

IMAGE EDGE ENHANCEMENT FILTERS IN FREQUENCY DOMAIN AND COMPARING FOURIER, COSINES AND WALSH TRANSFORMS

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Abstract

Image processing filtering helps to enhance the image by removing noise. Frequency filtering for image edge enhancement is presented. In frequency domain, comparison between techniques these are fast Fourier transform, cosine transform and Walsh transform. In frequency domain, low pass filters and high pass filters are considered. Changes in frequency estimate, brightness and contrast produced by the filtering measured. Low pass filtering functions are found to be more acceptable for image edge enhancement than high pass filtering functions. In frequency domain, both low pass and high pass filtering methods are performed. A low-pass filter is a filter that passes low-frequency signals and reduced signals with frequencies higher than the cut-off frequency. A high-pass filter is a filter that passes high frequencies well, but reduced frequencies lower than the cut-off frequency. Sharpening is fundamentally a high pass operation in the frequency domain

Key words: frequency domain, low pass and high pass filters, Fourier, cosine and Walsh transform

1. Introduction

Image processing is a method to convert an image into digital form and perform some operations on it, in order to get an enhanced image or to extract some useful information from it. It is among rapidly growing technologies today, with its applications in various aspects of a business. Image Processing forms core research area within engineering and computer science disciplines too. The principal objective of enhancement is to process an image so that the result is more suitable for a special process. Enhancement in Frequency domain processing techniques are based on which is direct manipulation of pixels modifying the Fourier transform, cosine transform and Walsh transform of an image.

2. Literature review

It is often necessary in medical image processing as well as other applications of image processing to identify the boundary between the objects in an image and separate the objects from each other [1,2,3,4,5]. Edges represent discontinuities of image intensity in an image. Image edge detection is the extraction of the edge of significant objects in an image and is the basis of high level image processing such as object recognition, image segmentation, feature extraction, space science and robot vision [1,6,7,8,9]. Image edge enhancement is the art of enhancing the edge of significant objects in an image. The objective of image edge enhancement is to improve visual perception of images.

Image processing filters are mainly used to suppress either the high frequencies in the image to smooth the image, or the low frequencies in the image to enhance or detect edges in the image [9].

D.R. Waghule and R.S. Ochawar, "Overview on Edge Detection Methods," Int. Conf. on Electronic Systems, Signal Processing and Computing Technologies, pp. 151-155, 2014. This paper presents an overview of the published work on edge detection.

Edge in an image is a contour across which the brightness of the image changes abruptly. Edge detection plays a vital role in image processing. Edge detection is a process that detects the presence and location of edges constituted by sharp changes in intensity of the image. An important property of the edge detection method is its ability to extract the accurate edge line with good orientation. Different edge detectors work better under different conditions. Comparative evaluation of different methods of edge detection makes it easy to decide which edge detection method is appropriate for image segmentation.

J. Li and X. Jing, "Edge Detection Based on Decision-Level Information Fusion and its Application in Hybrid Image Filtering," Int. Conf. on Image Processing (ICIP), pp. 251-254, 2004. A new edge detection method, based on decision-level information fusion, is proposed to classify image pixels into edge and non-edge categories. Traditional edge detection algorithms make the detection decision under a single criterion, which may perform inefficiently with a change of noise model. We use fusion entropy as a criterion to integrate decisions from different classifiers in order to improve the edge detection accuracy. The proposed decision fusion based edge detection method is applied to image filtering and leads to a weighted hybrid-filtering algorithm. Simulation results show that the new edge detection method has better performance than the single criterion edge detection methods.

S.J.H. Pirzada and A. Siddiqui, "Analysis of Edge Detection Algorithms for Feature Extraction in Satellite Images," Proceeding of the 2013 IEEE Int. Conf. on Space Science and Communication (Icon Space), 1-3 July 2013, Melaka, Malaysia, pp. 238-242, 2013. In field of image processing and pattern recognition, the use of edges as a feature is significant for feature extraction owing to its simplicity and accuracy. Its areas of application vary from object recognition to satellite based terrain recognition. There are many edge detection techniques like the canny edge detector and Sobel edge detector etc. However, the quality of an efficient algorithm depends on its capability to generate well localized edges of real images. Noise is inherent in all real images. To reduce its effect, various smoothing low pass filters are used prior to edge detection. This work, compares edge detection based on bilateral filtering with canny edge detection technique for satellite Images. Bilateral filtering based edge detection not only generates well localized edges but also simultaneously reduces considerable noise from real life images. The results show that the bilateral filtering based edge detection provides better edge maps than other comparable techniques.

3. General overview

The principal objective of enhancement is to process an image so that the result is more suitable for a special process Image Enhancement Fall into two categories: Enhancement in spatial domain and Frequency domain the term spatial domain refers to the Image Plane itself. Frequency domain processing techniques are based on which is direct manipulation of pixels modifying the Fourier transform of an image.

3.1. Fast Fourier Transform in Frequency Domain:

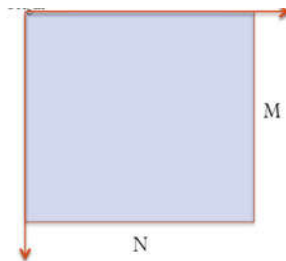
The discrete Fourier transform (DFT) is one of the most powerful tools in digital signal processing. The DFT enables us to conveniently analyze and design systems in frequency domain; however, part of the versatility of the DFT arises from the fact that there are efficient algorithms to calculate the DFT of a sequence. A class of these algorithms is called the Fast Fourier Transform (FFT).

