Different Types of Ideals in Partially Ordered Ternary Semigroups

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Abstract: Partially Ordered ternary semi group, Partially Ordered left(right) ternary semi group Partially ordered lateral ternary ideal, Partially Ordered ternary ideal are the terms that are defined in this paper. Some properties of these ternary semi groups are discussed. Completely prime partially ordered ideals and prime partially ordered ideals are defined and their properties established. Prime partially ordered radical and completely prime partially ordered radical are defined and their properties discussed.

Introduction:

CLIFFORD, PETRICH, LYARIN and ANJANEYULU studied the algebraic theory of semi groups. In 1932 LEHMER introduced the theory of ternary algebraic system .he worked on certain algebraic systems called triplexes which turn out to be commutative ternary groups with one associative ternary operation the ternary semi groups are universal algebras. The nation of a ternary semi group with examples was given by BANACH. The theory of ternary semi groups was developed by SANTIAGO. And the ideal theory in ternary semi groups was introduce the notions of completely prime ideal, prime ideal, completely semi prime ideal, prime ideal, prime radical and completely prime radical and characterize completely prime ideals, completely semi prime ideals, prime radicals and completely prime radicals in partially ordered ternary semi group.

2. Preliminaries:

Definition 2.1: A Ternary semi group T is said to be a partially ordered ternary semi group If T is a partially ordered set such that
$a \leq b \Rightarrow [a\, a_1\, a_2] \leq [b\, a_1\, a_2], [\, a_1\, a\, a_2] \leq [\, a_1\, b\, a_2], [\, a_1\, b\, a_2] \leq [\, a_1\, a_2\, b]$ for all $a, b, a_{1\,,} a_2 \in T$.

Definition 2.2: A Non empty subset A of a partially ordered ternary semi group T is said to be partially ordered left ternary ideal or partially ordered left ideal of T.

If (i). $b, c \in T, a \in T \Rightarrow bca \in A.$

(ii). If $a \in A\ and\ t \in T\ such\ that\ t \leq a\ then\ t \in A.$

Definition 2.3: A Non empty subset A of a partially ordered ternary semi group T is said to br partially ordered lateral ternary ideal or partially ordered lateral ideal of T .

If (i). $b, c \in T, a \in A \Rightarrow bac \in A.$

(ii). If $a \in A$ and $t \in T$ such that $t \leq a$ then $t \in A.$

Definition 2.4: A non empty subset A of a partially ordered ternary semi group T is said to be partially ordered right ternary ideal or partially ordered right ideal of T.

If (i). $b, c \in T, a \in T \Rightarrow abc \in A.$

(ii). If $a \in A$ and $t \in T$ such that $t \leq a$ then $t \in A.$

Definition 2.5: A non empty subset A of a partially ordered ternary semi group T is said to be partially ordered two sided ternary ideal or partially ordered two sided ideal of T.

If (i). $b, c \in T, a \in T \Rightarrow bca \in A, abc \in A.$

(ii). If $a \in A$ and $t \in T$ such that $t \leq a$ then $t \in A.$

Definition 2.6: A non empty subset A of a partially ordered ternary semi group T is said to be partially ordered ternary ideal or partially ordered ideal of T.

If (i). $b, c \in T, a \in A \Rightarrow bca \in A, bac \in A, abc \in A.$

(ii). If $a \in A$ and $t \in T$ such that $t \leq a$ then $t \in A.$

Note 2.7: Let T be a partially ordered ternary semi group and $A \subseteq T, B \subseteq T. Then$

$$(i). A \subseteq (A],$$

$$(ii). \big((A]\big] = (A],$$

$$(iii). (A](B](C] \subseteq (ABC],$$

$(iv). A \subseteq B \Rightarrow A \subseteq (B]$ and $(v). A \subseteq B \Rightarrow (A] \subseteq (B].$

Note 2.8: The non empty intersection of any family of partially ordered left ideals of a partially ordered ternary semi group T is a partially ordered left ideal of T.

3. Completely Prime Partially ordered ideals and Prime partially Ordered ideals:

Definition 3.1: A partially ordered (left/lateral/right) ideal A of a partially ordered ternary semi group t is said to be a completely prime (left/lateral/right) ideal of T provided

$x, y, z \in T and\ xyz \in A\ impplies\ either\ x\ \in A, or\ y \in A\ or\ z \in A.$

Theorem3.2: A partially ordered ideal A of a partially ordered ternary semi group T is completely prime if and only if $x_1, x_2, x_3 \dots x_n \in T$, n is an odd natural number, $, x_2, x_3 \dots x_n \in A \Rightarrow x_i \in A\ for\ some\ i = 1,2, \dots n.$

Theorem3.3: A partially ordered ideal A of a partially ordered ternary semi group t is completely prime if and only if T\A is either a sub semi group of T or empty.

