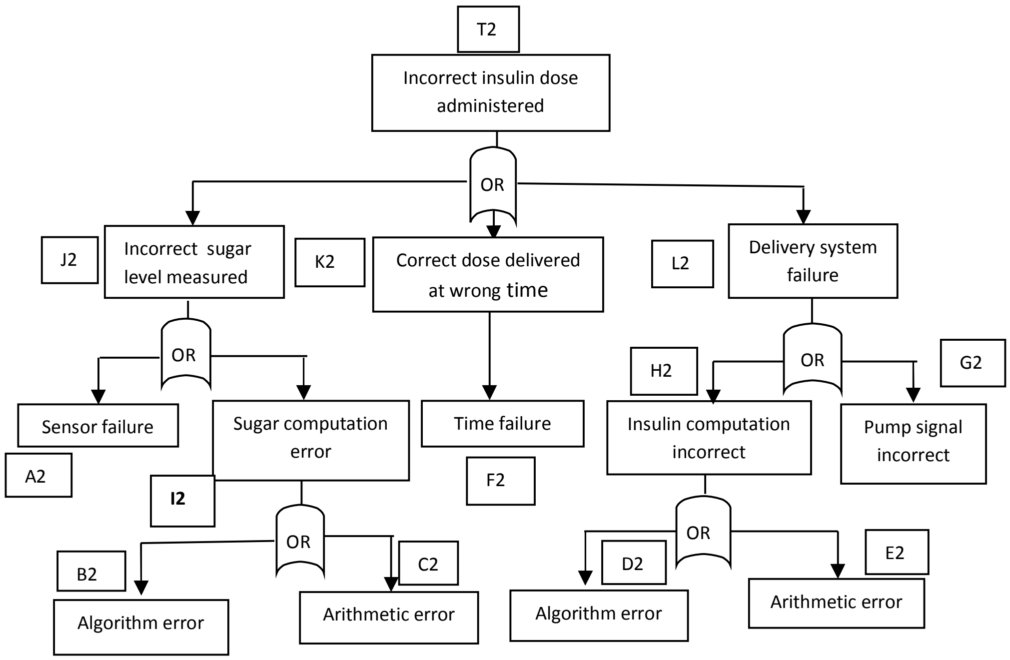
$E_1(i=5)$	$\begin{pmatrix} 0.0018504762, 0.0020502058, 0.0038997175, 0.004198945 \\ 0.0016507464, 0.0020502058, 0.0038997175, 0.004398675 \end{pmatrix}$	0.000025234	3
$F_1(i=6)$	$\begin{pmatrix} 0.0011528878, 0.0013527571, 0.0026075189, 0.002907323 \\ 0.0009530184, 0.0013527571, 0.0026075189, 0.002907323 \end{pmatrix}$	0.007982265	1
$G_1(i=7)$	$\begin{pmatrix} 0.0012028426, 0.0014027019, 0.0026075189, 0.002907308 \\ 0.0010029832, 0.0014027019, 0.0026075189, 0.003107168 \end{pmatrix}$	0.0077824332	2
$H_1(i=8)$	$\begin{pmatrix} 0.0011528878, 0.0013527571, 0.0026075189, 0.002907323 \\ 0.0009530184, 0.0013527571, 0.0026075189, 0.002907323 \end{pmatrix}$	0.007982265	1

7. CONCLUSION: For obtaining the critical basic events of top event "Medication not delivered to the patient". We calculated the difference $V\left(\tilde{q}_{T_1}, \tilde{q}_{T_{1i}}\right)$ for each basic event using Eq (10) and results are given in table 5. Based on the value of index V in table 5, it is analyzed that the most critical basic events are F_1 and H_1 whereas least critical basic events is C_1. The order of all critical basic events are given below in decreasing manner

