g*- CLOSED SETS in TOPOLOGICAL ORDERED SPACES

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Abstract : In this paper, we introduced a new class of sets called g*-closed sets in topological ordered spaces, also we discuss some of their properties and investigate the relationship among this separation properties along with some counter examples.

Keywords : w-g*-c_o space, w-c_o space and Topological ordered spaces.

I. INTRODUCTION

A topological ordered spaces [17] is a triple $(X,\tau,\leq)$, where τ is a topology on X And $\leq$ is a partial order on X. let (X,τ) be a topological space and A be a subset of X. the interior of A (denoted by int (A)) is the union of all open Subsets of A and closure of A (denoted by cl (A)) is the intersection of all Closed super sets of A. C(A) denotes the complement of A. Let $((X,\tau,\leq)$ be Topological ordered space .for any $x\in X$, $[x,\rightarrow] = \{y\in X/x\leq y\}$ and $[\leftarrow, x] = \{y\in x/y\leq x\}$ [13]. A subset A of a topological ordered space $(X,\tau,\leq)$ is said to be increasing [13] if A =i(A) and decreasing[18] if A=d(A), where $i(A)=\cup_{a\in A}[a,\rightarrow]$ and $d(A)= \cup_{a\in A}[\leftarrow,a]$. A subset of a topological ordered space $(X,\tau,\leq)$ is said to be balanced [13] if it is both increasing and decreasing.

II. PRELIMINARIES

DEFINITION 2.1: A subset A of a Topological space (x,τ) is called a

1. Weakly - C_o [10] if $\cap_{x\in X}$ ker(X) =Φ, Where ker(X) = $\cap\{G/x\in G\in\tau\}$.
2. g*-closed set [16] if cl (A) $\subseteq$U when A$\subseteq$U and U is g-open [12] in (x,τ).

DEFINITION 2.2: Let A be a subset of a topological space (x,τ) is called

1. Generalized closed set (g-closed) [12] if cl (A) $\subseteq$U whenever A$\subseteq$U and U is open in (X,τ).
2. A generalized semi-closed set (gs-closed) [4] if scl (A) $\subseteq$U whenever A$\subseteq$U and U is open in (X,τ).
3. A regular generalized closed set (rg-closed) [15] if cl (A) $\subseteq$U whenever A$\subseteq$U and U is regular open in (X,τ).
4. A generalized pre regular closed set (gpr-closed) [11] if pcl (A) $\subseteq$U whenever A$\subseteq$U and U is regular open in (X,τ).
5. An α-generalized closed set (αg-closed) [6] if αcl (A) $\subseteq$U whenever A$\subseteq$U and U is open in (X,τ).
6. A generalized semi-pre closed set (gsp-closed) [9] if spcl (A) $\subseteq$U whenever A$\subseteq$U and U is open in (X,τ).
7. An α**-generalized closed set (α**g-closed) [19] if αcl (A) $\subseteq$int (cl (U)) whenever A$\subseteq$U and U is open in (X,τ).
8. α-open set [14] if A $\subseteq$int (cl (int (A)), α-closed set if cl (int (cl (A))) $\subseteq$A.

DEFINITION 2.3: A subset A of a Topological ordered space $(x,\tau, \leq)$ is called a

1. g*i (resp. g*d, g*b)-closed set [16] if it is both g*-closed and i (resp. decreasing, balanced) - Closed sets.
2. gi (resp. gd, gb)-closed set [16] if it is both g-closed and i (resp. decreasing, balanced)-closed sets.
3. gsi (resp. gsd, gsb)-closed set [10] if it is both gs-closed and i (resp. decreasing, balanced)-closed sets.

4. rgi (resp. rgd, rgb)-closed set [1] if it is both rg-closed and i (resp. decreasing, balanced)-closed sets.
5. gpri (resp. gprd, gprb) -closed set [2] if it is both gpr-closed and i (resp. decreasing, balanced) - closed sets.
6. αgi (resp. αgd, αgb)-closed set [8] if it is both αg - closed and i (resp. decreasing, balanced) -closed sets.
7. gspi (resp. gspd, gspb)-closed set [3] if it is both gsp - closed and i (resp. decreasing, balanced) -closed sets.
8. α**gi (resp. α**gd,α**gb)-closed set [19] if it is both α**g - closed and i (resp. decreasing, balanced)-closed sets.
9. Weakly i-C_o (resp. d – C_o , b – C_o) [18] if $\cap_{x\in X}$i Ker{x} =Φ, (resp. $\cap_{x\in X}$ dKer{x} =Φ, $\cap_{x\in X}$ bKer{x} =Φ) where i Ker(x) = $\cap$\{G/x$\in$G$\in$I (O(x))\} (resp; dKer(x) = $\cap$\{G/x$\in$G$\in$D (O(x))\},
bKer(x) = $\cap$\{G/x$\in$G$\in$B (O(x))\}).

