Implementation of Extended Kalman Filter on Stochastic Model of LPF

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Abstract

This paper presents output voltage estimation of RC low pass filter (LPF). For this, extended Kalman filter (EKF) has been used on stochastic model of RC LPF. At first, deterministic model of RC low pass circuit has been derived and it is transformed into stochastic model by adding white Gaussian noise to input source and circuit elements. Thereafter, EKF is applied for output voltage estimation. MATLAB simulation results show that estimated output of RC circuit using noisy input gives approximately same output as PSPICE simulated output (actual output).

Keywords: RC LPF; EKF; stochastic parameter estimation

1. INTRODUCTION

RC circuit is an important circuit component in different electronic circuits. Jiang [1] used RC low pass circuits in class D amplifier for loop stability. Farajollahi*et al.* [2] proposed a RC circuit based transmission line model for prediction of polymer based electrode and actuators behavior. It is also used in resistance switching interface circuit [3]. Besides these it is also used in CMOS bulk lowpass analogue filter [4], flexible voltage controlled oscillators [5], flyback inverter [6], Chebyshev ladder active RC band pass filter [7], wireless receiver [8] etc.

Stochastic modelling is an approach for optimization problems by including uncertainty. As electronic circuits are affected by different types of noise, the stochastic modelling helps to study the effect of random fluctuations of individual circuit elements and voltage sources. Stochastic modelling of nonlinear rectifier circuit is given in [9]. Bonnin [10] presented the amplitude and phase description for nonlinear oscillator under white Gaussian noise. Tao *et al.* [11] proposed a stochastic approach to accelerate the design of lithium ion battery capacity fading dynamics model. Li-ion battery. Djurhuus*et al.* [12] analyzed the stochastic resonance of bistable electrical circuit. Rawat *et al.*[13] proposed stochastic modelling of linear RLC circuit.

EKF is broadly used for parameter estimation of nonlinear systems in different applications. Stojanovic and Nedic [14] presented a EKF based joint state and parameter estimation of stochastic nonlinear system with time varying parameters. Chatzis*et al.* [15] proposed stochastic parameter estimation of radar tracking using continuous-discrete EKF. In [16], Liu et al. studied the stochastic stability condition for EKF. In [17], Baccouche*et al.* proposed online state of charge estimation of aLi-ion battery using EKF. Yin et al. [18] studied the effect of observability properties of non-smooth systems in the convergence of EKF and Unscented Kalman Filter (UKF).

This paper is organized as follows: Deterministic and stochastic model of RC low pass filter circuit has been derived in Section 2. Brief introduction to EKF is presented in section 3. Implementation of EKF in RC circuit stochastic model is given in Section 4. Simulation results are given in section 5. Section 6 concludes the paper.

2. State Space Model of RC Circuit

2. A. Deterministic Model of RC circuit

Fig. 1 (a) shows the second order RC low pass circuit havinginput voltage source $u_1(t)$, resistors R_1, R_2 and capacitors C_1 , C_2. V_{C_1} and V_{C_2} are the capacitor voltages.

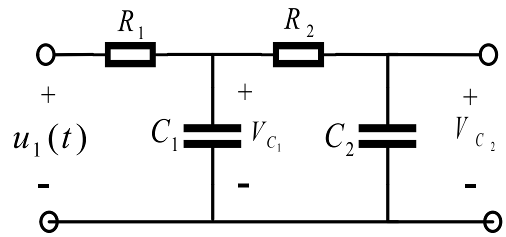

Fig. 1. RC low pass filter

Applying Kirchhoff's law to the circuit in Fig 1, we have

$$\frac{v_{c_1}(t)-u_1(t)}{R_1}+\frac{v_{c_1}(t)-v_{c_2}(t)}{R_2}+C_1\frac{dv_{c_1}}{dt}=0 \quad (1)$$

$$\frac{v_{c_2}(t)-v_{c_1}(t)}{R_2}+C_2\frac{dv_{c_2}}{dt}=0 \quad (2)$$

Or, (1) and (2) can be written as

$$\frac{dv_{c_1}}{dt}=v_{c_1}(t)\left(-\frac{1}{R_1C_1}-\frac{1}{R_2C_1}\right)+v_{c_2}(t)\left(\frac{1}{R_2C_1}\right)+u_1(t)\left(\frac{1}{R_1C_1}\right) \quad (3)$$