$$(F_1, H_1) > G_1 > E_1 > D_1 > (A_1, B_1) > C_1 \quad (20)$$

8.0. FFTA of insulin delivery system

The FTA diagram sample "Fault tree analysis- Insulin delivery system" was redesigned from the illustration of "CMSI 641: Introduction to software Engineering. Design of critical systems. B.J. Johnson. 2205. Loyola Marymount University". "Another way of assessing hazard is using fault tree analysis. In this process, each of the identified hazards is covered by a detailed analysis to find out what might cause it. Either inductive or deductive reasoning is applied. In the case of software hazards, the usual focus is to determine faults that will system cause the system to fail to deliver a system service, such as a monitoring system. A "fault tree" is constructed to link all the possible situations together, to help identify the interrelationship of the failures, which modules may cause them, and what " Trickle-down effects" there might be. Here is an example of a Fault tree ,as applied to the insulin delivery system from sommerville..Note that this tree is only partially complete, since only the potential software faults are shown on the diagram. The potential failure involving hardware, such as low battery, blood monitor or sensor failure, patient over-exertion or inattention, or medical staff failure are noticeable by their absence. The fault tree and safety specification processes are two ways of helping with system risk assessments that need to take place. First the likelihood of the risk occurrence must be assessed. This is often quantifiable, so numbers may be assigned based on things like MTBF, latency effect, and other known entities. There may be other non-quantifiable contributors to the risk likelihood, however, such that these must be assessed and estimated by experts in the domain. (Don't short-change this process when dealing with critical systems!)Finally, the risk assessment must include the severity of the risk, meaning an estimation of the cost to the project in the event the risk item actually does occur. "Cost to the project" means all associated costs, including schedule delays, human injury, damage to hardware, corruption of data, and so on."



$$T_2 = J_2 \cup K_2 \cup L_2$$

$$T_2 = (A_2 \cup I_2) \cup F_2 \cup (H_2 \cup G_2)$$

$$T_2 = (A_2 \cup (B_2 \cup C_2)) \cup F_2 \cup ((D_2 \cup E_2) \cup G_2)) \quad (21)$$

Using Eq 20 and expressions gives in Table3 (first row), the formulated expression for failure possibility (q_{T_2})of the top event "Incorrect insulin dose administered" is as follows:

$$q_{T_2} = [1-(1-q_{J_2}) \times (1-q_{K_2}) \times (1-q_{L_2})$$

$$q_{T_2} = [1-(1-(1-(1-q_{A_2}) \times (1-q_{I_2})))$$
$$\times (1-q_{F_2}) \times (1-(1-(1-q_{H_2}) \times (1-q_{G_2})))]$$

$$q_{T_2} = [1-(1-q_{A_2}) \times (1-(1-(1-q_{B_2}) \times (1-q_{C_2})))$$
$$\times (1-q_{F_2}) \times (1-(1-(1-q_{D_2}) \times (1-q_{E_2}))) \times (1-q_{G_2})]$$

$$q_{T_2} = [1-(1-q_{A_2}) \times (1-q_{B_2}) \times (1-q_{C_2}) \times (1-q_{F_2}) \times (1-q_{D_2}) \times (1-q_{E_2}) \times (1-q_{G_2})] \quad (22)$$

Table 6

Basic Event	Failure Possibility	Crisp value	TPFN representation
A_2	q_{A_2}	0.060	$\begin{pmatrix} 0.05400, 0.05800, 0.06200, 0.06600 : 0.8 \\ 0.05200, 0.05800, 0.06200, 0.06800 : 1.0 \end{pmatrix}$
B_2	q_{B_2}	0.052	$\begin{pmatrix} 0.04600, 0.05000, 0.05400, 0.05800 : 0.8 \\ 0.04400, 0.05000, 0.05400, 0.06000 : 1.0 \end{pmatrix}$