WEAKLY g*-i-C_o SPACES

Definition 3.1: A Topological space (X,τ) is called a

1. Weakly g*-i-C_o space (resp. g*-d-C_o, g*-b-C_o) if $\cap_{x\in X}$ g*i Ker{x}=Φ,(resp. $\cap_{x\in X}$ g*dKer{x}=Φ,$\cap_{x\in X}$ g*bKer{x}=Φ) where g*iKer(x)=$\cap$\{G/x$\in$G$\in$g*I(O(x))\}(resp. g*dKer(x) = $\cap$\{G/x$\in$G$\in$g*D(O(x)), g*bKer(x) = $\cap$\{G/x$\in$G$\in$g*B(O(x))\}.
2. Weakly g-i-C_o space (resp. g-d-C_o, g-b-C_o) if $\cap_{x\in X}$ gi Ker{x}=Φ,(resp.$\cap_{x\in X}$ gdKer{x}=Φ, $\cap_{x\in X}$ gbKer{x}=Φ) where gi Ker(x)=$\cap$\{G/x$\in$G$\in$gI(O(x))\}(resp. gdKer(x) = $\cap$\{G/x$\in$G$\in$gD(O(x)), gbKer(x) =$\cap$\{G/x$\in$G$\in$gB(O(x))\}.
3. Weakly gs-i-C_o space (resp. gs-d-C_o, gs-b-C_o) if $\cap_{x\in X}$ gsi Ker{x}=Φ,(resp.$\cap_{x\in X}$ gsdKer{x}=Φ, $\cap_{x\in X}$ gsbKer{x}=Φ) where gsi Ker(x)=$\cap$\{G/x$\in$G$\in$gsI(O(x))\}(resp. gsdKer(x) = $\cap$\{G/x$\in$G$\in$gsD(O(x)), gsbKer(x) = $\cap$\{G/x$\in$G$\in$gsB(O(x))\}.
4. Weakly rg-i-C_o space (resp. rg-d-C_o, rg-b-C_o) if $\cap_{x\in X}$ rgi Ker{x}=Φ,(resp.$\cap_{x\in X}$ rgdKer{x}=Φ, $\cap_{x\in X}$ rgbKer{x}=Φ) where rgi Ker(x)=$\cap$\{G/x$\in$G$\in$rgI(O(x))\} (resp. rgdKer(x) = $\cap$\{G/x$\in$G$\in$rgD(O(x)), rgbKer(x) = $\cap$\{G/x$\in$G$\in$rgB(O(x))\}.
5. Weakly gpr-i-C_o space (resp. gpr-d-C_o, gpr-b-C_o) if $\cap_{x\in X}$ gpri Ker{x} =Φ, (resp. $\cap_{x\in X}$ gprdKer{x}=Φ,$\cap_{x\in X}$ gprbKer{x}=Φ) where gpri Ker(x)=$\cap$\{G/x$\in$G$\in$gprI(O(x))\} (resp. gprdKer(x) = $\cap$\{G/x$\in$G$\in$gprD(O(x)), gprbKer(x) = $\cap$\{G/x$\in$G$\in$gprB(O(x))\}.
6. Weakly αg-i-C_o space (resp. αg-d-C_o, αg-b-C_o) if $\cap_{x\in X}$ αgi Ker{x} =Φ, (resp. $\cap_{x\in X}$ αgdKer{x}=Φ,$\cap_{x\in X}$ αgbKer{x}=Φ) where αgi Ker(x)=$\cap$\{G/x$\in$G$\in$$\alpha$gI(O(x))\} (resp. αgdKer(x) = $\cap$\{G/x$\in$G$\in$$\alpha$gD(O(x)), αgbKer(x) = $\cap$\{G/x$\in$G$\in$$\alpha$gB(O(x))\}.
7. Weakly gsp-i-C_o space (resp. gsp-d-C_o, gsp-b-C_o) if $\cap_{x\in X}$ gspi Ker{x} =Φ, (resp. $\cap_{x\in X}$ gspdKer{x}=Φ,$\cap_{x\in X}$ gspbKer{x}=Φ) where gspi Ker(x)=$\cap$\{G/x$\in$G$\in$gspI(O(x))\} (resp. gspdKer(x) = $\cap$\{G/x$\in$G$\in$gspD(O(x)), gspbKer(x) = $\cap$\{G/x$\in$G$\in$gspB(O(x))\}.
8. Weakly α**g-i-C_o space (resp. α**g-d-C_o, α**g-b-C_o) if $\cap_{x\in X}$ α**gi Ker{x} =Φ, (resp. $\cap_{x\in X}$ α**gdKer{x}=Φ,$\cap_{x\in X}$ α**gbKer{x}=Φ) where α**gi Ker(x)=$\cap$\{G/x$\in$G$\in$$\alpha$**gI(O(x))\} (resp. α**gdKer(x) = $\cap$\{G/x$\in$G$\in$$\alpha$**gD(O(x)),$\alpha$**gbKer(x) = $\cap$\{G/x$\in$G$\in$$\alpha$**gB(O(x))\}.

Theorem 3.2: Every weakly g*-i- C_o space is a weakly g-i - C_o space.
Proof: Let (X,τ, $\leq$) is a weakly g*-i - C_o space.
$\Rightarrow\cap_{x\in X}$ g*i Ker{x} =Φ.
Every g*i – open set is gi – open set in (X,τ, $\leq$).
Then g*i Ker{x} $\subseteq$ gi Ker{x}; $\forall$x$\in$X.
$\Rightarrow\cap_{x\in X}$ gi Ker{x} $\subseteq$ $\cap_{x\in X}$ g*i Ker{x}
Since $\cap_{x\in X}$ g*i Ker{x} =Φ, we have $\cap_{x\in X}$ gi Ker{x} = Φ
Therefore, (X,τ, $\leq$) is weakly g-i – C_o space.

Hence every weakly g*-i – C_o space is weakly g-i-C_o Space.
The converse of the above theorem is not true as it can be seen by the following example.

Example 3.3: Let X = {a, b, c} τ= {Φ,X,{a},{b, c}} ≤ = {(a,a),(b,b),(c,c),(a,b),(c,b)}.
Then (X,τ, ≤) is a Topological ordered space.
Now gi- open sets are - Φ, X, {a, c}, {c}, {a}.
g*i -open sets are - Φ, X, {a}.

gi ker {a} = {a}	g*i ker {a} = {a}
gi ker {b} =X	g*i ker {b} =X
gi ker {c} ={c}	g*i ker {c} =X
$\cap_{x\in X}$ gi ker {x} = Φ	$\cap_{x\in X}$ g*i ker {x} ≠ Φ

Thus (X,τ, ≤) is a weakly g-i-C_o space but not weakly g*-i-C_o space.

Theorem 3.4: Every weakly g*-i - C_o space is a weakly gs-i - C_o space.
Proof: Let (X,τ, ≤) is a weakly g*-i - C_o space.
$\Rightarrow \cap_{x\in X}$ g*i Ker{x} =Φ.
Every g*i – open set is gsi – open set in (X,τ, ≤).
Then g*i Ker{x} ⊆ gsi Ker{x}; ∀x∈X.
$\Rightarrow \cap_{x\in X}$ gsi Ker{x} ⊆ $\cap_{x\in X}$ g*i Ker{x}
Since $\cap_{x\in X}$ g*i Ker{x} =Φ, we have $\cap_{x\in X}$ gsi Ker{x} = Φ
Therefore, (X,τ, ≤) is a weakly gs-i – C_o space.
Hence every weakly g*-i – C_o space is weakly gs-i -C_o space.
The converse of the above theorem is not true as it can be seen by the following example.

