Domination Number of Sierpinski Wheel Graph of order n

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Abstract------*Domination in Graph has been widely researched branch of graph theory. In a graph, a domination set is a subset* S *of vertices such that every vertex is either in* S *or adjacent to a vertex in* S. *The domination number is the number of vertices in a smallest dominating set for graph* $G(V, E)$ *Sierpinski graphs are self similar pattern and repeats itself at different scales. In this paper we have investigated the domination number of Sierpinski Wheel graph of order* $n \geq 3$.

Keywords : *Domination Number, Generalised Sierpinski Graph, Wheel Graph*

1.1 Introduction

The domination number in Graphs has various important application in School bus routing, computer Communication networks, Radio Station, Land Surveying etc. The idea of dominating set in Graph came into light when the problem in the game of chess was being arisen during **1850**. The problem was, to determine minimum number of queens which can be placed on chess board such that all the squares are either occupied by a queen or attacked by a queen. In **1962 Ore** and **Berge** introduced the concept of domination in graph. The domination number in Graph has been further investigated by **Cockayne** and **Hedetniemi[12].**

Sierpinski Graphs are extremely Complex-meaning we can zoom in and find the same shapes forever. Generalised Sierpinski Graphs are kind of Sierpinski Graphs which were introduced by **Klavzar** and **Miltunovic** in **1997[11]**. They took Complete Graph K_k and formed Sierpinski Graph $S(n, K_k)$. They gave definition and iteration of Sierpinski Graph. The stage one Graph $S(1, K_k)$ is simply complete Graph, stage two Graph $S(2, K_k)$ is formed by copying $S(1, K_k)$ Graph k times and add one edge between each pair of $S(1, K_k)$. Repeating this process they found $S(3, K_k)$, $S(4, K_k)$, $S(5, K_k) \cdots\cdots$ $S(n, K_k)$. They also discussed Tower of Hanoi Problems. The Graph of Tower of Hanoi was $S(n, K_3)$ and it was isomorphic to Sierpinski triangle. In **2011 Gravier, Kovse** and **Aline** introduced new Graphs known as Generalised Sierpinski Graphs**[10]**. They replaced Complete Graph K_k by any Graph. In this paper, we have taken wheel Graph W_3, W_4, W_5, W_6, W_7, W_8, W_9, W_{10}, W_{11}, W_{12} and their Sierpinski graph.

2 Preliminaries

2.1 GENERALISED SIERPINSKI GRAPH

Genaralised Sierpinski graph was introduced by **Klavžar** and **Milutinovic** in **1997**. The Generalised Sierpinski graph of $\mathbf{G}$ of dimension $\mathbf{"n"}$ denoted by $\mathbf{S(n, G)}$ is the graph with vertex set $\{1, 2, 3, \cdots\cdots, n\}^n$ and edge set defined by : $\{u, v\}$ is an edge if and only if there exists $i \in \{1, 2, 3, \cdots\cdots, n\}$ such that :

- **i.** $u_j = v_j$ if $j < i$
- **ii.** $u_i = v_i$ and $(u_i, v_i) \in E(G)$
- **iii.** $u_j = v_j$ and $v_j = u_i$ if $j > i$

In other words, if $\{u, v\}$ is an edge of $S(n, G)$, there is an edge $\{x, y\}$ of G and a word "w" such that $u = wxy \ldots\ldots\ldots y$ and $v = wyx \ldots\ldots\ldots x$. We say that edge $\{u, v\}$ is using edge $\{x, y\}$ of G. Graphs $S(n, G)$ is can be constructed recursively from G with the following process: $S(1, G)$ is isomorphic to G. To construct $S(n, G)$ for $n > 1$, copy k times $S(n-1, G)$ and add to labels of vertices in copy x of $S(n-1, G)$ the letter x at the beginning. Then for any edge $\{x, y\}$ of G, add an edge between vertex $xy \cdots y$ and vertex $yx \cdots x$. For any word u of length d, with $1 \le d \le n$, the subgraph of $S(n, G)$ induced by vertices with label beginning by u, is isomorphic to $S(n-d, G)$. For a vertex x of G, we call extreme vertex x of $S(n, G)$ the vertex with label $x \cdots x$.

Example:-

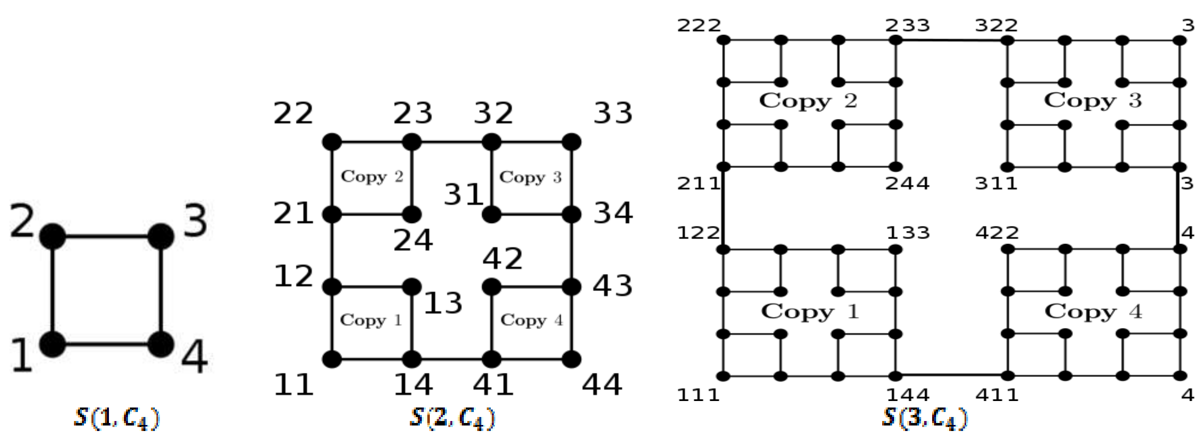


Fig 2.1 : Sierpinski 4 cycle Graph.

In this paper we have taken wheel graph of order 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 and following the above definition of Generalized Sierpinski graph we have drawn their Sierpinski Graph which are shown in **Table- 3.1 to 3.9.**

2.2 Domination number : A dominating Set for a graph $G = (V, E)$ is a subset D of vertex set V such that every vertex not in D is adjacent to at least one member of D. The domination number $\gamma(G)$ is the number of vertices in a smallest dominating set for G.