C_2	q_{C_2}	0.044	$\begin{pmatrix} 0.03800, 0.04200, 0.04600, 0.05000 : 0.8 \\ 0.03600, 0.04200, 0.04600, 0.05200 : 1,0 \end{pmatrix}$
D_2	q_{D_2}	0.024	$\begin{pmatrix} 0.01800, 0.02200, 0.02600, 0.03000 : 0.8 \\ 0.01600, 0.02200, 0.02600, 0.03200 : 1.0 \end{pmatrix}$
E_2	q_{E_2}	0.064	$\begin{pmatrix} 0.05800, 0.06200, 0.06600, 0.07000 : 0.8 \\ 0.05600, 0.06200, 0.06600, 0.07200 : 1.0 \end{pmatrix}$
F_2	q_{F_2}	0.055	$\begin{pmatrix} 0.04900, 0.05300, 0.05700, 0.06100 : 0.8 \\ 0.04700, 0.05300, 0.05700, 0.06300 : 1.0 \end{pmatrix}$
G_2	q_{G_2}	0.034	$\begin{pmatrix} 0.02800, 0.03200, 0.03600, 0.04000 : 0.8 \\ 0.02600, 0.03200, 0.03600, 0.04200 : 1.0 \end{pmatrix}$

8.1 Traditional FTA

Using the data given in table (column 3) and utilizing Eq.(21), the failure possibility of top event "Incorrect insulin dose administered" is computed as follows;

$$q_{T_2} = [1-(1-0.060)\times(1-0.052)\times(1-0.044)\times(1-0.055)\times(1-0.024)\times(1-0.064)\times(1-0.034)]$$

$$q_{T_2} = \mathbf{0.2895579078} \qquad \textbf{(23)}$$

The failure possibility of top event "Incorrect insulin dose administered" is 0.2102218399 and the reliability is **0.710442092**

8.2. Huang et al's method

Applying huang et.al approach, the failure possibility of top event "Incorrect Insulin dose administered" can be computed using Eq.20 . The calculations are as follows:

$$P_{oss}(I_2) = \max(P_{oss}(B_2), P_{oss}(C_2))$$

=max(0.052,0.044)

= 0.052

$$P_{oss}(H_2) = \max(P_{oss}(D_2), P_{oss}(E_2))$$

=max (0.024, 0.064)

= 0.064

$$P_{oss}(J_2) = \max(P_{oss}(A_2), P_{oss}(I_2))$$

= max (0.060, 0.052)

= 0.060

$$P_{oss}(L_2) = \max(P_{oss}(H_2), P_{oss}(G_2))$$

=max (0.064, 0.034)

= 0.064

$$P_{oss}(T_2) = \max(P_{oss}(J_2), P_{oss}(K_2), P_{oss}(L_2))$$

= max (0.060, 0.055, 0.064)

= **0.064** **(24)**

8.3. Tanaka et.al's approach

To apply Tanaka et,al approach for computing the failure possibility of top event, Eq (21), fuzzy arithmetic operations given in table 1, the definition of OR and AND gates given in table 3 (second row) and fuzzy failure probabilities of basic events given in table 7 (column 4) have been used. The mathematical calculations are given below.

$$\tilde{q}_{T_2} = [\tilde{1}\Theta(\tilde{1}\Theta\tilde{q}_{J_2})\otimes(\tilde{1}\Theta\tilde{q}_{K_2})\otimes(\tilde{1}\Theta\tilde{q}_{L_2})$$

$$\tilde{q}_{T_2} = [\tilde{1}\Theta(\tilde{1}\Theta(\tilde{1}\Theta\tilde{q}_{A_2})\otimes(\tilde{1}\Theta\tilde{q}_{I_2}))\otimes(\tilde{1}\Theta\tilde{q}_{F_2})\otimes(\tilde{1}\Theta(\tilde{1}\Theta\tilde{q}_{H_2})\otimes(\tilde{1}-\tilde{q}_{G_2}))]$$

$$q_{T_2} = [\tilde{1}\Theta(\tilde{1}\Theta(\tilde{1}\Theta\tilde{q}_{A_2})\otimes(\tilde{1}\Theta(\tilde{1}\Theta\tilde{q}_{B_2})\otimes(\tilde{1}\Theta\tilde{q}_{C_2})))\otimes(\tilde{1}\Theta\tilde{q}_{F_2})\otimes(\tilde{1}\Theta(\tilde{1}\Theta(\tilde{1}\Theta\tilde{q}_{D_2})\otimes(\tilde{1}\Theta\tilde{q}_{E_2}))(\tilde{1}\Theta\tilde{q}_{G_2})]$$