Example 3.5: Let X= {a, b, c} τ= {Φ, X,{a},{b, c}} ≤ = {(a, a), (b, b), (c, c), (a, b), (c, b)}.
Then (X,τ, ≤) is a Topological ordered space.
Now gsi - open sets are - Φ, X, {a, c}, {c}, {a}.
g*i - open sets are - Φ, X, {a}.

gsi ker {a} = {a}	g*i ker {a} = {a}
gsi ker {b} = X	g*i ker {b} = X
gsi ker {c} = {c}	g*i ker {c} = X
$\cap_{x\in X}$ gsi ker {x} = Φ	$\cap_{x\in X}$ g*i ker {x} ≠ Φ

Thus (X,τ, ≤) is a weakly gs-i-C_o space but not weakly g*-i-C_o space.
Theorem 3.6: Every weakly g*-i - C_o space is a weakly rg-i - C_o space.
Proof: Let (X,τ, ≤) is a weakly g*-i - C_o space.
$\Rightarrow \cap_{x\in X}$ g*i Ker{x} =Φ.
Every g*i – open set is rgi – open set in (X,τ, ≤).
Then g*i Ker{x} ⊆ rgi Ker{x}; ∀x∈X.
$\Rightarrow \cap_{x\in X}$ rgi Ker{x} ⊆ $\cap_{x\in X}$ g*i Ker{x}
Since $\cap_{x\in X}$ g*i Ker{x} =Φ, we have $\cap_{x\in X}$ rgi Ker{x} = Φ
Therefore, (X,τ, ≤) is a weakly rg-i – C_o space.
Hence every weakly g*-i – C_o space is weakly rg-i- C_o space.
The converse of the above theorem is not true as it can be seen by the following example.

Example 3.7: Let X= {a, b, c} τ= {Φ, X,{a},{b},{a,b}} ≤ = {(a, a), (b, b), (c, c), (a, b), (c, b)}.
Then (X,τ, ≤) is a Topological ordered space.
Now rgi - open sets are - Φ, X, {c}, {a}.
g*i - open sets are - Φ,X,{a}.

rgi ker {a} = {a}	g*i ker {a} = {a}
rgi ker {b} = X	g*i ker {b} = X
rgi ker {c} = {c}	g*i ker {c} = X
$\cap_{x\in X}$ rgi ker {X} = Φ	$\cap_{x\in X}$ g*i ker {X} ≠ Φ

Thus (X,τ, ≤) is a weakly rg-i-C_o space but not weakly g*-i-C_o space.

Theorem 3.8: Every weakly g*-i - C_o space is a weakly gpr- i - C_o space.
Proof: Let (X,τ, ≤) is a weakly g*-i - C_o space.
⇒$\cap_{x\in X}$ g*i Ker{x} =Φ.
Every g*i – open set is gpri – open set in (X,τ, ≤).
Then g*i Ker{x} ⊆ gpri Ker{x}; ∀x∈X.
⇒$\cap_{x\in X}$ gpri Ker{x} ⊆ $\cap_{x\in X}$ g*i Ker{x}
Since $\cap_{x\in X}$ g*i Ker{x} =Φ, we have $\cap_{x\in X}$ gpri Ker{x} = Φ
Therefore, (X,τ, ≤) is weakly gpr-i – C_o space.
Hence every weakly g*-i – C_o space is a weakly gpr-i- C_o space.
The converse of the above theorem is not true as it can be seen by the following example.

Example 3.9: Let X ={ a, b,c } τ={ Φ,X {a},{b},{a,b}} ≤ = {(a,a),(b,b),(c,c),(a,b),(c,b)}.
Then (X,τ, ≤) is a Topological ordered space.
Now gpri- open sets are - Φ, X, {c}, {a}.
g*i - open sets are - Φ, X, {a}.

gpri ker {a} = {a}	g*i ker {a} = {a}
gpri ker {b} =X	g*i ker {b} =X
gpri ker {c} ={c}	g*i ker {c} =X
$\cap_{x\in X}$ gpri ker {x} = Φ	$\cap_{x\in X}$ g*i ker {x} ≠ Φ

Thus (X,τ, ≤) is a weakly gpr-i-C_o space but not weakly g*-i-C_o space.

Theorem 3.10: Every weakly g*-i - C_o space is a weakly αg-i - C_o space.
Proof: Let (X,τ, ≤) is a weakly g*-i - C_o space.
⇒$\cap_{x\in X}$ g*i Ker{x} =Φ.
Every g*i – open set is αgi – open set in (X,τ, ≤).
Then g*i Ker{x} ⊆ αgi Ker{x}; ∀x∈X.
⇒$\cap_{x\in X}$ αgi Ker{x} ⊆ $\cap_{x\in X}$ g*i Ker{x}
Since $\cap_{x\in X}$ g*i Ker{x} =Φ, we have $\cap_{x\in X}$ αgi Ker{x} = Φ
Therefore, (X,τ, ≤) is a weakly αg-i – C_o space.
Hence every weakly g*-i - C_o space is a weakly αg-i- C_o space.
The converse of the above theorem is not true as it can be seen by the following example.

Example 3.11: Let X ={a, b, c} τ={Φ, X, {a},{b, c}} ≤ = {(a,a),(b,b),(c,c),(a,b),(c,b)}
Then (X,τ,≤) is a Topological ordered space.

Now αgi- open sets are - Φ, X, {a, c}, {c}, {a}.
g*i - open sets are - Φ,X,{a }

αgi ker {a} = {a}	g*i ker {a} = {a}
αgi ker {b} = X	g*i ker {b} =X
αgi ker {c} ={c}	g*i ker {c} =X
$\cap_{x\in X}$ αgi ker {x} = Φ	$\cap_{x\in X}$ g*i ker {x} $\neq \Phi$

Thus (X,τ, $\leq$) is a weakly αg-i-C_o space but not weakly g*-i-C_o space.