Definition 3.4: A partially ordered ideal A of a partially ordered ternary semi group T is said to be a prime ideal of T provided X,Y,Z are ideals of T and $XYZ \subseteq A \Rightarrow X \subseteq A\ or\ Y \subseteq A\ or\ Z \subseteq A$.

Theorem3.5: In a partially ordered ternary semi group T ,the following conditions are equivalent:

(i). A is a prime partially ordered ideal of T.

(ii). $a, b, c\ \in T; \langle a \rangle \langle b \rangle \langle c \rangle \subseteq A\ implies\ a \in A\ or\ b \in or\ c \in A.$

(iii). $a, b, c\ \in T; T^1T^1aT^1T^1bT^1T^1cT^1T^1 \subseteq A\ implies\ a \in A\ or\ b \in or\ c \in A.$

4. Completely semi prime Partially ordered ideals and semi prime partially ordered ideals

Definition 4.1: A partially ordered ideal A of a partially ordered ternary semi group t is said to be a completely semi prime partially ordered ideal provided
$x \in A\ , x^n \in A\ for\ some\ odd\ natural\ number\ n > 1\ implies\ x\epsilon A.$

Theorem4.2: A partially ordered ideal A of a partially ordered ternary semi group T is completely semi prime if and only if $x \in T\,, x^3 \in A\; implies\; x \in A$.

5. Prime partially ordered radical and completely prime partially ordered radical:

Notation 5.1: If A is a partially ordered ideal of a partially ordered ternary semi group T. Then we associate the following four types of sets

A_1= the intersection of all completely prime partially ordered ideals of T containing A.

$A_2 = \{x \in T : x^n \in A\; for\; some\; odd\; natural\; numbers\; n\}$.

A_3= The intersection off all prime partially ordered ideals of T containing A.

$A_4 = \{x \in T : \langle x^n \rangle \subseteq A\; for\; some\; odd\; natural\; numbers\; n\}$.

Theorem5.2: If A is partially ordered ideal of a partially ordered ternary semi group T, then $A \subseteq A_4 \subseteq A_3 \subseteq A_2 \subseteq A_1$.

Proof: i. $A \subseteq A_4$:let $x \in$A. Then $\langle x \rangle \subseteq A$ and hence x ∈ A_4

Therefore $A \subseteq A_4$.

ii. $A_4 \subseteq A_3$: let $x \in A_4$. Then $\langle x \rangle^n \subseteq A$ for some odd natural number n.

Let P be any prime partially ordered ideal of T containing A.

Then $\langle x \rangle^n \subseteq A$ for some odd natural number n $\Rightarrow \langle x \rangle^n \subseteq P$

Since P is prime, $\langle x \rangle \subseteq P$ and hence $x \in$P

Since this is true for all prime ideals of p containing A, $x \in A_3$

Therefore $A_4 \subseteq A_3$

iii. $A_3 \subseteq A_2$:Let $x \in A_3$. Suppose if possible $x \notin A_2$.

Then $x^n \notin A$ for all odd natural number n, and $x \in T$

Consider Q=U x^n for all odd natural number n, $x \in T$.

Let $a, b, c \in Q$. Then $a = (x)^r$, $b = (x)^s, c = (x)^t$ for some odd natural numbers r,s,t.

Therefore $abc = (x)^r\ (x)^s\ (x)^t = x^{r+s+t} \in Q$and hence Q is a sub semi group of T.

By theorem 3.3, P=T\Q is a completely prime partially ordered ideal of T and $x \notin P$.

By theorem 3.8, P is a prime partially ordered ideal of T and $x \notin P$. Therefore $x \notin A_3$.

It is contradiction. Therefore $x \in A_2$ and hence $A_3 \subseteq A_2$.

iv. $A_2 \subseteq A_1$**.** Now $x \in A_2 \Rightarrow x^n \in A$ for some odd natural number n.

Let p be any completely prime partially ordered ideal of T containing A.

Then $x^n \in A \subseteq P \Rightarrow x^n \in P \Rightarrow x \in P$. Therefore $x \in A_1$. Therefore $A_2 \subseteq A_1$**.**

Hence $A \subseteq A_4 \subseteq A_3 \subseteq A_2 \subseteq A_1$.

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