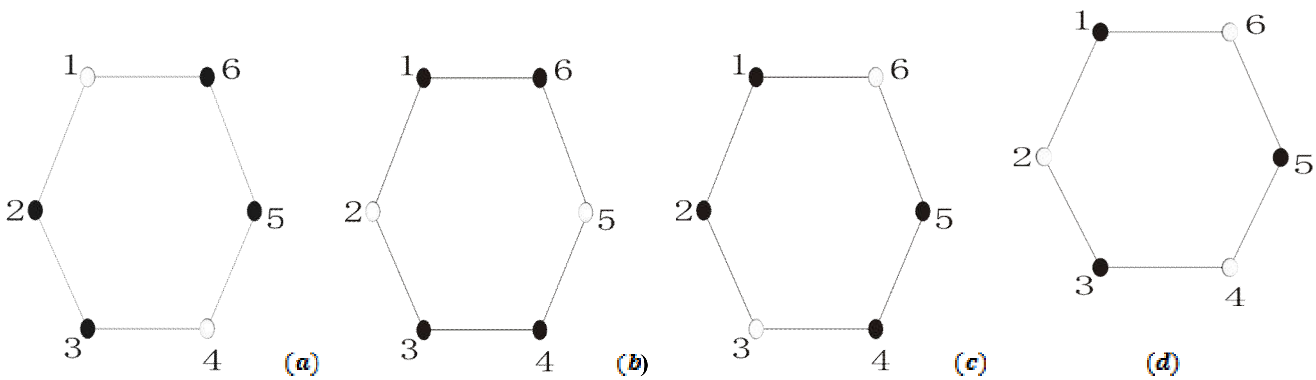


Fig 2.3: cycle Graph of order 6.

In this example, vertex set $V(G) = \{1, 2, 3, 4, 5, 6\}$ and subset for Graph (a), (b), (c) and (d) are $\{1, 4\}$, $\{2, 5\}$, $\{3, 6\}$, $\{2, 4, 6\}$. So its domination number $\gamma(G) = 2$.

2.3 Wheel Graph :

The Wheel Graph $W_n (n \geq 3)$ is obtained from Cycle Graph C_n by adding a vertex $'v'$ inside Cycle Graph C_n and connecting it to every vertex in C_n. It has $n + 1$ vertices and $2n$ edges.

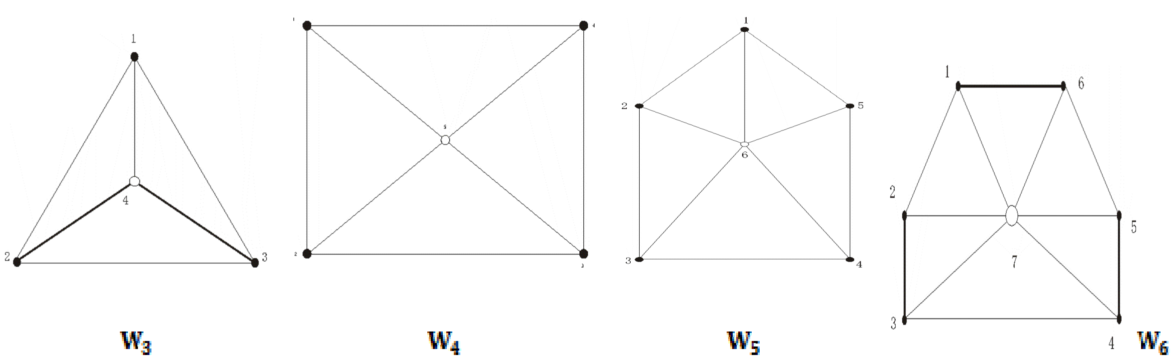
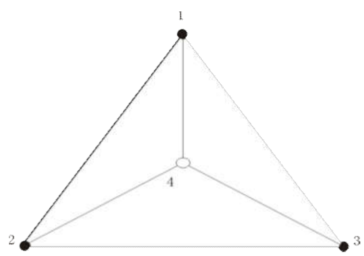
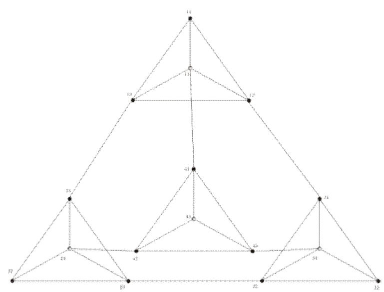


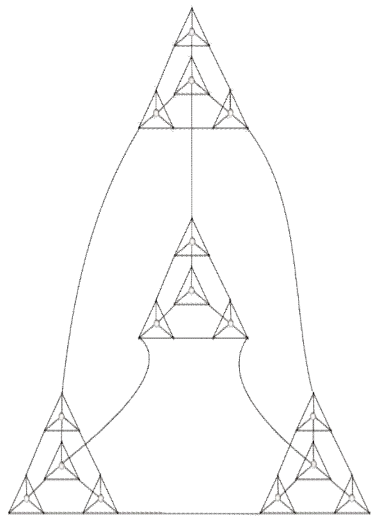
Fig 2.4: Wheel Graph

3 Domination Number of Sierpinski Wheel Graph of order 'n'

3.1 Sierpinski Wheel Graph of order '3' i.e. $S(m, W_3)$

Table 3.1

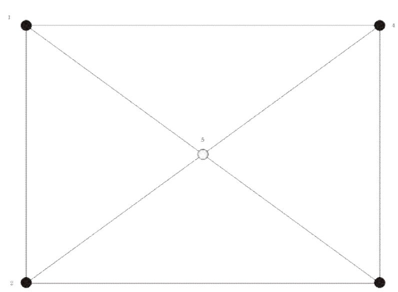
S. No.	Notation	Sierpinski Wheel Graph of order 3.	Dominating Set	Domination Number $(\gamma(S(m, W_3))$
1.	$S(1, W_3)$		In $S(1, W_3)$, the vertex Set is $V = \{1, 2, 3, 4\}$. Here we have taken a subset $D = \{4\} = \{white\ coloured\ vertex\}$ of vertex set $V\{S(1, W_3)\}$. D has minimum number of cardinality which dominates all vertices.	$\gamma(S(1, W_3)) = 1$
2.	$S(2, W_3)$		Similarly, In $S(2, W_3)$, Here we have taken a subset $D = \{All\ white\ coloured\ vertices\ of\ S(2, W_3)\}$ of vertex set $V\{S(2, W_3)\}$. D has minimum number of cardinality which dominate all vertices of $S(2, W_3)$. Here total number of white coloured vertices is **4** which is the domination number.	$\gamma(S(2, W_3)) = 4$