Theorem 3.12: Every weakly g*-i - C_o space is a weakly gsp-i - C_o space.
Proof: Let (X,τ, $\leq$) is a weakly g*-i - C_o space.
$\Rightarrow \cap_{x\in X}$ g*i Ker{x} =Φ.
Every g*i – open set is gspi – open set in (X,τ, $\leq$).
Then g*i Ker{x} $\subseteq$ gspi Ker{x}; $\forall x\in X$.
$\Rightarrow \cap_{x\in X}$ gspi Ker{x} $\subseteq \cap_{x\in X}$ g*i Ker{x}
Since $\cap_{x\in X}$ g*i Ker{x} = Φ, we have $\cap_{x\in X}$ gspi Ker{x} = Φ
Therefore, (X,τ, $\leq$) is weakly gsp-i – C_o space.
Hence every weakly g*-i -C_o space is a weakly gsp-i- C_o space.
The converse of the above theorem is not true as it can be seen by the following example.

Example 3.13: Let X ={a, b, c} τ={Φ, X, {a},{b, c}} $\leq$ = {(a,a),(b,b),(c,c),(a,c),(b,c)}.
Then (X,τ, $\leq$) is a Topological ordered space.
Now gspi- open sets are - Φ, X, {a, b}, {b}, {a}.
g*i - open sets are - Φ, X, {a}.

gspi ker {a} = {a}	g*i ker {a} = {a}
gspi ker {b} = {b}	g*i ker {b} =X
gspi ker {c} = X	g*i ker {c} =X
$\cap_{x\in X}$ gspi ker {x} = Φ	$\cap_{x\in X}$ g*i ker {x} $\neq \Phi$

Thus (X,τ, $\leq$) is a weakly gsp-i-C_o space but not weakly g*-i-C_o space.

Theorem 3.14: Every weakly g*-i - C_o space is a weakly α**g-i - C_o space.
Proof: Let (X,τ, $\leq$) is a weakly g*-i - C_o space.
$\Rightarrow \cap_{x\in X}$ g*i Ker{x} =Φ.
Every g*i – open set is α**gi – open set in (X,τ, $\leq$).
Then g*i Ker{x} $\subseteq$ α**gi Ker{x}; $\forall x\in X$.
$\Rightarrow \cap_{x\in X}$ α**gi Ker{x} $\subseteq \cap_{x\in X}$ g*i Ker{x}
Since $\cap_{x\in X}$ g*i Ker{x} =Φ, we have $\cap_{x\in X}$ α**gi Ker{x} = Φ
Therefore, (X,τ, $\leq$) is weakly α**g-i – C_o space.
Hence every weakly g*-i – C_o space is a weakly α**g-i – C_o space.
The converse of the above theorem is not true as it can be seen by the following example.

Example 3.15: Let X ={ a, b,c } τ={ Φ,X,{a},{b},{a,b}} $\leq$ = {(a,a),(b,b),(c,c),(a,b),(c,b)}.
Then (X,τ, $\leq$) is a Topological ordered space.
Now α**gi- open sets are - Φ, X, {c}, {a}.
g*i - open sets are - Φ, X, {a}.

α**gi ker {a} = {a}   g*i ker {a} = {a}
α**gi ker {b} =X   g*i ker {b} =X
α**gi ker {c} ={c}   g*i ker {c} =X
$\cap_{x\in X}$ α**gi ker {x} = Φ $\cap_{x\in X}$ g*i ker {x} $\neq \Phi$
Thus (X,τ, $\leq$) is a weakly α**g-i-C_o space but not weakly g*-i-C_o space.

Theorem 3.16: Every weakly g*-d- C_o space is a weakly g-d - C_o space.
Proof: Let (X,τ, $\leq$) is a weakly g*-d - C_o space.
$\Rightarrow\cap_{x\in X}$ g*d Ker{x} =Φ.
Every g*d – open set is gd – open set in (X,τ, $\leq$).
Then g*d Ker{x} $\subseteq$ gd Ker{x}; $\forall x\in X$.
$\Rightarrow\cap_{x\in X}$ gd Ker{x} $\subseteq$ $\cap_{x\in X}$ g*d Ker{x}
Since $\cap_{x\in X}$ g*d Ker{x} =Φ, we have $\cap_{x\in X}$ gd Ker{x} = Φ
Therefore, (X,τ, $\leq$) is weakly g-d -C_o space.
Hence every weakly g*-d-C_o space is weakly g-d-C_o space.
The converse of the above theorem is not true as it can be seen by the following example.

Example 3.17: Let X ={a, b, c} τ={Φ, X,{a},{b, c}} $\leq$ = {(a,a),(b,b),(c,c),(a,b),(a,c)}.
Then (X,τ, $\leq$) is a Topological ordered Space.
Now gd- open sets are - Φ, X, {b, c}, {c}, {b}.
g*d - open sets are - Φ, X, {b, c}.
gd ker {a} = X   g*d ker {a} =X
gd ker {b} = {b}   g*d ker {b} = {b, c}
gd ker {c} = {c}   g*d ker {c} = {b,c}
$\cap_{x\in X}$ gd ker {x} = Φ $\cap_{x\in X}$ g*d ker {x} $\neq \Phi$
Thus (X,τ, $\leq$) is a weakly g-d-C_o space but not weakly g*-d-C_o space.

Theorem 3.18: Every weakly g*-d - C_o space is a weakly gs-d - C_o space.
Proof: Let (X,τ, $\leq$) is a weakly g*-d - C_o space.
$\Rightarrow\cap_{x\in X}$ g*d Ker{x} =Φ.
Every g*d – open set is gsd – open set in (X,τ, $\leq$).
Then g*d Ker{x} $\subseteq$ gsd Ker{x}; $\forall x\in X$.
$\Rightarrow\cap_{x\in X}$ gsd Ker{x} $\subseteq$ $\cap_{x\in X}$ g*d Ker{x}
Since $\cap_{x\in X}$ g*d Ker{x} =Φ, we have $\cap_{x\in X}$ gsd Ker{x} =Φ
Therefore, (X,τ, $\leq$) is a weakly gs-d -C_o space.
Hence every weakly g*-d-C_o space is weakly gs-d-C_o space.
The converse of the above theorem is not true as it can be seen by the following example.