Fuzzy probability of top event "Incorrect insulin dose administered" is $q_{T_2} =$ $\begin{pmatrix} 0.2576242867, 0.2790468451, 0.2999373474, 0.3203068510; 0.8 \\ 0.2474914011, 0.2790468451, 0.2999373474, 0.3302996332; 1.0 \end{pmatrix}$ and fuzzy reliability of "Incorrect insulin dose administered" is

$$\begin{pmatrix} 0.6796931490, 0.7000626526, 0.0.7209531549, 0.7437571330 \\ 0.6697003668, 0.7000626526, 0.0.7209531549, 0.7525085989 \end{pmatrix} \quad (25)$$

Defuzzified value of fuzzy failure probability and fuzzy reliability is **0.2891603747 and 0.710839625**

8.4. Proposed Method

$$\tilde{q}_{T_2} = [\tilde{1}\Theta_{T_w}(\tilde{1}\Theta_{T_w}\tilde{q}_{J_2})\otimes_{T_w}(\tilde{1}\Theta_{T_w}\tilde{q}_{K_2})\otimes_{T_w}(\tilde{1}\Theta_{T_w}\tilde{q}_{L_2})$$

$$\tilde{q}_{T_2} = [\tilde{1}\Theta_{T_w}(\tilde{1}\Theta_{T_w}(\tilde{1}\Theta_{T_w}\tilde{q}_{A_2})\otimes_{T_w}(\tilde{1}\Theta_{T_w}\tilde{q}_{I_2}))\otimes_{T_w}(\tilde{1}\Theta_{T_w}\tilde{q}_{F_2})$$
$$\otimes_{T_w}(\tilde{1}\Theta_{T_w}(\tilde{1}\Theta_{T_w}\tilde{q}_{H_2})\otimes_{T_w}(\tilde{1}\Theta_{T_w}\tilde{q}_{G_2}))]$$

$$q_{T_2} = [\tilde{1}\Theta_{T_w}(\tilde{1}\Theta_{T_w}(\tilde{1}\Theta_{T_w}\tilde{q}_{A_2})\otimes_{T_w}(\tilde{1}\Theta_{T_w}(\tilde{1}\Theta_{T_w}\tilde{q}_{B_2})\otimes_{T_w}(\tilde{1}\Theta_{T_w}\tilde{q}_{C_2})))$$
$$\otimes_{T_w}(\tilde{1}\Theta_{T_w}\tilde{q}_{F_2})\otimes_{T_w}(\tilde{1}\Theta_{T_w}(\tilde{1}\Theta_{T_w}(\tilde{1}\Theta_{T_w}\tilde{q}_{D_2})\otimes_{T_w}(\tilde{1}\Theta_{T_w}\tilde{q}_{E_2}))(\tilde{1}\Theta_{T_w}\tilde{q}_{G_2})]$$

$$= \begin{pmatrix} 0.2759448236, 0.2790468451, 0.2999373475, 0.303011775; 0.8 \\ 0.2744352045, 0.2790468451, 0.2999373475, 0.304548988; 1.0 \end{pmatrix} \quad (26)$$

The Fuzzy probability of top event "Incorrect insulin dose administered" is $\begin{pmatrix} 0.2759448236, 0.2790468451, 0.2999373475, 0.303011775; ;0,8 \\ 0.2744352045, 0.2790468451, 0.2999373475, 0.304548988; 1.0 \end{pmatrix}$ and fuzzy reliability of "Incorrect insulin dose administered" is $\begin{pmatrix} 0.696988225, 0.7000626525, 0.7209531549, 0.7240551764; 0.8 \\ 0.695451012, 0.7000626525, 0.7209531549, 0.7255647955; 1.0 \end{pmatrix}$

