Solution of Maxwell's Equation by Kamal Transform

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Abstract- In this paper we have developed a new solution of maxwell's equations. There is electric and magnetic field. We are using kamal transformation for both fields of maxwell's equations describing TEMP waves traveling in conducting but lossy media is derived.

Keywords: Maxwell's Equation, Kamal transform.

1.INTRODUCTION

We apply Kamal transform to Maxwell's equations to determine the transient electric and magnetic field solution. Using the natural transform, Samudu transform Maxwell's equation were solved by R. solambarsn et al. [1]

Now we can take set A, the function is defined in the form

$A=\{f:|f(t)|<p^{t/\theta j}$ if $t\in (-1)^j \times [0\ \infty), j=1,2;\ \varphi_j>0\}$

Where φ_1 & φ_2 may be finite or infinite and the constant p must be finite.

Then Kamal transform is defined as

$$K\{ f(t) \} = G(v)=\int_0^\infty f(t)\, e^{-t/v}\, dt\ ,\ t\geq 0\ ,\ \varphi_1 \leq v \leq \varphi_2 \qquad ...(1)$$

Meanwhile, we can take advantage to certain properties of Kamal transform without any proof which will be useful to obtain the solution of Maxwell's equation.

The Kamal transform of function derivate f(t) w.r.t. "t" , n times.

$$K[f^n(t)] = \frac{1}{v^n} G(v)- \sum_{k=0}^{n-1} v^{k-n+1} f^k(0) \qquad …(2)$$

For n=1,2 in Eq.(2) gives Kamal transform of First and second derivative of f(t) w.r.t. " t " .

$$K\ \{ f'(t)\} = \frac{1}{v} G(v)-f(0) \qquad …(3)$$

$$K\ \{ f''(t)\} = \frac{1}{v^2}G(v)-\frac{1}{v}f(0)- f'(0) \qquad …(4)$$

Let f(t) and g(t) are two functions then Kamal transform of convolution of two functions is given by

$$K(f*g) = \frac{1}{v} k(f)k(g) \qquad …(5)$$

The inverse of Kamal transform is related with Bromwich contour integral which is given by

$$K^{-1}[G(v)] = f(t) = lim_{T\to\infty} \frac{1}{2\pi i}\int_{\alpha-iT}^{\alpha+iT} e^{t/v} f(t)\, dt \qquad …(6)$$

Next we apply the above properties to Maxwell's equation.

2.SOLUTION OF MAXWELL'S EQUATIONS

The electric field vector E and Magnetic field vector H are related by Maxwell's equations. The permeability μ and conductivity σ with constant permittivity ϵ in the planer transverse electromagnetic wave (TEMP) propagate in x direction in lossy medium. Then Maxwell's equations are given by

$$\nabla \text{ X } E = -\mu\frac{\partial H}{\partial t} \quad \ldots(7)$$

$$\nabla \text{ X } H = -\mu\frac{\partial E}{\partial t} + \sigma E \quad \ldots(8)$$

If the electric field vector is polarized along y direction then $E = E_y(x,t)$ and magnetic field along z direction $H = H_z(x,t)$ then Maxwell equation of Eq.(7) and (8) written as in differential equation form

$$\frac{\partial Ey}{\partial x} + \mu\frac{\partial HZ}{\partial t} = 0 \quad \ldots(9)$$

$$\frac{\partial \text{HZ}}{\partial x} + \epsilon\frac{\partial \text{Ey}}{\partial t} + \sigma\text{Ey} = 0 \quad \ldots(10)$$

Applying Kamal transform to the Eq.(9) and (10) and setting

$$K[E_y\,(x,t)] = F(x,v)\,, K[H_z\,(x,t)] = G\,(x,v)$$

Where v are kamal transform variable w.r.t. "t" gives

$$K[\frac{\partial Ey}{\partial x}] + \mu\, K[\frac{\partial HZ}{\partial t}] = 0$$

$$\frac{\partial F(x,v)}{\partial x} + \mu\left[\frac{1}{v}\,G(x,v) - H(x,0)\right] = 0$$

$$\frac{\partial F(x,v)}{\partial x} + \left[\frac{\mu}{v}\,G(x,v) - \mu H(x,0)\right] = 0 \quad \ldots(11)$$

Now again from Eq.(10)

$$k[\frac{\partial Hz}{\partial x}] + \epsilon\, K[\frac{\partial Ey}{\partial t}] + \sigma k[Ey\,] = 0$$

After solving we get

$$\frac{\partial G(x,v)}{\partial x} + \frac{\epsilon}{v}F(x,v) - \epsilon E(x,0) + \sigma F(x,v) = 0 \quad \ldots(12)$$