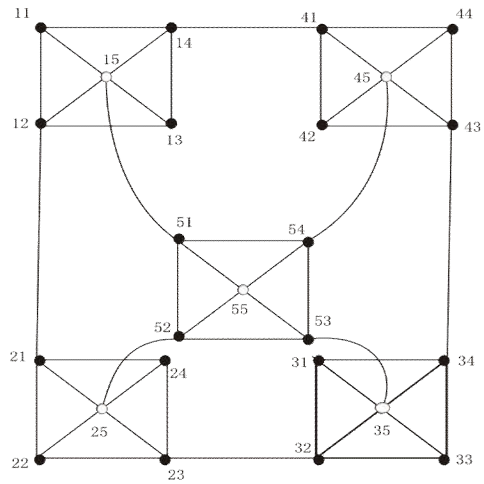
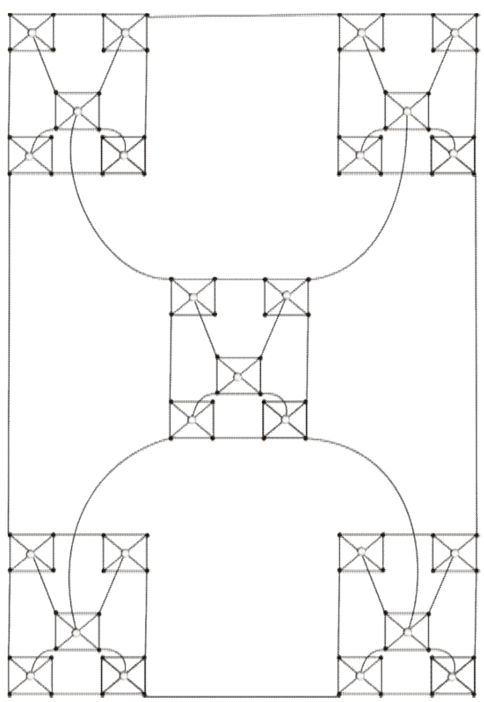
3.	$\mathbf{S(3, W_3)}$		Again, In $\mathbf{S(3, W_3)}$, Here we have taken a subset $\boldsymbol{D = \{All\ white\ coloured\ vertices\ of\ S(3, W_3)\}}$ of vertex set $\mathbf{V\{S(3, W_3)\}}$. $\boldsymbol{D}$ has minimum number of cardinality which dominate all vertices of $\mathbf{S(3, W_3)}$. Here each $\mathbf{S(2, W_3)}$ contains **4** white coloured vertices. Therefore, the total number of white coloured vertices of $\mathbf{S(3, W_3)}$ will be **16** which is Domination number.	$\boldsymbol{\gamma(S(3, W_3)) = 16}$

Since the figure of $\mathbf{S(4, W_3)}$ is very large. But we can see in figure $\mathbf{S(3, W_3)}$, it has four $\mathbf{S(2, W_3)}$ and each has **4** white coloured vertices. Therefore, total number of white coloured vertices of $\mathbf{S(3, W_3)}$ will be **16**. we have taken a subset $\boldsymbol{D = \{All\ white\ coloured\ vertices\ of\ S(3, W_3)\}}$ of vertex set $\boldsymbol{V\{S(3, W_3)\}}$. Since $\boldsymbol{D}$ has minimum number of cardinality which dominate all vertices of $\mathbf{S(3, W_3)}$, therefore the domination number of $\mathbf{S(3, W_3)}$ will be $\mathbf{4 \times 4 = 16}$ = all white coloured vertices of $\mathbf{S(3, W_3)}$. Similarly we can find the domination number of $\mathbf{S(4, W_3)}$. In figure $\mathbf{S(4, W_3)}$ there will be **four** $\mathbf{S(3, W_3)}$ and each has **16** white coloured vertices. Therefore the domination number of $\mathbf{S(4, W_3)}$ will be $\mathbf{4 \times 16 = 64}$. Proceeding in this way the domination number of Sierpinski wheel graph of order **3 i.e.** $\mathbf{S(m, W_3)}$ will be $\mathbf{(3+1)^{m-1}}$ .The domination number of $\mathbf{S(1, W_3)}$ **to** $\mathbf{S(m, W_3)}$ has been tabulated in **Table- 4.1.**

3. 2 Sierpinski Wheel Graph of order '4' i.e. $\boldsymbol{S(m, W_4)}$.

Table 3.2

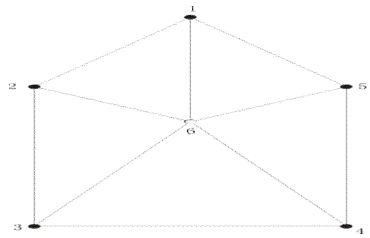
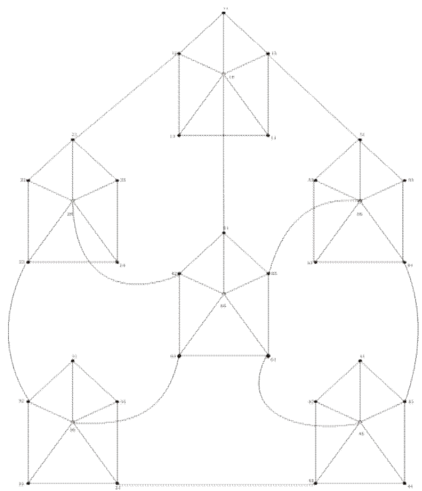
S. No .	**Notation**	**Sierpinski Wheel Graph of order '4**	**Dominating Set**	$\boldsymbol{(\gamma(S(m, W_4))}$
1.	$\mathbf{S(1, W_4)}$		**In** $\mathbf{S(1, W_4)}$, the vertex Set is $\boldsymbol{V = \{1, 2, 3, 4, 5\}}$. Here we have taken a subset $\boldsymbol{D = \{5\} = \{white\ coloured\ vertex\}}$ of vertex set $\mathbf{V\{S(1, W_3)\}}$. $\boldsymbol{D}$ has minimum number of	$\boldsymbol{\gamma(S(1, W_4)) = 1}$

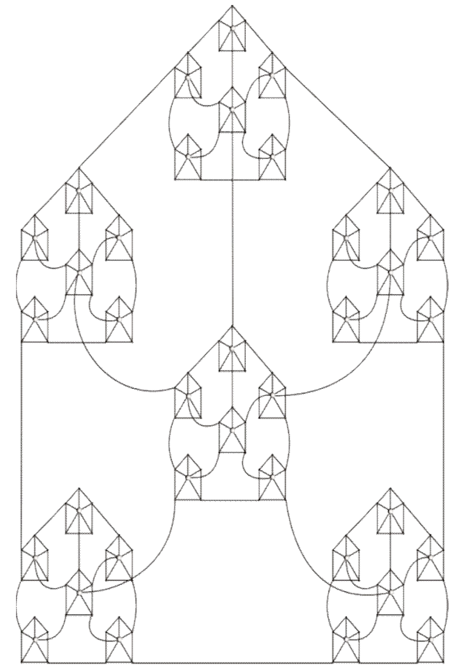
			cardinality which dominate all vertices.	
2.	$\mathbf{S(2, W_4)}$		**Similarly, In $S(2, W_4)$, Here we have taken a subset $D = \{All\ white\ coloured\ vertices\ of\ S(2, W_4)\}$ of vertex set $V\{S(2, W_4)\}$. D has minimum number of cardinality which dominate all vertices of $S(2, W_4)$. Here each $S(1, W_4)$ contains 1 white coloured vertex. Therefore.The total number of white coloured vertices of $S(2, W_4)$ will be 5 which is the domination number.**	$\gamma(S(2, W_4)) = 5$
3.	$\mathbf{S(3, W_4)}$		Again, In $S(3, W_4)$, Here we are taking a subset $D = \{All\ white\ coloured\ vertices\ of\ S(3, W_4)\}$ of vertex set $V\{S(3, W_4)\}$. D has minimum number of cardinality which dominate all vertices of $S(3, W_4)$. Here each $S(2, W_4)$ contains **5** white coloured vertices. Therefore, the total number of white coloured vertices of $S(3, W_4)$ will be **25** which is the Domination number.	$\gamma(S(3, W_4)) = 25$