Fourier Transform is an integral transform of one function into another. Jean Baptist Joseph Fourier (1768-1830), a French mathematician and physicist introduced Fourier transform. The FFT has played an important role in image processing for many years.

A fast *Fourier transform (FFT)* is an algorithm that samples a signal over a period of time (or space) and divides it into its frequency components.

An FFT algorithm computes the discrete Fourier transform (DFT) of a sequence, or its inverse (IFFT). Analysis converts a signal from its original domain to a representation in the frequency domain and vice versa.

The 2D Fourier Transform is a powerful tool and is used to enhance, restore, encode and describe the images. The FFT version is also available and is used in a number of image processing applications to reduce computational cost. The FFT is easy to implement by employing successive doubling technique and hence it finds an important place in image processing applications.



Given an image $f(x,y)$ with size $m \times n$

$$x=0, 1, 2, \dots, m-1,$$

$$y=0, 1, 2, 3, \dots, n-1$$

fourier transform of $f(x,y)$ in 2-D is given by:

$$F(u,v) = \sum_{x=0}^{m-1} \sum_{y=0}^{n-1} f(x,y) e^{-j2\pi(\frac{ux}{m} + \frac{vy}{n})}$$

where

$$u=0, 1, 2, \dots, m-1$$

$$v=0, 1, 2, 3, \dots, n-1.$$

$$j = \sqrt{-1}$$

the inverse Fourier transform in 2-D is given by:

$$f(x,y) = \frac{1}{mn} \sum_{u=0}^{m-1} \sum_{v=0}^{n-1} F(u,v) e^{j2\pi(\frac{ux}{m} + \frac{vy}{n})}$$

where

$$x=0, 1, 2, 3, \dots, m-1$$

$$y=0, 1, 2, 3, \dots, n-1$$

3.2. Cosine Transform In Frequency Domain:[14]

The discrete cosine transform (DCT) helps separate the image into parts (or spectral sub-bands) of differing importance (with respect to the image's visual quality). The DCT is similar to the discrete Fourier transform: it transforms a signal or image from the spatial domain to the frequency domain.

For N data items 1D DCT is defined by:

$$F(u) = \left(\frac{2}{N}\right)^{\frac{1}{2}} \sum_{i=0}^{N-1} \wedge(u) \cos \left[\frac{\pi u}{2N} (2i+1) \right] f(i)$$

And the corresponding *inverse* 1D DCT transform is simply $F^{-1}(u)$, i.e.:

$$= \left(\frac{2}{N}\right)^{\frac{1}{2}} \sum_{n=0} \wedge(u) \cos \left[\frac{\pi u}{2N} (2i+1) \right] f(u)$$

Where

$$\Lambda(\varepsilon) = \begin{cases} \frac{1}{2} & \text{for } \varepsilon = 0 \\ 1 & \text{otherwise} \end{cases}$$

For a 2D N by M image 2D DCT is defined:

$$F(u,v) = \left(\frac{2}{N}\right)^{\frac{1}{2}} \left(\frac{2}{M}\right)^{\frac{1}{2}} \sum_{i=0}^{N-1} \sum_{j=0}^{M-1} \Lambda(u) \Lambda(v) \times \cos \frac{\pi u}{2M} (2i+1) \cos \frac{\pi v}{2M} (2j+1) \cdot f(i,j)$$

and the corresponding **inverse** 2D DCT transform is simply $F^{-1}(u,v)$, i.e.:

$$= \left(\frac{2}{N}\right)^{\frac{1}{2}} \left(\frac{2}{M}\right)^{\frac{1}{2}} \sum_{i=0}^{N-1} \sum_{j=0}^{M-1} \Lambda(u) \Lambda(v) \times \cos \frac{\pi u}{2M} (2i+1) \cos \frac{\pi v}{2M} (2j+1) \cdot F(u,v)$$

3.3 Walsh Transform in frequency domain:

Walsh functions are a complete, orthogonal set of square-wave functions defined over the unit interval. These functions form the basis for the Walsh transform - the analogue to the Fourier transform for abruptly changing signals.[15] We define now the **1-D Walsh transform** as follows:

$$W(u) = \frac{1}{N} \sum_{x=0}^{N-1} f(x) \left[\prod_{i=0}^{n-1} (-1)^{b_i(x) b_{n-1-i}(u)} \right]$$

The above is equivalent to:

$$W(u) = \frac{1}{N} \sum_{x=0}^{N-1} f(x) (-1)^{\sum_{i=1}^{n-1} b_i(x) b_{n-1-i}(u)}$$

Inverse Walsh transform is almost identical to the forward transform.

$$f(x) = \sum_{u=0}^{N-1} W(u) \left[\prod_{i=0}^{n-1} (-1)^{b_i(x) b_{n-1-i}(u)} \right]$$

The above is equivalent to:

$$f(x) = \sum_{u=0}^{N-1} W(u) (-1)^{\sum_{i=1}^{n-1} b_i(x) b_{n-1-i}(u)}$$

We define now the **2-D Walsh transform** as a straightforward extension of the 1-D transform:

$$W(u,v) = \frac{1}{N} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x,y) \left[\prod_{i=0}^{n-1} (-1)^{b_i(x) b_{n-1-i}(u) + b_i(y) b_{n-1-i}(v)} \right]$$