$$\frac{dv_{c_2}}{dt}=v_{c_1}(t)\left(\frac{1}{R_2C_2}\right)+v_{c_2}(t)\left(-\frac{1}{R_2C_2}\right) \quad (4)$$

$$y=v_{c_2}(t) \quad (5)$$

Where, y is the measured output. Above equations can be represented in following matrix notation as

$$\frac{dv(t)}{dt}=Av(t)+B(t) \quad (6)$$

Where $dv(t)$ is given as

$$dv(t)=[dv_{c_1}(t) \quad dv_{c_2}(t)]^T$$

$$A=\begin{bmatrix}\left(-\frac{1}{R_1C_1}-\frac{1}{R_2C_1}\right) & \frac{1}{R_2C_1}\\ \frac{1}{R_2C_2} & -\frac{1}{R_2C_2}\end{bmatrix} \quad (7)$$

$$B=\begin{bmatrix}\frac{v_1(t)}{R_1C_1} & 0\end{bmatrix} \quad (8)$$

I. 2. B. Stochastic model of RC circuit

White Gaussian noise has been added to input source and elements of the RC circuit to convert deterministic ordinary differential equation into stochastic differential equation. The correlated process i.e. colored noise is added to circuit elements as

$$A_j^* = A_j + k_j w_j(t), \qquad 1 \le j \le n$$

Where A_j^* is the noisy circuit element, A_j is circuit element, k_j is the constant that denotes the intensity of noise, $w_j(t)$ is the zero-mean correlated process and u_1 is input to the circuit.

$$R_1^* = R_1 + k_1 w_1(t) \tag{9}$$

$$R_2^* = R_2 + k_2 w_2(t) \tag{10}$$

$$C_1^* = C_1 + k_3 w_3(t) \tag{11}$$

$$C_2^* = C_2 + k_4 w_4(t) \tag{12}$$

$$u_1^*(t) = u_1(t) + k_5 w_5(t) \tag{13}$$

The correlated process $w_j(t)$ is mathematically represented by stochastic differential equation (SDE) in terms of white Gaussian noise.

$$dw_j(t) = \rho_j w_j(t)dt + \sigma_j \rho_j dB_j(t) \tag{14}$$

Where, $B_j(t)$ is the Brownian motion process. $B_j(t)$ is continuously differentiable, therefore equation (14) becomes

$$\frac{dw_j(t)}{dt} = \rho_j w_j(t) + \sigma_j \rho_j N_j(t) \tag{15}$$

Where, $N_j(t) = \frac{d}{dt} B_j(t)$ and defined as white Gaussian noise. Substituting (9) - (13) into (3) to (5), we get

$$\frac{dv_{c_1}}{dt} = v_{c_1}(t)\left(-\frac{1}{(R_1 + k_1 w_1(t))(C_1 + k_3 w_3(t)} - \frac{1}{(R_2 + k_2 w_2(t))(C_1 + k_3 w_3(t)}\right) + $$
$$+ v_{c_2}(t)\left(\frac{1}{(R_2 + k_2 w_2(t))(C_1 + k_3 w_3(t)}\right) + \left(\frac{u_1(t) + k_5 N_5(t)}{(R_1 + k_1 w_1(t))(C_1 + k_3 w_3(t)}\right) \tag{16}$$

$$\frac{dv_{c_2}}{dt} = v_{c_1}(t)\left(\frac{1}{(R_2 + k_2 w_2(t))} \times \frac{1}{(C_2 + k_4 w_4(t))}\right)$$
$$+ v_{c_2}(t)\left(-\frac{1}{(R_2 + k_2 w_2(t))} \times \frac{1}{(C_2 + k_4 w_4(t))}\right) \tag{17}$$