Example 3.19: Let X= {a, b, c} τ= {Φ, X, {a}, {b,c}} $\leq$ = {(a, a), (b, b), (c, c), (a, b), (a, c)}.
Then (X,τ, $\leq$) is a Topological ordered space.
Now gsd - open sets are - Φ, X, {b, c}, {c}, {b}
g*d - open sets are - Φ, X, {b, c}.
gsd ker {a} = X   g*d ker {a} =X
gsd ker {b} = {b}   g*d ker {b} = {b,c}
gsd ker {c} = {c}   g*d ker {c} = {b,c}

$\cap_{x\in X}$ gsd ker {x} = Φ $\cap_{x\in X}$ g*d ker {x} ≠ Φ

Thus (X,τ, ≤) is a weakly gs-d-C_o space but not weakly g*-d-C_o space.

Theorem 3.20: Every weakly g*-d - C_o space is a weakly rg-d - C_o space.

Proof: Let (X,τ, ≤) is a weakly g*-d - C_o space.

⇒$\cap_{x\in X}$ g*d Ker{x} =Φ.

Every g*d – open set is rgd – open set in (X,τ, ≤).

Then g*d Ker{x} ⊆ rgd Ker{x}; ∀x∈X.

⇒$\cap_{x\in X}$ rgd Ker{x} ⊆ $\cap_{x\in X}$ g*d Ker{x}

Since $\cap_{x\in X}$ g*d Ker{x} =Φ, we have $\cap_{x\in X}$ rgd Ker{x} = Φ

Therefore, (X,τ, ≤) is a weakly rg-d – C_o space.

Hence every weakly g*-d-C_o space is weakly rg-d-C_o space.

The converse of the above theorem is not true as it can be seen by the following example.

Example 3.21: Let X= {a, b,c} τ={Φ,X,{a},{b},{a,b}} ≤ = {(a,a),(b,b),(c,c),(a,b),(a,c)}.

Then (X,τ, ≤) is a Topological ordered space.

Now rgd - open sets are - Φ,X,{c},{b}.

g*d - open sets are - Φ,X,{b}

rgd ker {a} = X	g*d ker {a} = X
rgd ker {b} ={b}	g*d ker {b} = {b}
rgd ker {c} = {c}	g*d ker {c} = X
$\cap_{x\in X}$ rgd ker {x} = Φ	$\cap_{x\in X}$ g*d ker {x} ≠ Φ

Thus (X,τ,≤) is a weakly rg-d-C_o space but not weakly g*-d-C_o space.

Theorem 3.22: Every weakly g*-d - C_o space is a weakly gpr-d - C_o space.

Proof: Let (X,τ, ≤) is a weakly g*-d - C_o space.

⇒$\cap_{x\in X}$ g*d Ker{x} =Φ.

Every g*d– open set is gprd – open set in (X,τ, ≤).

Then g*d Ker{x} ⊆ gprd Ker{x}; ∀x∈X.

⇒$\cap_{x\in X}$ gprdKer{x} ⊆ $\cap_{x\in X}$ g*d Ker{x}

Since $\cap_{x\in X}$ g*dKer{x} =Φ, we have $\cap_{x\in X}$ gprd Ker{x} =Φ

Therefore, (X,τ, ≤) is weakly gpr-d – C_o space.

Hence every weakly g*-d-C_o space is weakly gpr-d-C_o space.

The converse of the above theorem is not true as it can be seen by the following example.

Example 3.23: Let X ={ a, b,c } , τ= { Φ,X, {a},{b},{a,b}} ≤ = {(a,a),(b,b),(c,c),(b,a)} .

Then (X,τ, ≤) is a Topological ordered space.

Now gprd- open sets are - Φ,X,{a,b},{c},{a}.

g*d - open sets are - Φ,X,{a,b},{a}.

gprd ker {a}={a}	g*d ker {a}={a}
gprd ker {b}={a,b}	g*d ker {b}={a,b}
gprd ker {c}={c}	g*d ker {c}=X
$\cap_{x\in X}$ gprd ker {x} = Φ	$\cap_{x\in X}$ g*d ker {x} ≠ Φ

Thus (X,τ,≤) is a weakly gpr-d-C_o space but not weakly g*-d-C_o space .

Theorem 3.24: Every weakly g*-d - C_o space is a weakly αg-d - C_o space.

Proof: Let $(X,\tau,\leq)$ is a weakly g*-d - C_o space.

$\Rightarrow \cap_{x\in X}$ g*d Ker{x} $=\Phi$.

Every g*d -open set is αgd -open set in $(X,\tau,\leq)$.

Then g*d Ker{x} $\subseteq$ αgd Ker{x}; $\forall x\in X$.

$\Rightarrow \cap_{x\in X}$ αgd Ker{x} $\subseteq \cap_{x\in X}$ g*d Ker{x}

Since $\cap_{x\in X}$ g*d Ker{x} $=\Phi$, we have $\cap_{x\in X}$ αgd Ker{x} $=\Phi$

Therefore, $(X,\tau,\leq)$ is a weakly αg-d – C_o space.

Hence every weakly g*-d-C_o space is weakly αg-d-C_o space.

The converse of the above theorem is not true as it can be seen by the following example.

Example 3.25: Let X ={ a, b,c } τ ={ Φ,X,{a},{b,c}} $\leq$ = {(a,a),(b,b),(c,c),(a,b),(a,c)}.

Then $(X,\tau,\leq)$ is a Topological ordered space.

Now αgd- open sets are - Φ,X,{b,c},{c},{b}.

g*d - open sets are - Φ,X,{b,c}.

αgd ker {a}=X	g*d ker {a}=X
αgd ker {b}={b}	g*d ker {b}={b,c}
αgd ker {c}= {c}	g*d ker {c}={b,c}
$\cap_{x\in X}$ αgd ker {x} = Φ	$\cap_{x\in X}$ g*d ker {x} $\neq \Phi$

Thus $(X,\tau,\leq)$ is a weakly αg-d-C_o space but not weakly g*-d-C_o space .

Theorem 3.26: Every weakly g*-d - C_o space is a weakly gsp-d - C_o space.

Proof: Let $(X,\tau,\leq)$ is a weakly g*-d - C_o space.

$\Rightarrow \cap_{x\in X}$ g*d Ker{x} $=\Phi$.