Since the figure of $S(4, W_4)$ is very large. But we can see in figure $S(3, W_4)$, it has **five** $S(2, W_4)$ and each has **5** white coloured vertices. Therefore, total number of white coloured vertices of $S(3, W_4)$ will be **25**. we have taken a subset $\boldsymbol{D = \{All\ white\ coloured\ vertices\ of\ S(3, W_4)\}}$ of vertex set $\boldsymbol{V\{S(3, W_4)\}}$. Since $\boldsymbol{D}$ has minimum number of cardinality which dominate all vertices of $\mathbf{S(3, W_4)}$, therefore the domination number of $\mathbf{S(3, W_4)}$ will be $\mathbf{5 \times 5 = 25}$ = all white coloured vertices of $\mathbf{S(3, W_4)}$. Similarly we can find the domination number of $\mathbf{S(4, W_4)}$. In figure $\mathbf{S(4, W_4)}$ there will be **five** $\mathbf{S(3, W_4)}$ and each has **25** white coloured vertices. Therefore the domination number of $\mathbf{S(4, W_4)}$ will be $\mathbf{5 \times 25 = 125}$. Proceeding in this way the domination number of Sierpinski wheel graph of order **4 i.e.** $\mathbf{S(m, W_4)}$ will be $\mathbf{(4 + 1)^{m-1}}$ .The domination number of $\mathbf{S(1, W_4)}$ **to** $\mathbf{S(m, W_4)}$ has been tabulated in **Table- 4.1.**

3.3 Sierpinski Wheel Graph of order '5' i.e. $S(m, W_5)$.

Table 3.3

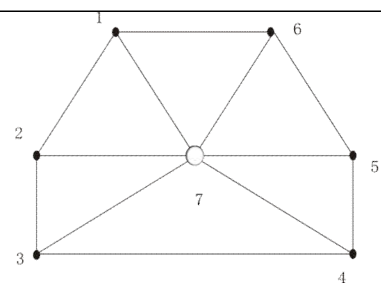
S. No.	Notation	Sierpinski Wheel Graph of order '5'	Dominating set	$(\gamma(S(m, W_5))$
1.	$S(1, W_5)$		In $S(1, W_5)$, the vertex Set is $V = \{1, 2, 3, 4, 5, 6\}$. Here we have taken a subset $D = \{6\} = \{white\ coloured\ vertex\}$ of vertex set $V\{S(1, W_5)\}$. D has minimum number of cardinality which dominate all vertices.	$\gamma(S(1, W_5)) = 1$
2.	$S(2, W_5)$		**Similarly,In $S(2, W_5)$, Here we have taken a subset $D = \{All\ white\ coloured\ vertices\ of\ S(2, W_5)\}$ of vertex set $V\{S(2, W_5)\}$. D has minimum number of cardinality which dominates all vertices of $S(2, W_5)$. Here each $S(1, W_5)$ contains 1 white coloured vertex. Therefore.The total number of white coloured vertices of $S(2, W_3)$ will be 6 which is the domination number.**	$\gamma(S(2, W_5)) = 6$

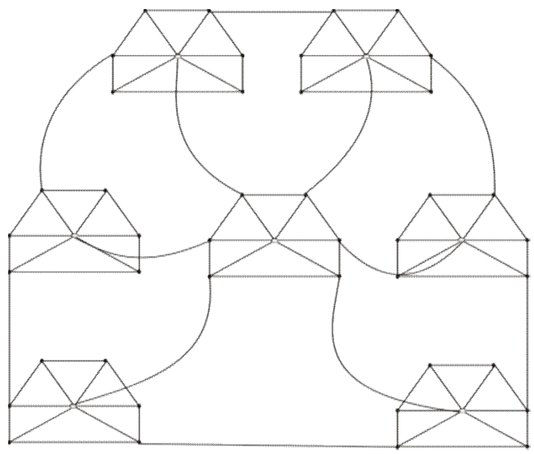
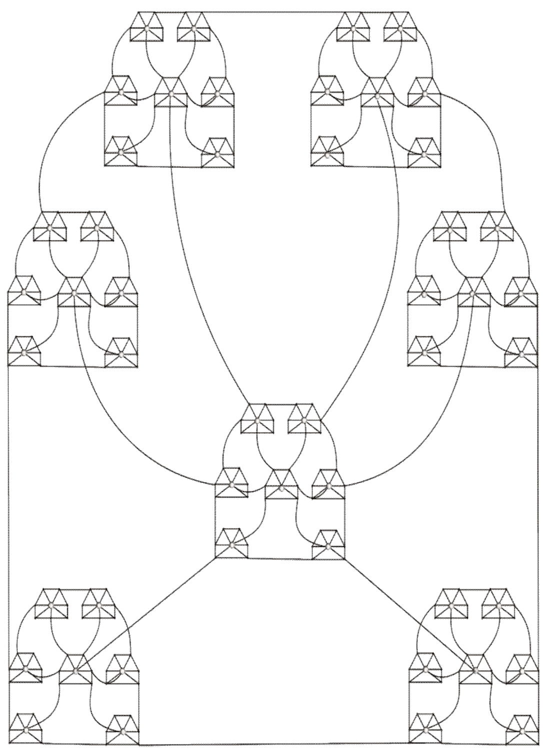
3.	$S(3, W_5)$		**Again, In $S(3, W_5)$, Here we have taken a subset $D = \left\{\begin{matrix} All\ white\ coloured \\ vertices\ of\ S(3, W_5)\end{matrix}\right\}$ of vertex set $V\{S(3, W_5)\}$. D has minimum number of cardinality which dominate all vertices of $S(3, W_5)$. Here each $S(2, W_5)$ contains 6 white coloured vertices. Therefore, the total number of white coloured vertices of $S(3, W_5)$ will be 36 which is the Domination number.**	$\gamma(S(3, W_5)) = 36$