Defuzzified value of fuzzy failure probability and fuzzy reliability **0.2894893132** and **0.710510687**

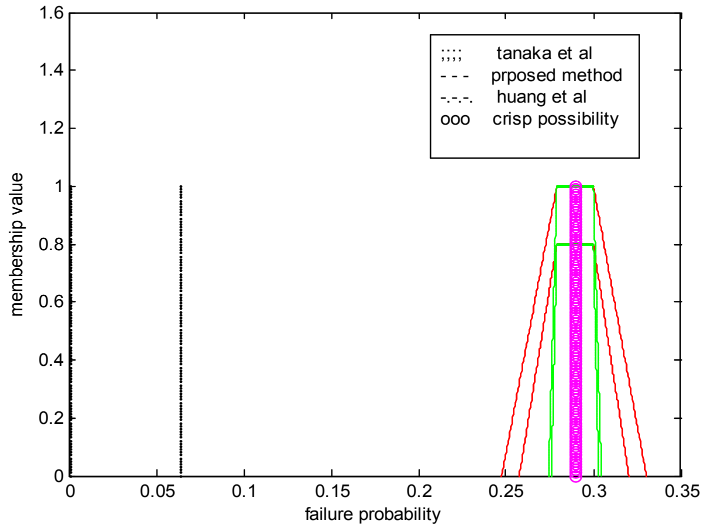


Table 7

Eliminated event	$q_{T_{2i}}$	$V(q_{T_2}, q_{T_{2i}})$	**Rank**
$A_2(i=1)$	$\begin{pmatrix} 0.2313932251, 0.2346569481, 0.2536645495, 0.256928273 \\ 0.2298078900, 0.2346569481, 0.2536645495, 0.258560134 \end{pmatrix}$	0.3625767865	2
$B_2(i=2)$	$\begin{pmatrix} 0.2378366564, 0.2411019422, 0.2599760544, 0.263212294 \\ 0.2362475836, 0.2411019422, 0.2599760544, 0.264830413 \end{pmatrix}$	0.3081525191	4
$C_2(i=3)$	$\begin{pmatrix} 0.2442012773, 0.2474392955, 0.2661817059, 0.269309200 \\ 0.2426254744, 0.2474392955, 0.2661817059, 0.270995527 \end{pmatrix}$	0.2616079498	6
$D_2(i=4)$	$\begin{pmatrix} 0.259685499, 0.262829085, 0.2812498433, 0.284393429 \\ 0.258113706, 0.262829085, 0.2812498433, 0.285963850 \end{pmatrix}$	0.1395948357	7
$E_2(i=5)$	$\begin{pmatrix} 0.2280861660, 0.2313932251, 0.2504682521, 0.253731975 \\ 0.2264976405, 0.2313932251, 0.2504682521, 0.255363837 \end{pmatrix}$	0.3869969843	1
$F_2(i=6)$	$\begin{pmatrix} 0.2355133900, 0.23869783011, 0.2576217895, 0.260868281 \\ 0.2338280934, 0.23869783011, 0.2576217895, 0.262491526 \end{pmatrix}$	0.3375369409	3
$G_2(i=7)$	$\begin{pmatrix} 0.2520376219\ , 0.255213683\ , 0.2273793928\ 9, 0.27696999 \\ 0.2504495914\ , 0.255213683\ , 0.2273793928\ 9, 0.27855802 \end{pmatrix}$	0.2926787852	5

CONCLUSION - For obtaining the critical basic events of top event “incorrect insulin dose administered” .We calculated the difference $V(q_{T_2}, q_{T_{2i}})$ for each basic events using Eq and results are given in table .Based on the values of index V in table .It is analysed that the most critical basic event is E_2, whereas least critical basic event is D_2. The order of the critical basic events are given below in decreasing manner:

E2 > A2 > F2 > B2 > G2 > C2 > D2 (27)

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