Every g*d – open set is gspd – open set in $(X,\tau,\leq)$.

Then g*d Ker{x} $\subseteq$ gspd Ker{x}; $\forall x\in X$.

$\Rightarrow \cap_{x\in X}$ gspd Ker{x} $\subseteq \cap_{x\in X}$ g*d Ker{x}

Since $\cap_{x\in X}$ g*d Ker{x} $=\Phi$, we have $\cap_{x\in X}$ gspd Ker{x} $=\Phi$

Therefore, $(X,\tau,\leq)$ is weakly gsp-d – C_o space.

Hence every weakly g*-d-C_o space is weakly gsp-d-C_o space.

The converse of the above theorem is not true as it can be seen by the following example.

Example 3.27: Let X ={ a, b,c } τ= {Φ,X, {a},{b,c}} $\leq$ = {(a,a),(b,b),(c,c),(a,b),(a,c)}.

Then $(X,\tau,\leq)$ is a Topological ordered space.

Now gspd- open sets are - Φ,X,{b,c},{c},{b}.

g*d - open sets are - Φ,X,{b,c}.

gspd ker {a}=X	g*d ker {a}=X
gspd ker {b}={b}	g*d ker {b}={b,c}
gspd ker {c}= {c}	g*d ker {c}={b,c}
$\cap_{x\in X}$ gspd ker {x} = Φ	$\cap_{x\in X}$ g*d ker {x}=Φ

Thus $(X,\tau,\leq)$ is a weakly gsp-d-C_o space but not weakly g*-d-C_o space .

Theorem 3.28: Every weakly g*-d - C_o space is a weakly α**g-d - C_o space.

Proof: Let $(X,\tau,\leq)$ is a weakly g*-d - C_o space.

$\Rightarrow \cap_{x\in X}$ g*d Ker{x} =Φ.
Every g*d – open set is α**gd – open set in (X,τ, ≤).
Then g*d Ker{x} ⊆ α**gd Ker{x}; ∀x∈X.
$\Rightarrow \cap_{x\in X}$ α**gd Ker{x} ⊆ $\cap_{x\in X}$ g*d Ker{x}
Since $\cap_{x\in X}$ g*d Ker{x} =Φ, we have $\cap_{x\in X}$ α**gd Ker{x} = Φ
Therefore, (X,τ, ≤) is weakly α**g-d – C_o space.
Hence every weakly g*-d – C_o space is a weakly α**g-d – C_o space.
The converse of the above theorem is not true as it can be seen by the following example.

Example 3.29: Let X = { a, b,c } τ= { Φ,X, {a},{b},{a,b}} ≤ = {(a,a),(b,b),(c,c),(a,b),(a,c)}.
Then (X,τ, ≤) is a Topological ordered space.
Now α**gd- open sets are - Φ,X,{c},{b}.
g*d - open sets are - Φ,X,{b}.

α**gd ker {a}=X	g*d ker {a}=X
α**gd ker {b}={b}	g*d ker {b}={b}
α**gd ker {c}={c}	g*d ker {c}=X
$\cap_{x\in X}$ α**gd ker {x} = Φ	$\cap_{x\in X}$ g*d ker {x} ≠ Φ

Thus (X,τ,≤) is a weakly α**g-d-C_o space but not weakly g*-d-C_o space .

Theorem 3.30: Every weakly g*-b- C_o space is a weakly g-b - C_o space.
Proof: Let (X,τ, ≤) is a weakly g*-b - C_o space.
$\Rightarrow \cap_{x\in X}$ g*b Ker{x} =Φ.
Every g*b – open set is gb – open set in (X,τ, ≤).
Then g*b Ker{x} ⊆ gb Ker{x}; ∀x∈X.
$\Rightarrow \cap_{x\in X}$ gb Ker{x} ⊆ $\cap_{x\in X}$ g*b Ker{x}
Since $\cap_{x\in X}$ g*b Ker{x} =Φ, we have $\cap_{x\in X}$ gb Ker{x} = Φ
Therefore, (X,τ, ≤) is weakly g-b – C_o space.
Hence every weakly g*-b-C_o space is weakly g-b-C_o space.
The converse of the above theorem is not true as it can be seen by the following example.

Example 3.31: Let X ={ a, b,c} τ={ Φ,X,{a},{a,b},{a,c}} ≤ = {(a,a),(b,b),(c,c)}.
Then (X,τ, ≤) is a Topological ordered Space.
Now gb- open sets are - Φ,X,{a,c},{a},{b}.
g*b - open sets are - Φ,X,{a,c},{a,b},{a}.

gb ker {a}= {a}	g*b ker {a}={a}
gb ker {b}={b}	g*b ker {b}={a,b}
gb ker {c}= {a,c}	g*b ker {c}={a,c}
$\cap_{x\in X}$ gb ker {x} = Φ	$\cap_{x\in X}$ g*b ker {x} ≠ Φ

Thus (X,τ,≤) is a weakly g-b-C_o space but not weakly g*-b-C_o space .

Theorem 3.32: Every weakly g*-b - C_o space is a weakly gs-b - C_o space.
Proof: Let (X,τ, ≤) is a weakly g*-b - C_o space.
$\Rightarrow \cap_{x\in X}$ g*b Ker{x} =Φ.
Every g*b – open set is gsb – open set in (X,τ, ≤).

Then g*b Ker{x} $\subseteq$ gsb Ker{x}; $\forall x \in X$.
$\Rightarrow \cap_{x\in X}$ gsb Ker{x} $\subseteq \cap_{x\in X}$ g*b Ker{x}
Since $\cap_{x\in X}$ g*b Ker{x} =Φ, we have $\cap_{x\in X}$ gsb Ker{x} = Φ
Therefore, (X,τ, ≤) is a weakly gs-b – C_o space.
Hence every weakly g*-b-C_o space is weakly gs-b-C_o space.
The converse of the above theorem is not true as it can be seen by the following example.