Since the figure of $S(4, W_5)$ is very large. But we can see in figure $S(3, W_5)$, it has **six** $S(2, W_5)$ and each has **6** white coloured vertices. Therefore, total number of white coloured vertices of $S(3, W_5)$ will be **36**. we have taken a subset $D = \{All\ white\ coloured\ vertices\ of\ S(3, W_5)\}$ of vertex set $V\{S(3, W_5)\}$. Since D has minimum number of cardinality which dominates all vertices of $S(3, W_5)$, therefore the domination number of $S(3, W_5)$ will be $6 \times 6 = 36$ = all white coloured vertices of $S(3, W_5)$. Similarly we can find the domination number of $S(4, W_5)$. In figure $S(4, W_5)$ there will be **six** $S(3, W_5)$ and each has **36** white coloured vertices. Therefore the domination number of $S(4, W_5)$ will be $6 \times 36 = 216$. Proceeding in this way the domination number of Sierpinski wheel graph of order **5** **i.e.** $S(m, W_5)$ will be $(5 + 1)^{m-1}$ .The domination number of $S(1, W_5)$ **to** $S(m, W_5)$ has been tabulated in **Table- 4.1.**

3.4 Sierpinski Wheel Graph of order '6' i.e. $S(m, W_6)$.

Table-3.4

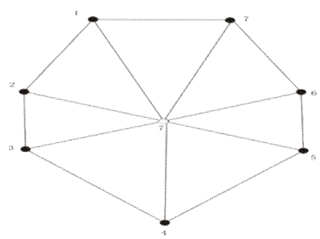
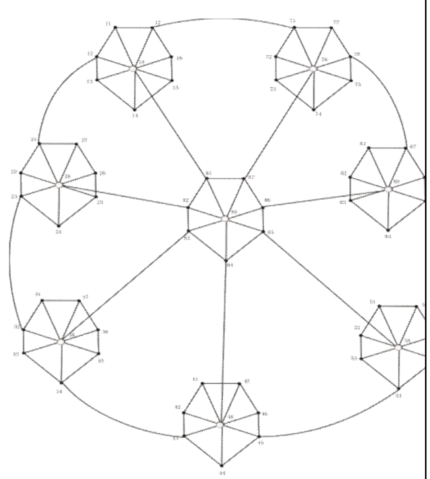
S. No.	Notation	Sierpinski Wheel Graph of order '6'	Dominating set	$(\gamma(S(m, W_6))$
1.	$S(1, W_6)$		In $S(1, W_6)$, the vertex Set is $V = \{1, 2, 3, 4, 5, 6, 7\}$. Here we have taken a subset $D = \{7\} = \{white\ coloured\ vertex\}$ of vertex set $V\{S(1, W_6)\}$. D has minimum number of cardinality which dominate all vertices.	$\gamma(S(1, W_6)) = 1$

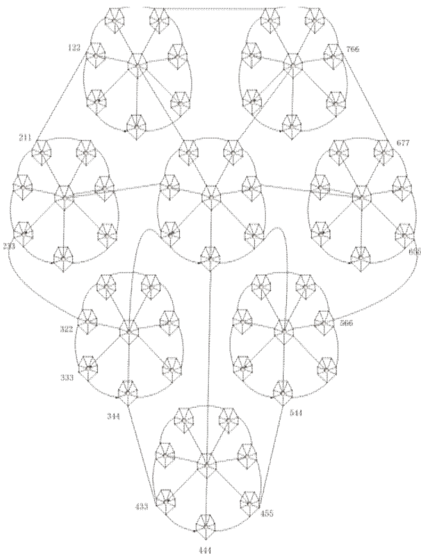
2.	$S(2,W_6)$		Similarly,In $S(2,W_6)$, Here we have taken a subset $D = \left\{\begin{matrix} All\ white\ coloured\ vertices \\ of\ S(2,W_6) \end{matrix}\right\}$ of vertex set $V\{S(2,W_6)\}$. D has minimum number of cardinality which dominate all vertices of $S(2,W_6)$. Here each $S(1,W_6)$ contains 1 white coloured vertex. Therefore.The total number of white coloured vertices of $S(2,W_6)$ will be 7 which is the domination number.	$\gamma(S(2,W_6)) = 7$
3.	$S(3,W_6)$		Again, In $S(3,W_6)$, Here we have taken a subset $D = \left\{\begin{matrix} All\ white\ coloured \\ vertices\ of\ S(3,W_6) \end{matrix}\right\}$ of vertex set $V\{S(3,W_6)\}$. D has minimum number of cardinality which dominate all vertices of $S(3,W_6)$. Here each $S(2,W_6)$ contains 4 white coloured vertices. Therefore, the total number of white coloured vertices of $S(3,W_6)$ will be 49 which is the Domination number.	$\gamma(S(3,W_6)) = 49$

Since the figure of $S(4,W_6)$ is very large. But we can see in figure $S(3,W_6)$, it has **seven** $S(2,W_6)$ and each has **7** white coloured vertices. Therefore, total number of white coloured vertices of $S(3,W_6)$ will be **49**. we have taken a subset $D = \{All\ white\ coloured\ vertices\ of\ S(3,W_6)\}$ of vertex set $V\{S(3,W_6)\}$. Since D has minimum number of cardinality which dominates all vertices of $S(3,W_6)$, therefore the domination number of $S(3,W_6)$ will be $\mathbf{7 \times 7 = 49}$ = all white coloured vertices of $S(3,W_6)$. Similarly we can find the domination number of $S(4,W_6)$. In figure $S(4,W_6)$ there will be **seven** $S(3,W_6)$ and each has **49** white coloured vertices. Therefore the domination number of $S(4,W_6)$ will be $\mathbf{7 \times 49 = 343}$. Proceeding in this way the domination number of Sierpinski wheel graph of order **6 i.e.** $S(m, W_6)$ will be $(6+1)^{m-1}$ .The domination number of $S(1, W_6)$ **to** $S(m, W_6)$ has been tabulated in **Table- 4.1.**

3.5 Sierpinski Wheel Graph of order '7' i.e. $S(m, W_7)$.

Table-3.5

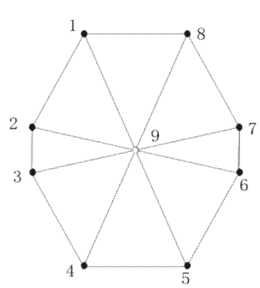
S. No	Notation	Sierpinski Wheel Graph of order '7'	Dominating set	$(\gamma(S(m, W_7))$
1.	$S(1, W_7)$		In $S(1, W_7)$, the vertex Set is $V = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Here we have taken a subset $D = \{8\} = \{white\ coloured\ verte$ of vertex set $V\{S(1, W_7)\}$. D has minimum number of cardinality which dominate all vertices.	$\gamma(S(1, W_7)) = 1$
2.	$S(2, W_7)$		**Similarly,In $S(2, W_7)$, Here we have taken a subset $D = \begin{Bmatrix} All\ white\ coloured \\ vertices\ of\ S(2, W_7) \end{Bmatrix}$ of vertex set $V\{S(2, W_7)\}$. D has minimum number of cardinality which dominates all vertices of $S(2, W_7)$. Here each $S(1, W_7)$ contains 1 white coloured vertex. Therefore.The total number of white coloured vertices of $S(2, W_7)$ will be 8 which is the domination number.**	$\gamma(S(2, W_7)) = 8$