To convert (16) and (17) into stochastic differential equation (SDE), we multiply these equations by dt, therefore get

$$dv_{c_1}(t) = v_{c_1}(t)\left(-\frac{1}{(R_1+k_1w_1(t))(C_1+k_3w_3(t)}-\frac{1}{(R_2+k_2w_2(t))(C_1+k_3w_3(t)}\right)dt$$
$$+v_{c_2}(t)\left(\frac{1}{(C_1+k_3w_3(t)}\times\frac{1}{(C_1+k_3w_3(t)}\right)dt$$
$$+\left(\frac{u_1(t)+k_5w_5(t)}{(R_1+k_1w_1(t))(C_1+k_3w_3(t)}\right)dt \tag{18}$$

$$dv_{c_2}(t) = v_{c_1}(t)\left(\frac{1}{(R_2+k_2w_2(t))}\times\frac{1}{(C_2+k_4w_4(t))}\right)dt$$
$$+v_{c_2}(t)\left(-\frac{1}{(R_2+k_2w_2(t))}\times\frac{1}{(C_2+k_4w_4(t))}\right)dt \tag{19}$$

Combining (14), (18) and (19), we have

$$dX(t) = A(t)X(t)dt + z(t)dt + K(t)dB(t) \tag{20}$$

Where

$$X(t) = [w_1(t) \quad w_2(t) \quad w_3(t) \quad w_4(t) \quad v_{c_1}(t) \quad v_{c_2}(t)]^T \tag{21}$$

$$A(t) = \begin{bmatrix} -\rho_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\rho_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\rho_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\rho_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & A_{55} & A_{56} \\ 0 & 0 & 0 & 0 & A_{65} & A_{66} \end{bmatrix} \tag{22}$$

where

$$A_{55} = -\frac{1}{(R_1+k_1w_1(t))(C_1+k_3w_3(t)}-\frac{1}{(R_2+k_2w_2(t))(C_1+k_3w_3(t)}$$
$$A_{56} = \frac{1}{(R_2+k_2w_2(t))(C_1+k_3w_3(t)}$$
$$A_{65} = \frac{1}{(R_2+k_2w_2(t))(C_2+k_4w_4(t))}$$
$$A_{66} = -\frac{1}{(R_2+k_2w_2(t))(C_2+k_4w_4(t))}$$

$$Z(t) = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{u_1(t)}{(R_1+k_1w_1(t))(C_1+k_3w_3(t)} & 0 \end{bmatrix}^T \tag{23}$$

$$dB(t) = [dB_1(t) \quad dB_2(t) \quad dB_3(t) \quad dB_4(t) \quad dB_5(t) \quad 0]^T \tag{24}$$

The discrete time state space matrix is obtained by using Euler-Maruyama method. Therefore, the discrete version of A and B are

$A_D(k) = e^{A\Delta T}$, $B_D(k) = \left(\int_0^{\Delta T} e^{A\tau}d\tau\right)B$ respectively.

The discrete state space model of above SDE becomes

$$X(k+1) = A(k)X(k) + z(k) + K(k)B(k) \tag{26}$$

$$Y(k) = C(k)X(k) \tag{27}$$

Where

$$X(k) = [w_1(k) \quad w_2(k) \quad w_3(k) \quad w_4(k) \quad v_{c_1}(k) \quad v_{c_2}(k)]^T \tag{28}$$

$$A(k) = \begin{bmatrix} 1-\Delta T\rho_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1-\Delta T\rho_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1-\Delta T\rho_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1-\Delta T\rho_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1+\Delta TA_{55} & A_{56} \\ 0 & 0 & 0 & 0_1 & A_{65} & 1+\Delta TA_{66} \end{bmatrix} \tag{29}$$

$$A_{55} = -\frac{1}{(R_1 + k_1 w_1(k))(C_1 + k_3 w_3(k)} - \frac{1}{(R_2 + k_2 w_2(k))(C_1 + k_3 w_3(k)}$$

$$A_{56} = \frac{1}{(R_2 + k_2 w_2(k))(C_1 + k_3 w_3(k)}$$

$$A_{65} = \frac{1}{(R_2 + k_2 w_2(k))(C_2 + k_4 w_4(k))}$$

$$A_{66} = -\frac{1}{(R_2 + k_2 w_2(k))(C_2 + k_4 w_4(k))}$$

$$Z(k) = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{\Delta T u_1(k)}{(R_1 + k_1 w_1(k))(C_1 + k_3 w_3(k)} & 0 \end{bmatrix}^T \tag{30}$$