Example 3.33: Let X= {a, b,c} τ={Φ,X,{a},{b},{a,b}} ≤ = {(a,a),(b,b),(c,c),(a,c)}.
Then (X,τ, ≤) is a Topological Space.
Now gsb - open sets are - Φ,X,{a,c},{b}.
g*b - open sets are - Φ,X,{b}.

gsb ker {a} = {a,c}	g*b ker {a} =X
gsb ker {b} = {b}	g*b ker {b} = {b}
gsb ker {c} = {a,c}	g*b ker {c} = X
$\cap_{x\in X}$ gsb ker {X} = Φ	$\cap_{x\in X}$ g*b ker {X} ≠ Φ

Thus (X,τ,≤) is a weakly gs-b-C_o space but not weakly g*-b-C_o space.

Theorem 3.34: Every weakly g*-b - C_o space is a weakly rg-b - C_o space.
Proof: Let (X,τ, ≤) is a weakly g*-b - C_o space.
$\Rightarrow \cap_{x\in X}$ g*b Ker{x} =Φ.
Every g*b – open set is rgb – open set in (X,τ, ≤).
Then g*b Ker{x}$\subseteq$ rgb Ker{x}; $\forall x \in X$.
$\Rightarrow \cap_{x\in X}$ rgb Ker{x} $\subseteq \cap_{x\in X}$ g*b Ker{x}
Since $\cap_{x\in X}$ g*b Ker{x} =Φ, we have $\cap_{x\in X}$ rgb Ker{x} = Φ
Therefore, (X,τ, ≤) is a weakly rg-b – C_o space.
Hence every weakly g*-b-C_o space is weakly rg-b-C_o space.
The converse of the above theorem is not true as it can be seen by the following example.

Example 3.35: Let X= {a,b,c} τ={Φ,X,{a},{b},{a,b}} ≤ = {(a,a),(b,b),(c,c),(b,a)}
Then (X,τ,≤) is a Topological ordered Space.
Now rgb - open sets are - Φ,X,{a,b},{c}.
g*b - open sets are - Φ,X,{a,b}.

rgb ker {a} = {a,b}	g*b ker {a} ={a,b}
rgb ker {b} ={a,b}	g*b ker {b} ={a,b}
rgb ker {c} = {c}	g*b ker {c} = X
$\cap_{x\in X}$ rgb ker{x} = Φ	$\cap_{x\in X}$ g*b ker{x} ≠ Φ

Thus (X,τ, ≤) is a weakly rg-b-C_o space but not weakly g*-b-C_o space.

Theorem 3.36: Every weakly g*-b - C_o space is a weakly gpr-b - C_o space.
Proof: Let (X,τ, ≤) is a weekly g*-b - C_o space.
$\Rightarrow \cap_{x\in X}$ g*b Ker{x} =Φ.
Every g*b – open set is gprb – open set in (X,τ, ≤).
Then g*b Ker{x} $\subseteq$ gprb Ker{x}; $\forall x \in X$.
$\Rightarrow \cap_{x\in X}$ gprbKer{x} $\subseteq \cap_{x\in X}$ g*b Ker{x}
Since $\cap_{x\in X}$ g*b Ker{x} =Φ, we have $\cap_{x\in X}$ gprb Ker{x} = Φ

Therefore, $(X,\tau, \leq)$ is weakly gpr-b – C_o space.
Hence every weakly g*-b-C_o space is a weakly gpr-b-C_o space.
The converse of the above theorem is not true as it can be seen by the following example.

Example 3.37: Let X = { a, b,c } τ= { Φ,X, {a},{b},{a,b}} ≤ = {(a,a),(b,b),(c,c),(b,a)}.
Then $(X,\tau, \leq)$ is a Topological ordered Space.
Now gprb- open sets are - Φ,X,{a,b},{c}.
g*b - open sets are - Φ,X,{a,b}.

gprb ker {a}={a,b}	g*b ker {a}={a,b}
gprb ker {b}={a,b}	g*b ker {b}={a,b}
gprb ker {c}={c}	g*b ker {c}=X
$\cap_{x\in X}$ gprb ker {x} = Φ	$\cap_{x\in X}$ g*b ker {x} ≠ Φ

Thus $(X,\tau, \leq)$ is a weakly gpr-b-C_o space but not weakly g*-b-C_o space .

Theorem 3.38: Every weakly g*-b - C_o space is a weakly αg-b - C_o space.
Proof: Let $(X,\tau, \leq)$ is a weakly g*-b - C_o space.
$\Rightarrow \cap_{x\in X}$ g*b Ker{x} =Φ.
Every g*b – open set is αgb – open set in $(X,\tau, \leq)$.
Then g*b Ker{x} ⊆ αgb Ker{x}; ∀x∈X.
$\Rightarrow \cap_{x\in X}$ αgb Ker{x} ⊆ $\cap_{x\in X}$ g*b Ker{x}
Since $\cap_{x\in X}$ g*b Ker{x} =Φ, we have $\cap_{x\in X}$ αgb Ker{x} = Φ
Therefore, $(X,\tau, \leq)$ is a weakly αg-b – C_o space.
Hence every weakly g*-b-C_o space is weakly αg-b-C_o space.
The converse of the above theorem is not true as it can be seen by the following example.

Example 3.39: Let X ={ a, b,c } τ={ Φ,X, {a},{b,c}} ≤ = {(a,a),(b,b),(c,c),(b,a)}.
Then $(X,\tau, \leq)$ is a Topological ordered Space.
Now αgb- open sets are - Φ,X,{a,b},{c}.
g*b - open sets are - Φ, X.

αgb ker {a}={a,b}	g*b ker {a}=X
αgb ker {b}={a,b}	g*b ker {b}=X
αgb ker {c}= {c}	g*b ker {c}= X
$\cap_{x\in X}$ αgb ker {x} = Φ	$\cap_{x\in X}$ g*b ker {x} ≠ Φ

Thus $(X,\tau, \leq)$ is a weakly αg-b-C_o space but not weakly g*-b-C_o space .