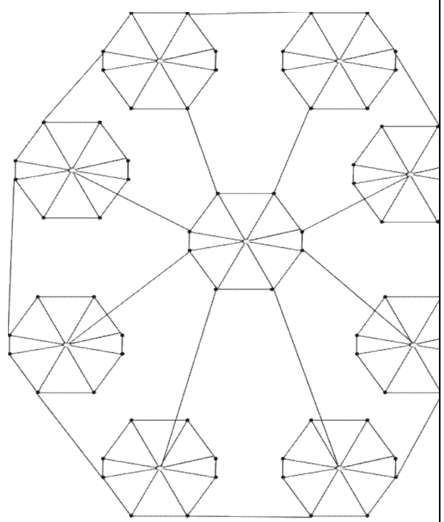
3.	$S(3,W_7)$		**Again, In $S(3,W_7)$, Here we have taken a subset $D = \left\{\begin{matrix} All\ white\ coloured \\ vertices\ of\ S(3,W_7) \end{matrix}\right\}$ of vertex set $V\{S(3,W_7)\}$. D has minimum number of cardinality which dominate all vertices of $S(3,W_7)$. Here each $S(2,W_7)$ contains 8 white coloured vertices. Therefore, the total number of white coloured vertices of $S(3,W_7)$ will be 64 which is the Domination number.**	$\gamma(S(3,W_7)) = 6$

Since the figure of $S(4,W_7)$ is very large. But we can see in figure $S(3,W_7)$, it has **eight** $S(2,W_7)$ and each has **8** white coloured vertices. Therefore, total number of white coloured vertices of $S(3,W_7)$ will be **64**. we have taken a subset $D = \{All\ white\ coloured\ vertices\ of\ S(3,W_7)\}$ of vertex set $V\{S(3,W_7)\}$. Since D has minimum number of cardinality which dominate all vertices of $S(3,W_7)$, therefore the domination number of $S(3,W_7)$ will be $8 \times 8 = 64$ = all white coloured vertices of $S(3,W_7)$. Similarly we can find the domination number of $S(4,W_7)$. In figure $S(4,W_7)$ there will be **eight** $S(3,W_7)$ and each has **64** incentre. Therefore the domination number of $S(4,W_7)$ will be $8 \times 64 = 512$. Proceeding in this way the domination number of Sierpinski wheel graph of order **7 i.e.** $S(m,W_7)$ will be $(7+1)^{m-1}$.The domination number of $S(1,W_7)$ **to** $S(m,W_7)$ has been tabulated in **Table- 4.1.**

3.6 Sierpinski Wheel Graph of order '8' i.e. $S(m,W_8)$.

Table-3.6

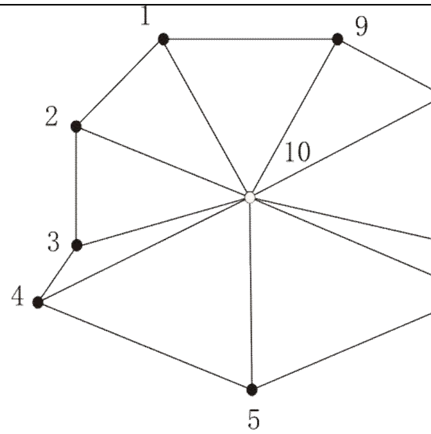
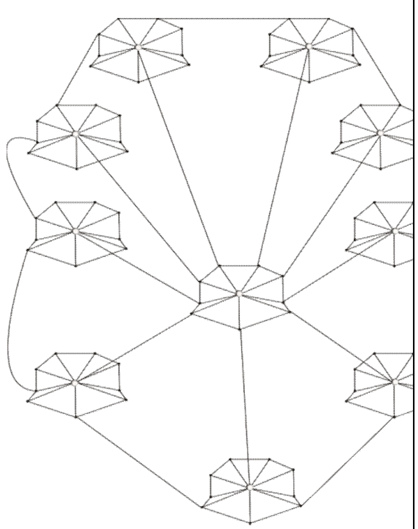
S. No.	Notation	Sierpinski Wheel Graph of order '8'	Dominating set	$(\gamma(S(m,W_8))$
1.	$S(1,W_8)$		In $S(1,W_8)$, the vertex Set is $V = \{1,2,3,4,5,6,7,8,9\}$. Here we have taken a subset $D = \{9\} = \{white\ coloured\ ver$ of vertex set $V\{S(1,W_8)\}$. D has minimum number of cardinality	$\gamma(S(1,W_8)) =$

			which dominate all vertices.	
2.	$S(2, W_8)$		Similarly,In $\mathbf{S(2,W_8)}$, Here we have taken a subset $D = \left\{ \begin{matrix} \textit{All white coloured} \\ \textit{vertices of } \mathbf{S(2,W_8)} \end{matrix} \right\}$ of vertex set $\mathbf{V\{S(2,W_8)\}}$. $\boldsymbol{D}$ has minimum number of cardinality which dominate all vertices of $\mathbf{S(2,W_8)}$. Here each $\mathbf{S(1,W_8)}$ contains 1 white coloured vertex. Therefore.The total number of white coloured vertices of $\mathbf{S(2,W_8)}$ will be **9** which is the domination number.	$\gamma(S(2,W_8)) =$

Since the figure of $\mathbf{S(3,W_8)}$ is very large. But we can see in figure $\mathbf{S(2,W_8)}$, it has **nine** $\mathbf{S(1,W_8)}$ and each has **1** white coloured vertex. Therefore, total number of white coloured vertices of $\mathbf{S(2,W_8)}$ will be **9**. we have taken a subset $\boldsymbol{D = \{All\ white\ coloured\ vertices\ of\ S(2,W_8)\}}$ of vertex set $\boldsymbol{V\{S(2,W_8)\}}$. Since $\boldsymbol{D}$ has minimum number of cardinality which dominate all vertices of $\mathbf{S(2,W_0)}$, therefore the domination number of $\mathbf{S(2,W_0)}$ will be $\mathbf{1 \times 9 = 9}$ = all white coloured vertices of $\mathbf{S(2,W_8)}$. Similarly we can find the domination number of $\mathbf{S(3,W_8)}$. In figure $\mathbf{S(3,W_8)}$ there will be **nine** $\mathbf{S(2,W_8)}$ and each has **9** white coloured vertices. Therefore the domination number of $\mathbf{S(3,W_8)}$ will be $\mathbf{9 \times 9 = 81}$. Proceeding in this way the domination number of Sierpinski wheel graph of order **8 i.e.** $\mathbf{S(m,W_8)}$ will be $\mathbf{(8+1)^{m-1}}$. The domination number of $\mathbf{S(1,W_8)}$ **to** $\mathbf{S(m,W_8)}$ has been tabulated in **Table- 4.2.**