$$K(k) = \begin{bmatrix} -\Delta T\sigma_1\rho_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -\Delta T\sigma_2\rho_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\Delta T\sigma_3\rho_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & -\Delta T\sigma_4\rho_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & \Delta Tk_{56} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{31}$$

$$K_{56} = \frac{k_5}{(R_1 + k_1 w_1(k))(C_1 + k_3 w_3(k)}$$

$$B(k) = [B_1(k) \quad B_2(k) \quad B_3(k) \quad B_4(k) \quad B_5(k) \quad 0]^T \tag{32}$$

$$C(k) = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{33}$$

3. EXTENDED KALMAN FILTER

EKF, an extension of KF, is used for nonlinear state estimation.

In general, nonlinear discrete time system is defined as

$$X(k) = f(X(k-1), u(k-1)) + w(k-1) \tag{33}$$

$$Y(k-1) = g(X(k-1), u(k-1)) + v(k-1) \tag{34}$$

where $f(.)$ and $g(.)$ are the nonlinear functions of input and previous state. $u(k)$ and $Y(k)$ are the control input and measured output respectively. $w(k)$ and $v(k)$ are the process and measurement noise (white Gaussian noise) with zero mean and covariance $R(k)$ and $Q(k)$.

Linear state space equation are obtained from (33)-(34) by applying partial derivatives of function f and g with respect to $x(k)$ and $u(k)$ to get Jacobian matrix.

The transformed equations are

$$X(k) = A(k-1)X(k-1) + B(k-1)u(k-1) + w(k-1) \tag{35}$$

$$Y(k-1) = C(k-1)X(k-1) + D(k-1)u(k-1) + v(k-1) \tag{36}$$

Where

$$A(k-1) = \frac{\partial f(X(k-1),u(k-1))}{\partial XP^{-}(k)}, B(k-1) = \frac{\partial f(X(k-1),u(k-1))}{\partial u(k-1)}$$

$$C(k-1) = \frac{\partial g(X(k-1),u(k-1))}{\partial x(k-1)}, \; D(k-1) = \frac{\partial g(X(k-1),u(k-1))}{\partial u(k-1)}$$

3. A. EKF general steps:

Estimation through EKF algorithm broadly consists of two steps

- Time update
- Measurement update

At first states $X(k-1)$ are initiated to some value $\hat{X}^{0}(k-1)$ with error covariance $\hat{P}^{0}(k-1)$.

1. Time update:

1 (a). State estimation time update:

$$\hat{X}^{-}(k) = A(k-1)\hat{X}^{+}(k-1) + B(k-1)u(k-1) + Q(k-1) \tag{37}$$

Where $\hat{X}^{-}(k)$ is the priori and posteriori state estimate at the time $k-1$ and k respectively. Superscript "–" and "+" represent priori and posteriori values that are approximated before and after measurement.

1(b).Error covariance time update:

$$P^{-}(k) = A(k-1)P^{+}(k-1)A^{T}(k-1) + Q(k-1) \tag{38}$$

Where $P^{-}(k)$ and $P^{+}(k-1)$ are the priori and posteriori error covariance time k and $k-1$ respectively.

1(c).Output state prediction:

$$\hat{Y}^{+}(k-1) = C(k-1)\hat{X}^{+}(k-1) + R(k-1) \tag{39}$$

2. Measurement update

2(a).Calculation of Kalman gain:

$$K_{k} = P^{-}(k)C^{T}(k-1)[C(k-1)P^{-}(k)C^{T}(k-1) + Q_{k-1}]^{-1} \tag{40}$$

Where K_k is the Kalman gain.

2(b).State estimate measurement update:

$$\hat{X}^{+}(k) = \hat{X}^{-}(k) + K_{k}\left[Y(k-1) - C(k-1)\hat{X}^{-}(k)\right] \tag{41}$$

Where, $\hat{X}^{+}(k)$ is the posteriori estimated state, $\hat{X}^{-}(k)$ is the priori estimated state and $Y(k)$ is the real time measured output.

2(c).Error covariance measurement update:

$$P^{+}(k) = [I - K_{k}C(k-1)]P^{-}(k) \tag{42}$$

In this step, posteriori error covariance, $P^{+}(k)$, is estimated. These steps are repeated until we get best approximated output.