Theorem 3.40: Every weakly g*-b - C_o space is a weakly gsp-b - C_o space.
Proof: Let $(X,\tau, \leq)$ is a weakly g*-b - C_o space.
$\Rightarrow \cap_{x\in X}$ g*b Ker{x} =Φ.
Every g*b – open set is gspb – open set in $(X,\tau, \leq)$.
Then g*b Ker{x} ⊆ gspb Ker{x}; ∀x∈X.
$\Rightarrow \cap_{x\in X}$ gspb Ker{x} ⊆ $\cap_{x\in X}$ g*b Ker{x}
Since $\cap_{x\in X}$ g*b Ker{x} =Φ, we have $\cap_{x\in X}$ gspb Ker{x} = Φ
Therefore, $(X,\tau, \leq)$ is weakly gsp-b – C_o space.

Hence every weakly g*-b-C_o space is weakly gsp-b-C_o space.
The converse of the above theorem is not true as it can be seen by the following example.

Example 3.41: Let X = { a, b,c } τ= { Φ,X, {a},{b},{a,b}} ≤ = {(a,a),(b,b),(c,c),(a,c)}.
Then (X,τ, ≤) is a Topological ordered Space.
Now gspb- open sets are - Φ,X,{a,c},{b}.
g*b - open sets are - Φ,X,{b}.

gspb ker {a}={a,c}	g*b ker {a}=X
gspb ker {b}={b}	g*b ker {b}={b}
gspb ker {c}= {a,c}	g*b ker {c}=X
$\cap_{x\in X}$ gspb ker {x} = Φ	$\cap_{x\in X}$ g*b ker {x} ≠ Φ

Thus (X,τ,≤) is a weakly gsp-b-C_o space but not weakly g*-b-C_o space .

Theorem 3.42: Every weakly g*-b - C_o space is a weakly α**g-b - C_o space.
Proof: Let (X,τ, ≤) is a weakly g*-b - C_o space.
⇒$\cap_{x\in X}$ g*b Ker{x} =Φ.
Every g*b – open set is α**gb – open set in (X,τ, ≤).
Then g*b Ker{x} ⊆ α**gb Ker{x}; ∀x∈X.
⇒$\cap_{x\in X}$ α**gb Ker{x} ⊆ $\cap_{x\in X}$ g*b Ker{x}
Since $\cap_{x\in X}$ g*b Ker{x} = Φ, we have $\cap_{x\in X}$ α**gb Ker{x} = Φ
Therefore, (X,τ, ≤) is weakly α**g-b -C_o space.
Hence every weakly g*-b – C_o space is a weakly α**g-b – C_o space.
The converse of the above theorem is not true as it can be seen by the following example.

Example 3.43: Let X ={ a, b,c } τ={ Φ,X, {a},{b},{a,b}} ≤ = {(a,a),(b,b),(c,c),(b,a)}.
Then (X,τ, ≤) is a Topological ordered Space.
Now α**gb- open sets are - Φ, X,{a, b},{c}.
g*b - open sets are - Φ,X,{a,b}.

α**gb ker {a}={a,b}	g*b ker {a}={a,b}
α**gb ker {b}={a,b}	g*b ker {b}={a,b}
α**gb ker {c}={c}	g*b ker {c}=X
$\cap_{x\in X}$ α**gb ker {x} = Φ	$\cap_{x\in X}$ g*b ker {x} ≠ Φ

Thus (X,τ, ≤) is a weakly α**g-b-C_o space but not weakly g*-b-C_o space .

References

[1]. K.V.RamaRao Rao, V. Amarendra Babu and M.K.R.S.Veera Kumar, regular generalized closed type sets in topological ordered spaces, Diophantus Math.,4(1)(2015),11-16.

[2]. V. Amarendra Babu, K. Moses, and M.K.R.S. Veera Kumar some more applications of generalized pre regular closed type sets in topological ordered spaces. (Vol-1, December -2015), 21-46.

[3]. G.Banu Vinolia, V.Amarendra Babu and M.K.R.S.Veera Kumar, on some applications of generalized semi-pre closed type sets in topological ordered spaces, Bessel Journal of Mathematics Vol 5, sept-2015, 47-60.

[4]. S.P.Arya and T.Nour, characterizations of s-normal Indian J.Pure Appl Math., 21(8) (1990), 717-719.

[5].P.Bhattacharya and B.K.Lahiri, semi generalized closed sets in topology Indian J.Math, 29(3) (1983), 375-382.

[6]. Maki. H, Devi.R and Balachandran.K, Generalized closed sets and α-generalized closed sets, Mem, Fac.Sci.Kochi Sev., A., Math15 (1994), 51-63.

[7].S.G. Crossley and S.K.Hildebrand, semi-closure, Texas J.Sci., 22 (1971)99-112.

[8].K.KrishnaRao Rao and R.Chudamani, Pre-homomorphisms in topological ordered spaces, Technology and humanities, 41(2012), 408-421.

[9].Dontchev J., on generalization semi-pre open sets, Mem, fac Sci.Kochi Sev.A, Math., 16(1995), 35-48.

[10].G.Dimaio, separation axioms weaker than R_0, Indian J.Pure Math.16 (1985), 373-375.

[11].Y.Gnanambal, on gpr closed sets in topological spaces, Indian J.pure.appl.Math., 28(3) (1997), 351-360.

[12].N.levine, generalized closed sets in topological spaces, Rend.circ. Mat. Palermo, 19(1970), 89-96.

[13].L.Nachbin Topology and order, D.Van. Nostrand inc., Princeton New Jersey (1965).

[14].O.Njastand,on, on some classes of nearly open sets, Pacific J.Math, 15 (1965), 961-970.

[15].N.palaniappan and K.C.Rao regular generalized closed sets, Kyungpook math., 33(1993), 211-219.

[16].V.V.S .Ramachandram, B.Sankara Rao, M.K.R.S.Veera Kumar some more relationship of g- closed, g^*-closed and sg-closed type sets with other closed type sets in Topological ordered spaces, Indian J.Math. And nature, vol-1, (December-2015) (1-9).

[17].P. Sailaja, examples on counter examples of some separation properties using finite topologies, M.Phil dissertation, Nagarjuna University May (2002).

[18].M.K.R.S. Veera Kumar, homomorphisms in topological ordered spaces, Acta cinecia India, XXVIII [M][1][2002],67-76.

[19].P.Sivanagaraju, V.Sivanaga Malleswari and M.K.R.S.Veera Kumar- α** generalized closed sets in topological ordered space, Vessal J.Math., Vol-5, Sec-2015, 13-24.

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