The above is equivalent to:

$$W(u,v) = \frac{1}{N} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} f(x,y) (-1)^{\sum_{i=1}^{n-1} (b_i(x) b_{n-1-i}(u) + b_i(y) b_{n-1-i}(v))}$$

Inverse 2-D Walsh transform It is identical to the forward 2-D Walsh transform.

$$f(x,y)=\frac{1}{N}\sum_{x=0}^{N-1}\sum_{y=0}^{N-1}W(u,v)\left[\prod_{i=0}^{n-1}(-1)^{b_i(x)b_{n-1-i}(u)+b_i(y)b_{n-1-i}(v)}\right]$$

The above is equivalent to:

$$f(x,y)=\frac{1}{N}\sum_{x=0}^{N-1}\sum_{y=0}^{N-1}W(u,v)(-1)^{\sum_{i=1}^{n-1}(b_i(x)b_{n-1-i}(u)+b_i(x)b_{n-1-i}(u))}$$

Filters:

There are two types of enhancement techniques called spatial domain and frequency domain techniques which are categorized again for smoothing and sharpening the images the filtering in frequency domain using FFT, dct and fwht use of the terms frequency domain and frequency components is really no different from the terms time domain and time components, which we would use to express the domain and values of $f(x)$ if x where a time variable.

Low pass filter (smoothing): A low-pass filter is a filter that passes low-frequency signals and attenuates signals with frequencies higher than the cut-off frequency. The actual amount of attenuation for each frequency varies depending on specific filter design. Smoothing is fundamentally a low pass operation in the frequency domain.

$$H(u,v)=e^{-D^2(u,v)/2D_0^2}$$

High pass filters (sharpening): A high-pass filter is a filter that passes high frequencies well, but attenuates all the frequencies lower than the cut-off frequency. Sharpening is fundamentally a high pass operation in the frequency domain. High pass filter (*Hhp*) is often represented by its relationship to the low pass filter (*Hlp*):

$$H_{hp}=1-H_{lp}$$

4. Methodology

4.1. Edge Enhancement:

Image edge enhancement is the art of enhancing the edge of significant objects in an image. The objective of image edge enhancement is to improve visual perception of images. Image processing filters are mainly used to suppress either the high frequencies in the image to smooth the image, or the low frequencies in the image to enhance or detect edges in the image. An image can be filtered in order to enhance the edges either in the frequency or in the spatial domain.

4.2. Threshold for filters in frequency domain:

To find the threshold (D_0) for filters in frequency domain:

$$D_0=\sqrt{(R^2 + C^2)}$$

For example:

If image is 225*225 then $R(\text{rows})=0-224$ and $C(\text{columns})=0-224$
 $D_0=\sqrt{(224^2+224^2)}=316.7$

We can take threshold with difference 20 i.e. 100,120,140,160.....300. If threshold is 100 then a low pass filter will give frequency components that are below 100 and a high pass filter will give frequency components that are above 100. For these threshold values we can find frequency, brightness and contrast.

4.3. Performance Evaluation

For the given input image, after finding Image edge enhancement using low pass and high pass filters in both spatial and frequency domains respectively, some factors must be considered in order to choose the better filter in that domain. So, the three factors are frequency, brightness and contrast.

Measurements

The *Frequency Estimate F* of an image is a measure of the spatial frequency of the image and it is the rate of change of intensity across the image. F is given as

$$F = \frac{1}{3MF(NF-1)} f_{rows} + \frac{1}{3NF(MF-1)} f_{columns}$$

Where

$$f_{rows} = \sum_{i=1}^{MF} \sum_{j=2}^{NF} \sum_{t=1}^3 |x(i, j, t) - x(i, j - 1, t)|$$

$$\text{and } f_{columns} = \sum_{j=1}^{MF} \sum_{i=2}^{NF} \sum_{t=1}^3 |x(i, j, t) - x(i - 1, j, t)|$$

Contrast C of an image is the range from the darkest regions of the image to the lightest regions. High-contrast images have large regions of dark and light. Images with good contrast have a good representation of all luminance intensities. As the contrast of an image increases, the viewer perceives an increase in detail. This is purely a perception issue as the amount of information in the image does not increase. Human beings' perception is sensitive to luminance contrast rather than absolute luminance intensities.

$$C = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

Where I_{max} and I_{min} are the maximum and minimum intensities in the image.

Brightness B of an image is defined as the average of all the pixels within the image. B is given as

$$B = \frac{1}{NM} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} x(m, n)$$

Where M and N are the numbers of rows and columns respectively and NM is the total number of pixels in $x(m, n)$.

Brightness, Contrast and Frequency Estimate of all the filters that is spatial domain filters and frequency domain filters are calculated. Comparison is made in such a way that the filter with high frequency and brightness is given more importance than the other filters.

Algorithmic Approach of Frequency Domain:

Step 1: Consider an input image to perform edge enhancement.

Step 2: In the frequency domain, fast Fourier/cosine/Walsh transform is applied on input image

Step 3: After transformation, choose an edge enhancement filter. Ideal/Gaussian/Butterworth is some filter in frequency domain.

Step 4: For low pass filter, there are several standard forms of low pass filters suppose Gaussian low pass filters.

$$H(u, v) = e^{-D^2(u, v)/2D_0^2}$$

Step 5: To find the threshold (D_0) for filters in frequency domain: $D_0 = \sqrt{(R^2 + C^2)}$.