3. B. Applying EKF in RC circuit:

For applying EKF to RC circuit equations (26) and (27) are modelled as

$$X(k) = A(k-1)X(k-1) + z(k-1) + K(k-1)B(k-1) + w(k-1) \quad (43)$$

$$Y(k-1) = C(k-1)X(k-1) + v(k-1) \quad (44)$$

Where stochastic differential equation (26) and (27) are added with process noise $w(k)$ and measurement noise $v(k)$. After that, step-1 (time update) followed by step 2 (measurement update) from equation (37) - (42) is applied to get best estimated output.

4. SIMULATION RESULTS

Output voltage estimation of RC LPF has been done by applying EKF using stochastic model of the circuit. RC circuit element values are $R_1 = 1M\Omega$, $R_2 = 1M\Omega$, $C_1 = 1uF$, $C_2 = 1uF$, $\sigma_j = 1$ and $\rho_j = 1$. Estimated output voltage is compared to simulated PSPICE data, which shows that estimated output through EKF is immune to white Gaussian noise. Fig. 2(a) gives comparison of EKF estimated output to simulated output when resistance R_1 is affected by white Gaussian noise. Similarly, Fig. 2(b), Fig. 2(c), Fig. 2(d), Fig. 2(e) gives comparison of simulated output and circuit elements resistance R_2, capacitance R_1, capacitance R_1, input source modelled by zero mean correlated process. Table I shows the SNR for different circuit elements modelled by different noise intensity. It shows that SNR is least when input source is affected by white Gaussian noise as compared to other circuit elements added with zero mean correlated process.

Table I. Signal to Noise Ratio For Different Circuit Element Modelled By White Gaussian Noise

S. No.	Circuit element intensity of noise	SNR (dB)
1.	$k_1 = 1$	56.22
2.	$k_2 = 1$	58.39
3.	$k_3 = 1$	62.46
4.	$k_4 = 1$	65.95
5.	$k_5 = 1$	50.64

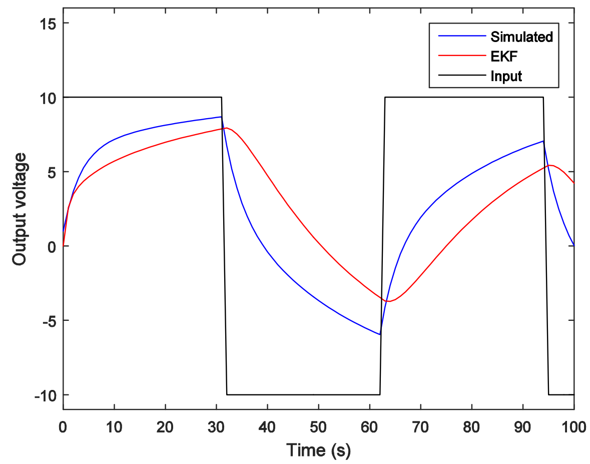

Fig. 2(a). RC LPF PSPICE simulated and stochastic estimated output for $k_1 = 1$ and other k's are zero.

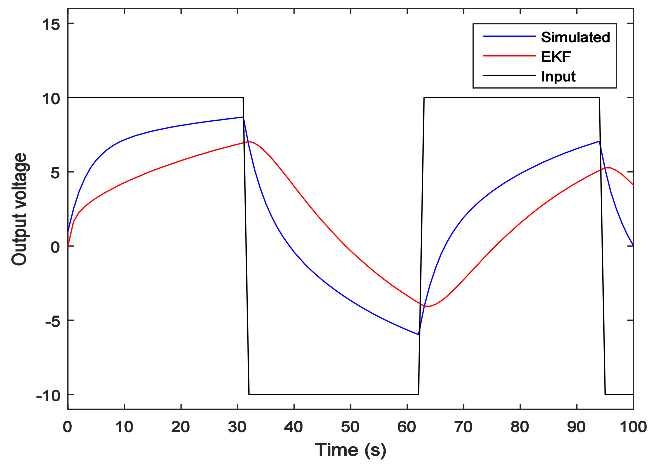

Fig. 2(b). RC LPF PSPICE simulated and stochastic estimated output for $k_2 = 1$ and other k's are zero.