3.7 Sierpinski Wheel Graph of order '9' i.e. $S(m, W_9)$.

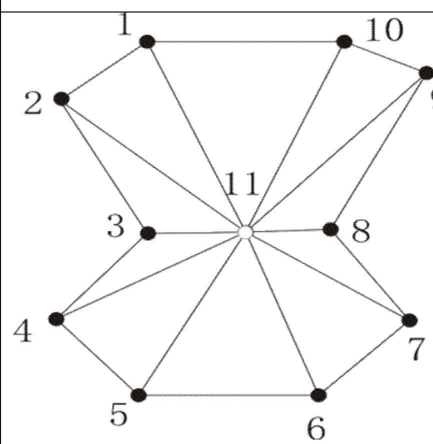
Table-3.7

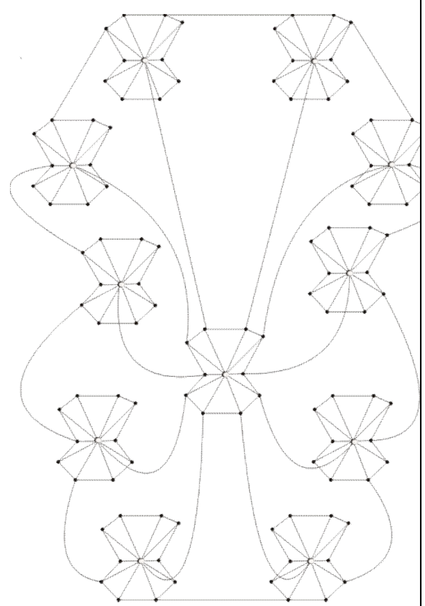
S. No.	Notation	Sierpinski Wheel Graph of order '9'	Dominating set	$\gamma(S(m,W_9))$
1.	$S(1,W_9)$		In $S(1,W_9)$, the vertex Set is $V = \{1,2,3,4,5,6,7,8,9,10\}$. Here we have taken a subset $D = \{10\} = \{white\ coloured\ ve$ of vertex set $V\{S(1,W_9)\}$. D has minimum number of cardinality which dominates all vertices of $S(1,W_9)$.	$\gamma(S(1,W_9)) = 1$
2.	$S(2,W_9)$		**Similarly,In $S(2,W_9)$, Here we have taken a subset $D = \begin{Bmatrix} All\ white\ coloured \\ vertices\ of\ S(2,W_9) \end{Bmatrix}$ of vertex set $V\{S(2,W_9)\}$. D has minimum number of cardinality which dominate all vertices of $S(2,W_9)$. Here each $S(1,W_9)$ contains 1 white coloured vertex. Therefore.The total number of white coloured vertices of $S(2,W_9)$ will be 10 which is the domination number.**	$\gamma(S(2,W_9)) = 1$

Since the figure of $\mathbf{S(3, W_9)}$ is very large. But we can see in figure $\mathbf{S(2, W_9)}$, it has **ten** $\mathbf{S(1, W_9)}$ and each has **1** white coloured vertex. Therefore, total number of white coloured vertices of $\mathbf{S(2, W_9)}$ will be **10**. we have taken a subset $\boldsymbol{D = \{All\ white\ coloured\ vertices\ of}\ \mathbf{S(2, W_9)}\}$ of vertex set $\boldsymbol{V}\{\mathbf{S(2, W_9)}\}$. Since $\boldsymbol{D}$ has minimum number of cardinality which dominates all vertices of $\mathbf{S(2, W_9)}$, therefore the domination number of $\mathbf{S(2, W_9)}$ will be $\mathbf{1 \times 10 - 10}$ = all white coloured vertices of $\mathbf{S(2, W_9)}$. Similarly we can find the domination number of $\mathbf{S(3, W_9)}$. In figure $\mathbf{S(3, W_9)}$ there will be **ten** $\mathbf{S(2, W_9)}$ and each has **10** white coloured vertices. Therefore the domination number of $\mathbf{S(3, W_9)}$ will be $\mathbf{10 \times 10 - 100}$. Proceeding in this way the domination number of Sierpinski wheel graph of order **9 i.e.** $\mathbf{S(m, W_9)}$ will be $\mathbf{(9 + 1)^{m-1}}$ .The domination number of $\mathbf{S(1, W_9)}$ **to** $\mathbf{S(m, W_9)}$ has been tabulated in **Table- 4.2.**

3.8 Sierpinski Wheel Graph of order '10' i.e. $S(m, W_{10})$.

Table-3.8

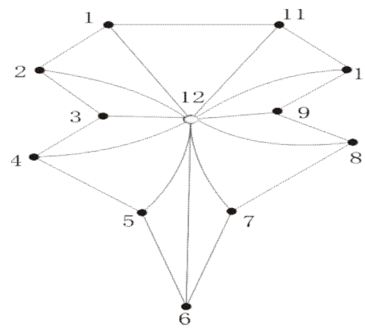
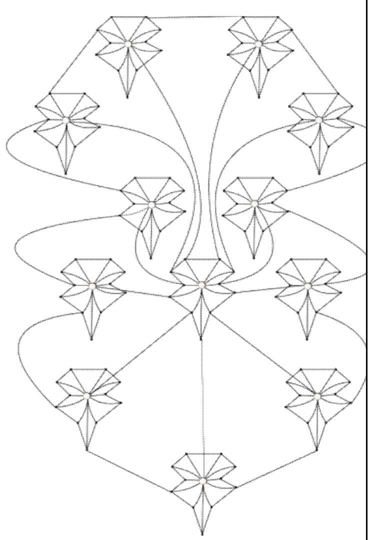
S. No.	Notation	Sierpinski Wheel Graph of order '10'	Dominating set	$(\gamma(S(m, W_{10}))$
1.	$S(1, W_{10})$		In $\mathbf{S(1, W_{10})}$, the vertex Set is $\boldsymbol{V} = \{\mathbf{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}\}$. Here we have taken a subset $\boldsymbol{D} = \{\mathbf{11}\} = \{$**white coloured ver** of vertex set $\mathbf{V}\{\mathbf{S(1, W_{10})}\}$. $\boldsymbol{D}$ has minimum number of cardinality which dominate all vertices.	$\gamma(S(1, W_{10})) = 1$

2.	$S(2, W_{10})$		Similarly,In $S(2, W_{10})$, Here we have taken a subset $D = \left\{ \begin{matrix} All \text{ white coloured} \\ \text{vertices } of\ S(2, W_{10}) \end{matrix} \right\}$ of vertex set $V\{S(2, W_{10})\}$. D has minimum number of cardinality which dominates all vertices of $S(2, W_{10})$. Here each $S(1, W_{10})$ contains 1 white coloured vertex. Therefore.The total number of white coloured vertices of $S(2, W_{10})$ will be 11 which is the domination number.	$\gamma(S(2, W_{10})) = 1$