Step 6: For high pass filter, there are several standard forms of high pass filters. All high pass filter (H_{hp}) is often represented by its relationship to the low pass filter (H_{lp}):

$$H_{hp} = 1 - H_{lp}$$

Step 7: Now inverse Fourier transform is applied.

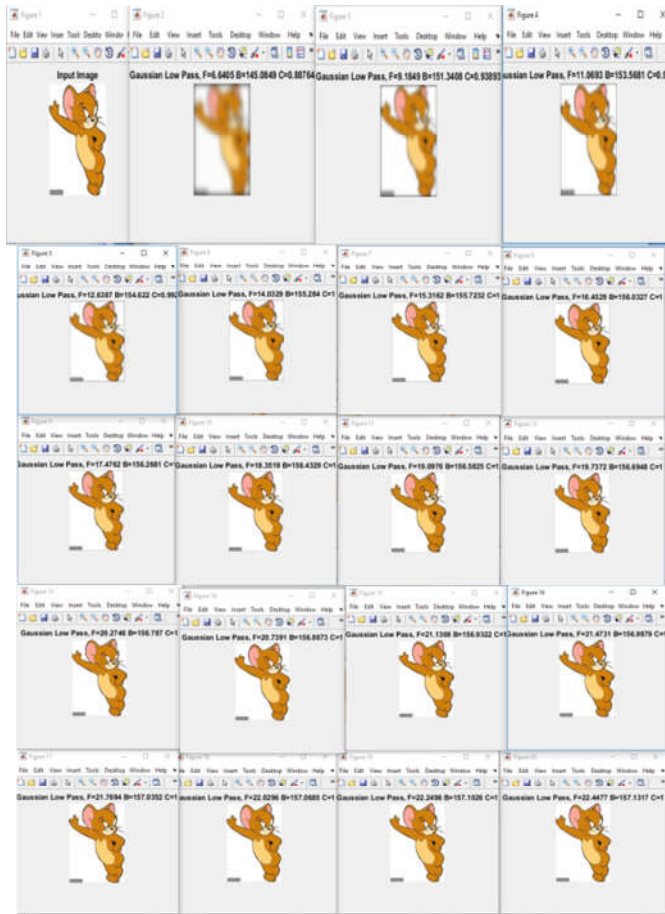
Step 8: Comparison for these filters is made on the basis of frequency, brightness and contrast.

5. Results

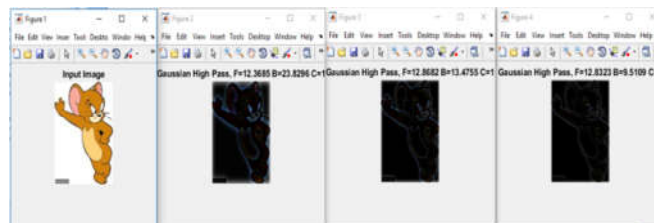
Image: jerry.bmp 115*160 pixels
 $D = \sqrt{(\text{row}^2 + \text{column}^2)}$
 $= \sqrt{(115^2 + 160^2)}$
 $= 197.0406049524$

Threshold D_0 is 10, 20, 30, ..., 100, ..., 190

*Fourier transform Frequency estimate, brightness and contrast:
 Low pass filter:*



High pass filter:

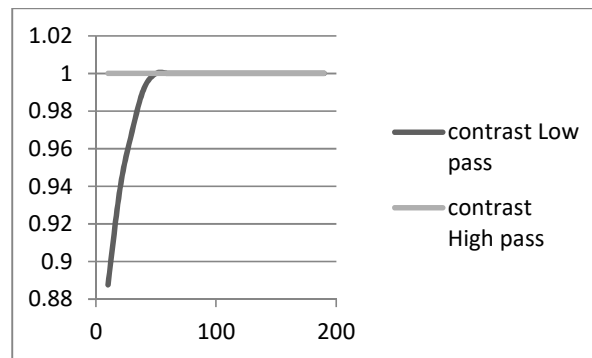
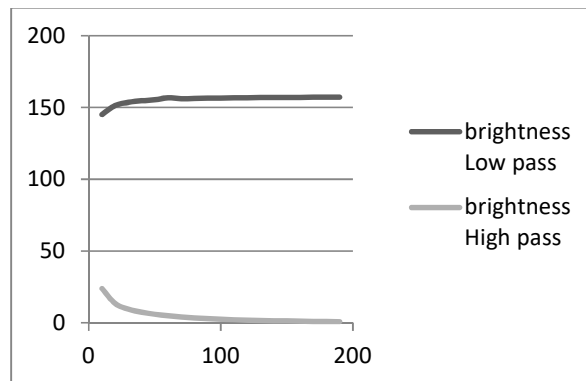
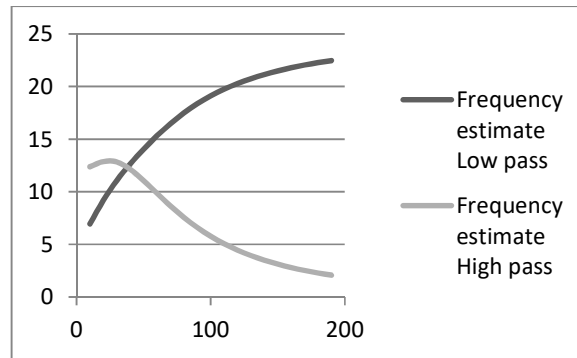