Differentiating Eq.(11) partially w.r.t " x " gives

$$\frac{\partial^2 F(x,v)}{\partial x^2} + \frac{\mu}{v}\frac{\partial G(x,v)}{\partial x} = \mu\,\frac{\partial H(x,0)}{\partial x} \quad \ldots(13)$$

$$\frac{\partial^2 F(x,v)}{\partial x^2} - F(x,v)\,[\frac{\mu\epsilon}{v^2} + \frac{\mu\sigma}{v}] = [\,\mu\frac{\partial H(x,t)}{\partial x}\,]_{t=0} - \frac{\mu\epsilon}{v}\,E(x,0) \quad \ldots(14)$$

Substituting Eq.(12) for $\frac{\partial G(x,v)}{\partial x}$, then simplifying and arranging

$$\frac{\partial^2 F(x,v)}{\partial x^2} - F(x,v)\,[\frac{\mu\epsilon}{v^2} + \frac{\mu\sigma}{v}] = [-\epsilon\,\mu\frac{\partial E(x,t)}{\partial t}\,]_{t=0} + E(x,0)[\,\mu\sigma - \frac{\mu t}{v}\,] \quad \ldots(15)$$

Now consider the initial and boundary conditions

$$E_y(\infty,t) = finite \quad \ldots(16)$$

$$E_y(\,x,0) = H_z(x,0) = 0 \quad \ldots(17)$$

$$\frac{\partial E_y(\,x,0)}{\partial x} = \frac{\partial H_z(x,0)}{\partial x} = 0 \quad \ldots(18)$$

$$[\frac{\partial \mathrm{Ey}(x,\mathrm{t})}{\partial x}]_{\mathrm{t}=0} = [\frac{\partial \mathrm{Hz}(x,\mathrm{t})}{\partial x}]_{\mathrm{t}=0} = 0 \quad \ldots (19)$$

$$\mathrm{E}(0,\mathrm{t}) = \mathrm{f}(\mathrm{t}) \; ; \; \mathrm{t} \geq 0 \text{ and } \mathrm{t} < 0 \quad \ldots (20)$$

Using the initial conditions Eq.(17) and Eq.(19).The RHS of Eq.(15) is zero.Then

$$\frac{d^2F(x,v)}{dx^2} - [\frac{\mu\epsilon}{v^2} + \frac{\mu\sigma}{v}]F(x,v) = 0 \quad \ldots (21)$$

Now we are substituting $\alpha^2 = \frac{\mu\epsilon}{v^2} + \frac{\mu\sigma}{v}$ and the resulting homogenous differential equation solution is

$$F(x\,,v\,) = A(v)\,e^{-\alpha x} + B(v)\,e^{-\alpha x} \quad \ldots (22)$$

Using the boundary condition Eq.(20) gives $B(v) = 0$ in Eq.(22),

$$A(v) = K\{f(t)\} = f(v) \quad \ldots (23)$$

$$F(x,v) = f(v)\,e^{-\alpha x} \quad \ldots (24)$$

Now multiplying and dividing the RHS of Eq.(24) by parameter v then we get

$$F(x,v) = \frac{vf(v)\,e^{-\alpha x}}{v} \quad \ldots (25)$$

By the application of Eq.(3) and Eq.(6),

$$F(x,v) = vk[\,f'(t) + f(0)\,]k[\frac{1}{2\pi i}\int_{\alpha-i\infty}^{\alpha+i\infty} \frac{e^{t/v}e^{-\alpha x}}{v}dt] \quad \ldots (26)$$

Applying inverse kamal transformation on both sides of Eq.(26) and using the convolution theorem,

$$\mathrm{E}(x,t) = \int_0^{\mathrm{t}} [f(t-\Phi) + f(0)]\frac{1}{2\pi i}\int_{\alpha-i\infty}^{\alpha+i\infty} \frac{e^{t/v}e^{-\alpha x}}{v}dt\,d\Phi \quad \ldots (27)$$

which is the transient electric field $E(x,t)$ of TEMP waves in lossy medium is given by

Similarly for magnetic field $H(x,t)$ of TEMP waves in Lossy medium is given by

$$H(x,t) = \int_0^{\mathrm{t}} [g'(t-\theta) + f(0)]\frac{1}{2\pi i}\int_{\alpha-i\infty}^{\alpha+i\infty} \frac{e^{t/v}e^{-\alpha x}}{v}dt\,d\theta\,] \quad \ldots (28)$$

Conclusion

We have observed that magnetic field solutions are imaginary and electric field solution are real from their reduced homogeneous differential equations as was expected.

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