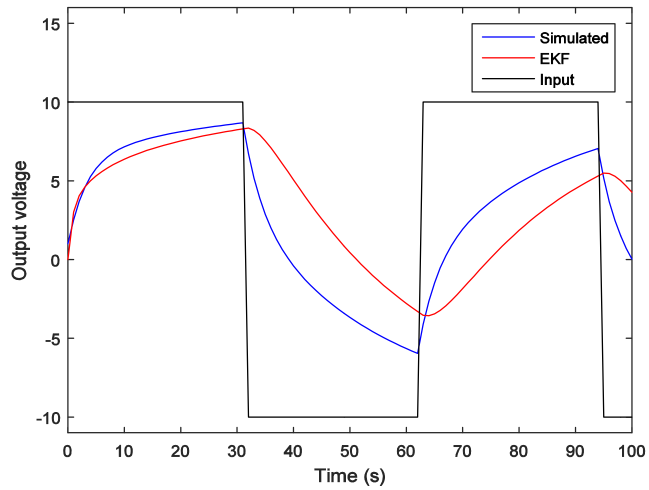

Fig. 2(d). RC LPF PSPICE simulated and stochastic estimated output for $k_3 = 1$ and other k's are zero.

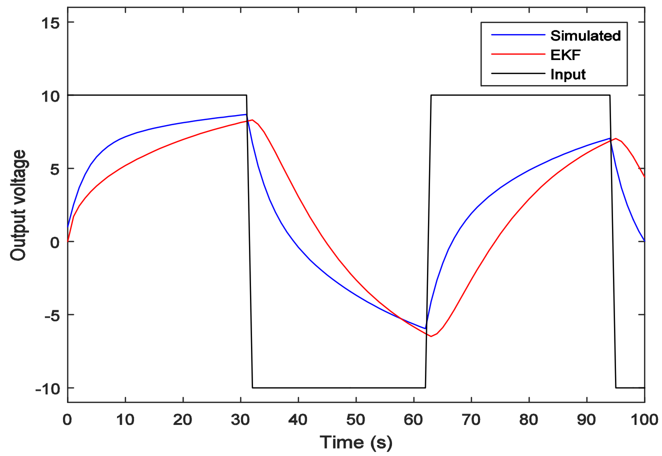

Fig. 2(e). RC LPF PSPICE simulated and stochastic estimated output for $k_4 = 1$ and other k's are zero.

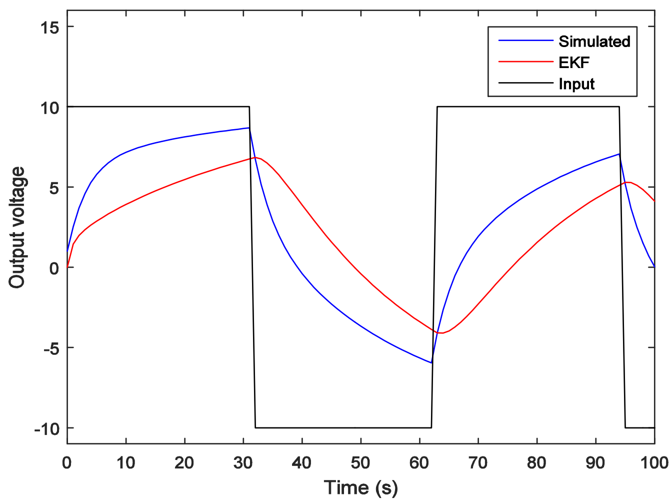

Fig. 2(c). RC LPF PSPICE simulated and stochastic estimated output for $k_5 = 1$ and other k's are zero.

5. CONCLUSION

Output voltage of RC LPF is estimated using EKF when input source and circuit elements are added with white Gaussian noise. MATLAB and PSPICE simulation results show that EKF work effectively in case of stochastic modelled RC circuit. SNR is computed for each case when either any of the circuit element is affected by white Gaussian noise. The advantage of the proposed method is that it gives good estimation as EKF is a stochastic approach of parameter estimation. Also, it requires small computations and easy to implement. Further, it can be used for real time implementation.

ACKNOWLEDGMENT

The first author would like to thank CSIR (Council for Scientific Industrial Research, Government of India) for financial support in terms of SRF (Senior Research Fellowship).

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