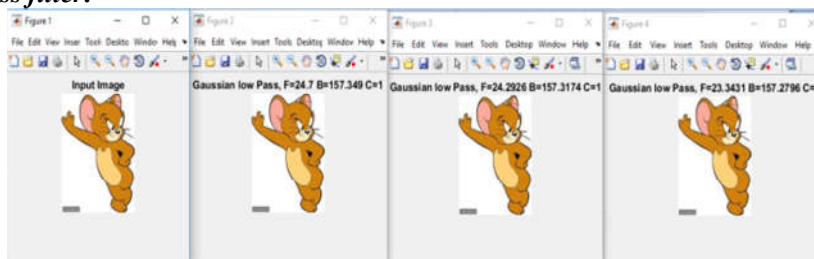
Tabular data:

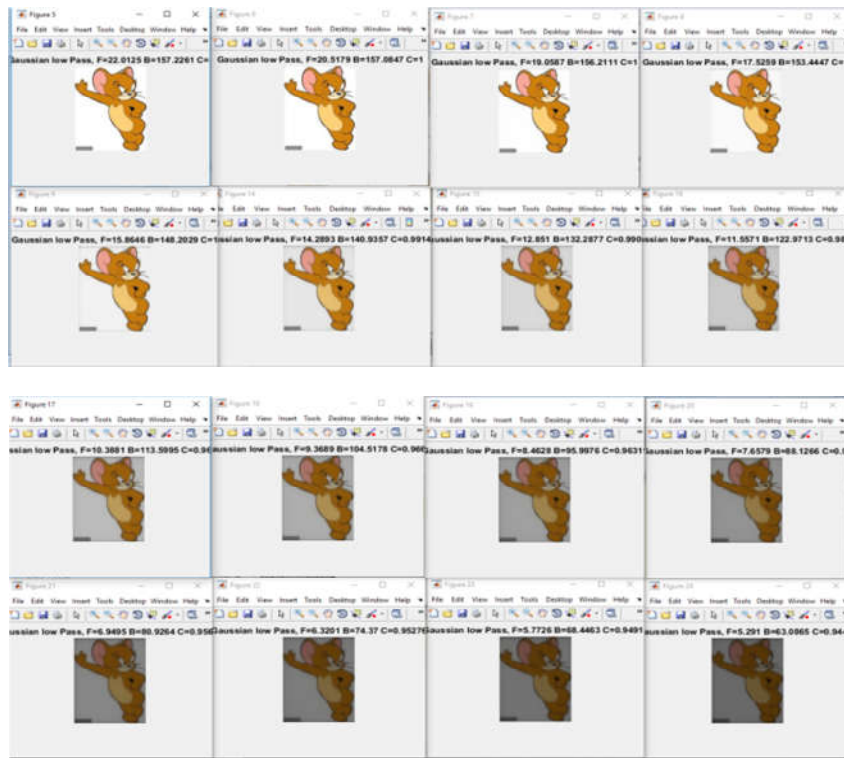
thres hold	Frequency estimate		brightness		contrast	
	Low pass	High pass	Low pass	High pass	Low pass	H p
10	6.941	12.369	145.09	23.83	0.888	1
20	9.1849	12.868	151.34	13.48	0.939	1
30	11.069	12.832	153.57	9.511	0.969	1
40	12.639	12.147	154.62	7.409	0.992	1
50	14.033	11.05	155.29	5.974	1	1
60	15.316	9.852	156.72	4.913	1	1
70	16.453	8.655	156.03	4.102	1	1
80	17.476	7.574	156.27	3.463	1	1
90	18.356	6.616	156.43	2.963	1	1
100	19.096	5.775	156.58	2.546	1	1
110	19.737	5.0714	156.69	2.207	1	1
120	20.275	4.467	156.79	1.926	1	1
130	20.739	3.947	156.867	1.689	1	1
140	21.131	3.5101	156.933	1.492	1	1
150	21.473	3.134	156.989	1.323	1	1
160	21.769	2.806	157.035	1.1803	1	1
170	22.0296	2.535	157.0685	1.0638	1	1
180	22.249	2.297	157.103	0.959	1	1
190	22.447	2.0859	157.132	0.868	1	1

Graphical data:

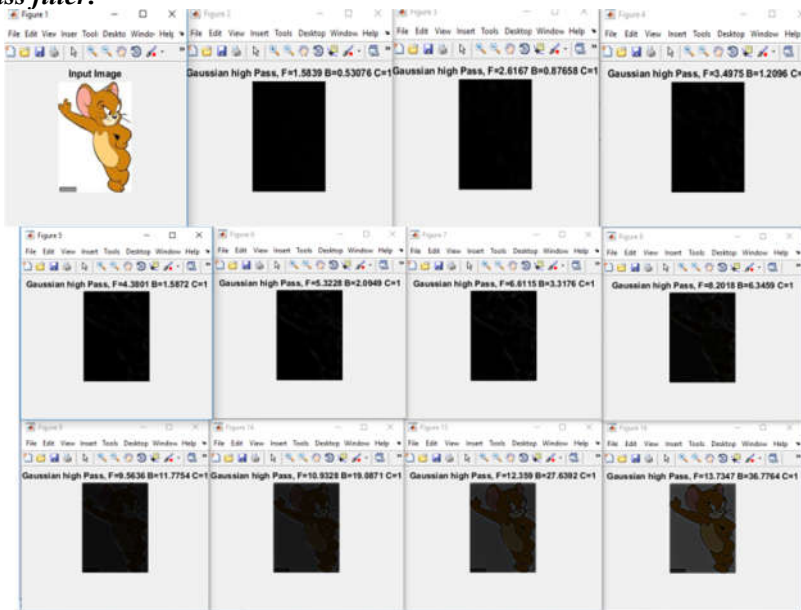


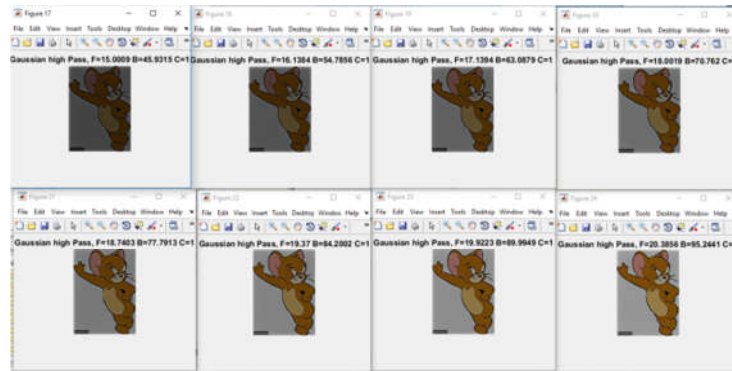
*Cosines transform frequency estimate, brightness and contrast:
Low pass filter:*





High pass filter:

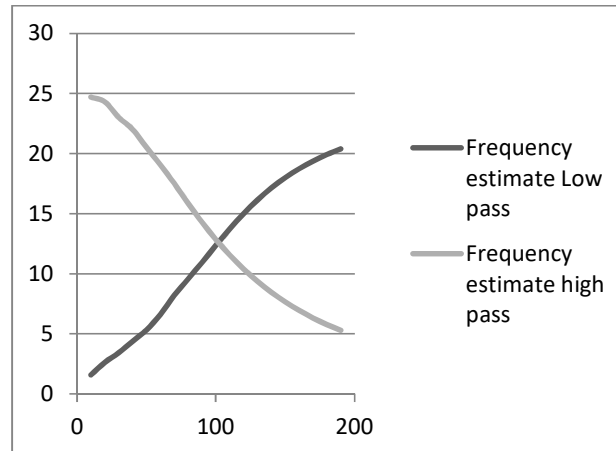


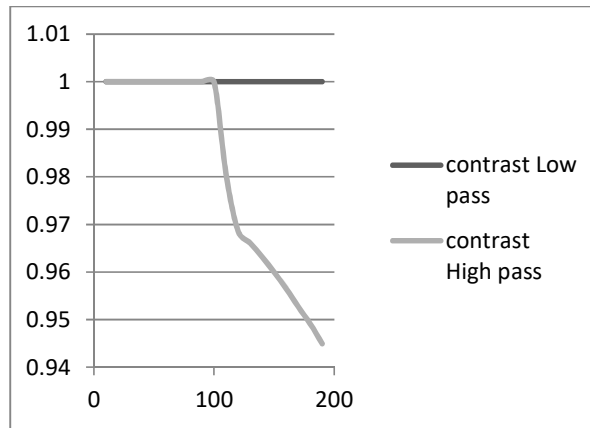
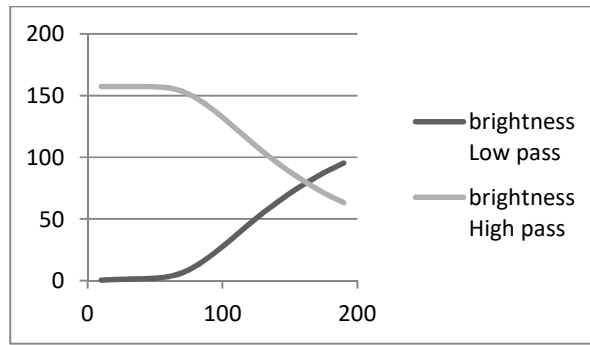


Tabular Data:

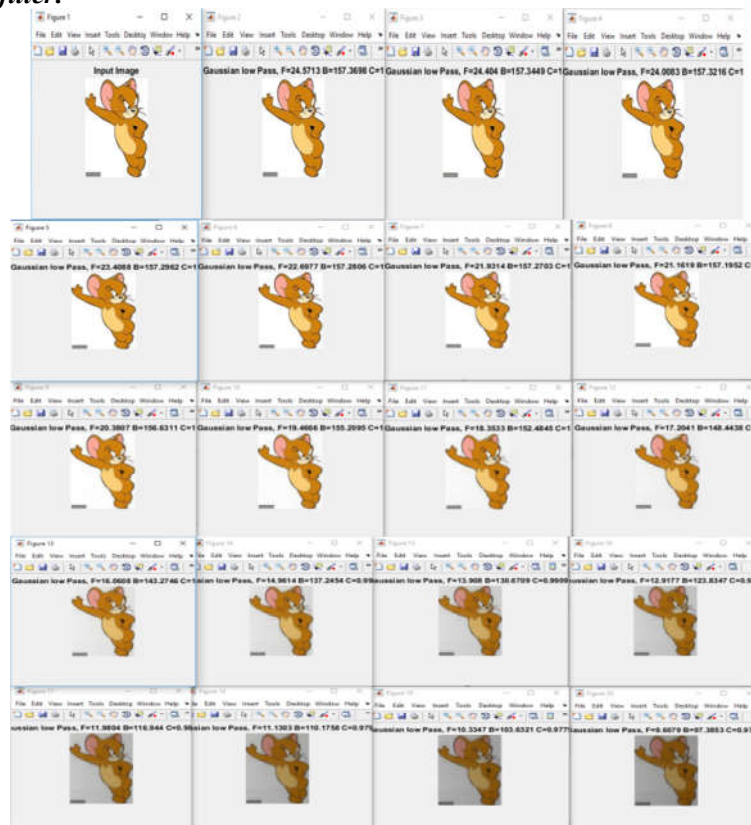
thres hold	Frequency estimate		brightness		contrast	
	Low pass	high pass	Low pass	High pass	Lp	Hp
10	1.584	24.700	0.531	157.35	1	1
20	2.617	24.293	0.8766	157.32	1	1
30	3.428	23.013	1.200	157.28	1	1
40	4.381	22.013	1.587	157.22	1	1
50	5.323	20.518	2.0949	157.09	1	1
60	6.612	19.058	3.3176	156.21	1	1
70	8.2018	17.526	6.3459	153.45	1	1
80	9.5635	15.865	11.775	148.20	1	1
90	10.933	14.289	19.087	140.94	1	1
100	12.359	12.851	27.639	132.29	1	0.999
110	13.735	11.557	36.776	122.97	1	0.981
120	15.000	10.388	45.932	113.6	1	0.969
130	16.138	9.3689	54.786	104.56	1	0.966
140	17.139	8.4628	63.088	95.998	1	0.963
150	18.002	7.658	70.762	88.126	1	0.960
160	18.740	6.9496	77.791	80.926	1	0.956
170	19.369	6.3201	84.200	74.37	1	0.953
180	19.922	5.7726	89.995	68.446	1	0.949
190	20.385	5.2910	95.244	63.086	1	0.945

Graphical data:

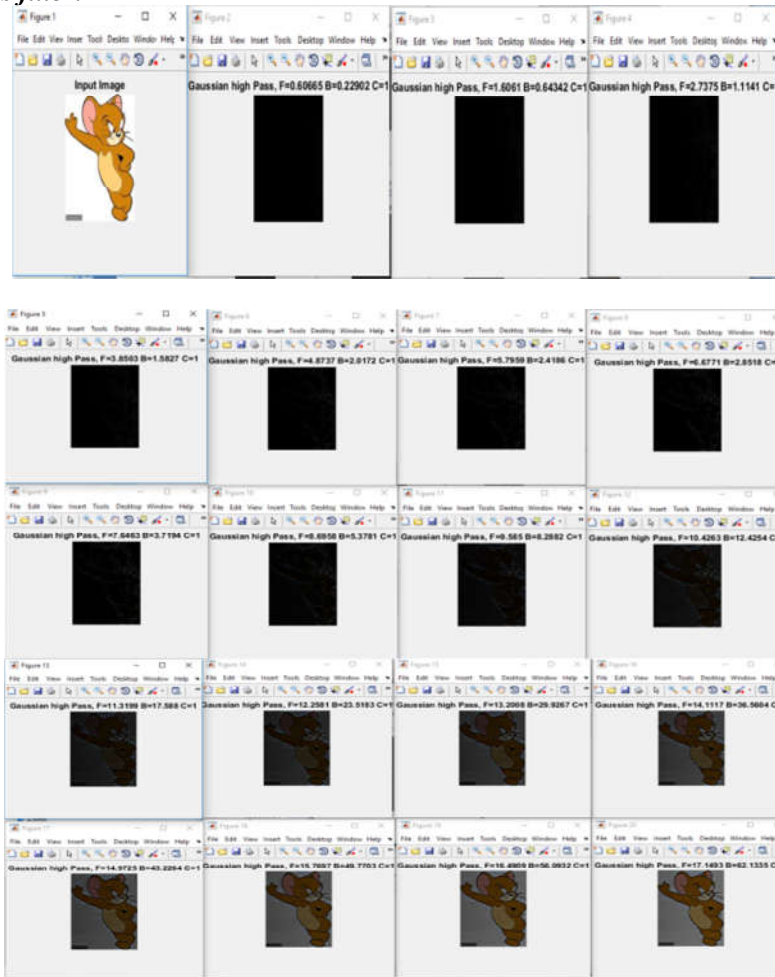




**Walsh transform frequency estimation, brightness and contrast:
Low pass filter:**



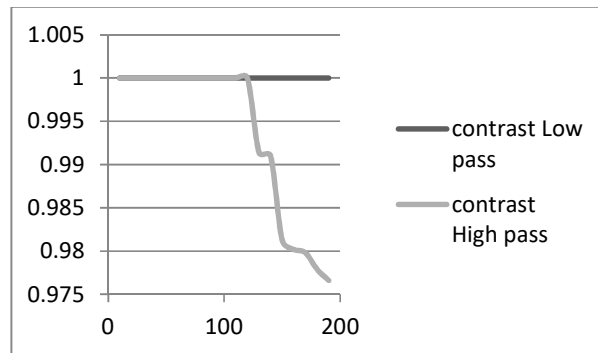
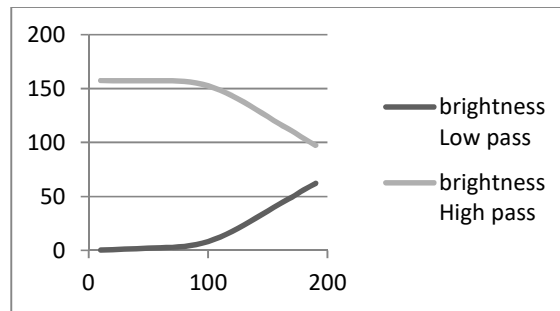
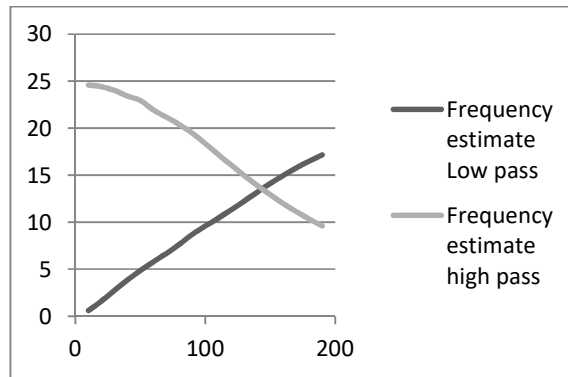
High pass filter:



Tabular Data:

thresh old	Frequency estimate		brightness		contrast	
	Low pass	high pass	Low pass	High pass	Lp	High pass
10	0.6067	24.57	0.229	157.369	1	1
20	1.6061	24.404	0.643	157.344	1	1
30	2.7374	24.008	1.114	157.322	1	1
40	3.8503	23.409	1.582	157.296	1	1
50	4.8736	22.931	2.017	157.28	1	1
60	5.7959	21.931	2.419	157.270	1	1
70	6.677	21.162	2.852	157.195	1	1
80	7.646	20.380	3.719	156.631	1	1
90	8.695	19.466	5.378	155.209	1	1
100	9.5849	18.353	8.288	152.485	1	1
110	10.426	17.204	12.42	148.444	1	1
120	11.319	16.060	17.58	143.274	1	1
130	12.258	14.961	23.52	137.245	1	0.991
140	13.200	13.908	29.92	130.670	1	0.990
150	14.111	12.917	36.56	123.83	1	0.981
160	14.973	11.980	43.22	116.943	1	0.980
170	15.769	11.130	49.23	110.758	1	0.979
180	16.49	10.334	56.09	103.632	1	0.977
190	17.149	9.6079	62.13	97.3852	1	0.978

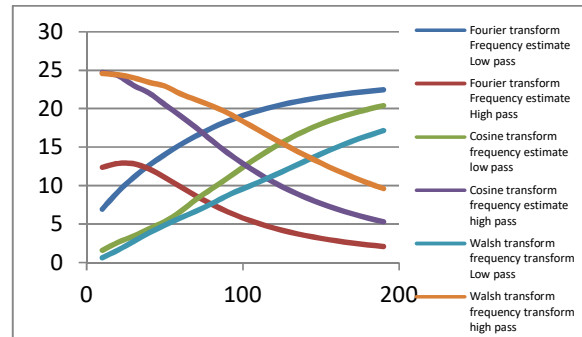
Graphical data:



Frequency Estimation Of Fourier, Cosine And Walsh Transforms:

threshold (D) 0	Fourier transform		Cosine transform		Walsh transform	
	Low pass	High pass	low pass	high pass	Low pass	high pass
10	6.9404	12.368472	1.583870	24.700037	0.606648	24.57129
20	9.1849	12.868237	2.616720	24.292576	1.606112	24.4039
30	11.069281	12.832295	3.427540	23.012526	2.7374	24.00828
40	12.638739	12.146640	4.3810131	22.012526	3.85033	23.4088
50	14.032863	11.053318	5.322793	20.517920	4.8736	22.9314
60	15.316184	9.851968	6.611499	19.058685	5.7959	21.9314
70	16.452862	8.655404	8.201800	17.525929	6.677	21.16185
80	17.476233	7.573900	9.563572	15.864642	7.646	20.38067
90	18.351917	6.616437	10.932819	14.289295	8.695	19.46657
100	19.097569	5.775262	12.358992	12.851044	9.5849	18.3533
110	19.737189	5.071418	13.734703	11.557056	10.4263	17.2040
120	20.274609	4.466679	15.000873	10.388055	11.3199	16.060768

130	20.739106	3.947618	16.138405	9.368890	12.2581	14.96138
140	21.130788	3.510113	17.139377	8.462764	13.20008	13.908019
150	21.473114	3.134049	18.00188	7.657959	14.1117	12.9177
160	21.769357	2.806173	18.740313	6.949545	14.9725	11.98035
170	22.029633	2.534973	19.369980	6.320103	15.769654	11.130254
180	22.249613	2.292632	19.922326	5.772622	16.4909	10.3347
190	22.447703	2.085931	20.385572	5.291001	17.149	9.607913



6. Conclusion

Image edge enhancement using filters in frequency domain have been studied. In frequency domain, for low pass filters, as the threshold increases the values of frequency and brightness are increased whereas for high pass filters, as the threshold increases the values of frequency and brightness are decreased but contrast remains same for all the filters with Fourier transform.

In this paper, when low pass and high filters applied on both cosines transform and Walsh transform, both are similar to Fourier transform. For low pass filter as the for low pass filters, as the threshold increases the values of frequency and brightness are increased whereas for high pass filters, as the threshold increases the values of frequency and brightness are decreased but contrast remains same for all the filters with both cosines and Walsh transforms.

Future scope:

Image enhancement is performed each within the spatial domain in addition as in time domain. Image enhancement is performed by using fast Fourier transform in frequency domain and can also be computed by using different transforms which are closely related to Sine and cosine transforms, Hartley transform and Fractional Fourier transform.

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