Since the figure of $\mathbf{S(3, W_{10})}$ is very large. But we can see in figure $\mathbf{S(2, W_{10})}$, it has **eleven** $\mathbf{S(1, W_{10})}$ and each has **1** white coloured vertex. Therefore, total number of white coloured vertices of $\mathbf{S(2, W_{10})}$ will be **11**. we have taken a subset $\boldsymbol{D = \{All\ white\ coloured\ vertices\ of}\ \mathbf{S(2, W_{10})\}}$ of vertex set $V\{\mathbf{S(2, W_{10})}\}$. Since $\boldsymbol{D}$ has minimum number of cardinality which dominates all vertices of $\mathbf{S(2, W_{10})}$, therefore the domination number of $\mathbf{S(2, W_{10})}$ will be $\mathbf{1 \times 11 = 11}$ = all white coloured vertices of $\mathbf{S(2, W_{10})}$. Similarly we can find the domination number of $\mathbf{S(3, W_{10})}$. In figure $\mathbf{S(3, W_{10})}$ there will be **eleven** $\mathbf{S(2, W_{10})}$ and each has **11** white coloured vertices. Therefore the domination number of $\mathbf{S(3, W_{10})}$ will be $\mathbf{11 \times 11 = 121}$. Proceeding in this way the domination number of Sierpinski wheel graph of order **10 i.e.** $\mathbf{S(m, W_{10})}$ will be $\mathbf{(10 + 1)^{m-1}}$. The domination number of $\mathbf{S(1, W_{10})}$ **to** $\mathbf{S(m, W_{10})}$ has been tabulated in **Table- 4.2.**

3.9 Sierpinski Wheel Graph of order ‘11’ i.e. $S(m, W_{11})$.

Table-3.9

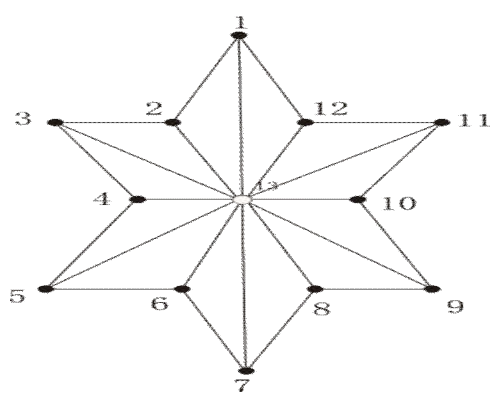
S. No.	Notation	Sierpinski Wheel Graph of order 11’	Dominating set	$(\gamma(S(m, W_{11}))$
1.	$S(1, W_{11})$		In $S(1, W_{11})$, the vertex Set is $V = \{1,2,3,4,5,6,7,8,9,10,11,12\}$. Here we have taken a subset $D = \{12\} = \{$white coloured ver of vertex set $V\{S(1, W_{11})\}$. D has minimum number of cardinality which dominates all vertices.	$\gamma(S(1, W_{11})) = 1$
2.	$S(2, W_{11})$		**Similarly,In $S(2, W_{11})$, Here we have taken a subset $D = \left\{\begin{matrix} All\ \text{white coloured} \\ \text{vertices } of\ S(2, W_{11}) \end{matrix}\right\}$ of vertex set $V\{S(2, W_{11})\}$. D has minimum number of cardinality which dominates all vertices of $S(2, W_{11})$. Here each $S(1, W_{11})$ contains 1 white coloured vertex. Therefore.The total number of white coloured vertices of $S(2, W_{11})$ will be 12 which is the domination number.**	$\gamma(S(2, W_{11})) = 1$

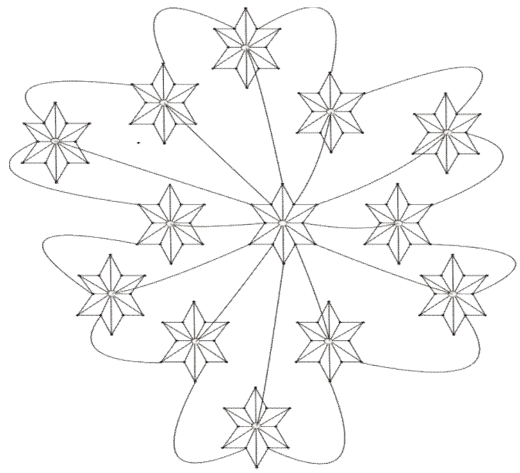
Since the figure of $\mathbf{S(3, W_{11})}$ is very large. But we can see in figure $\mathbf{S(2, W_{11})}$, it has **twelve** $\mathbf{S(1, W_{11})}$ and each has **1** white coloured vertex. Therefore, total number of white coloured vertices of $\mathbf{S(2, W_{11})}$ will be **12**. we have taken a subset

$D - \{All\ white\ coloured\ vertices\ of\ \mathbf{S(2,W_{11})}\}$ of vertex set $V\{\mathbf{S(2,W_{11})}\}$. Since D has minimum number of cardinality which dominates all vertices of $\mathbf{S(2,W_{11})}$, therefore the domination number of $\mathbf{S(2,W_{11})}$ will be $\mathbf{1 \times 12 = 12}$ = all white coloured vertices of $\mathbf{S(2,W_{11})}$. Similarly we can find the domination number of $\mathbf{S(3,W_{11})}$. In figure $\mathbf{S(3,W_{11})}$ there will be **twelve** $\mathbf{S(2,W_{11})}$ and each has **12** white coloured vertices. Therefore the domination number of $\mathbf{S(3,W_{11})}$ will be $\mathbf{12 \times 12 = 144}$. Proceeding in this way the domination number of Sierpinski wheel graph of order **11 i.e.** $\mathbf{S(m,W_{11})}$ will be $\mathbf{(11+1)^{m-1}}$. The domination number of $\mathbf{S(1,W_{11})}$ **to** $\mathbf{S(m,W_{11})}$ has been tabulated in **Table- 4.2.**

3.10 Sierpinski Wheel Graph of order '12' i.e. $S(m,W_{12})$.

Table-3.10

S. No.	Notation	Sierpinski Wheel Graph of order '12'	Dominating set	$(\gamma(S(m,W_{12})$
1.	$S(1,W_{12})$		In $\mathbf{S(1,W_{12})}$, the vertex Set is $V = \{\mathbf{1,2,3,4,5,6,7,8,9,10,11,12}\}$. Here we have taken a subset $D = \{4\} = \{\textbf{white coloured v}$ of vertex set $\mathbf{V\{S(1,W_{12})\}}$. D has minimum number of cardinality which dominates all vertices of $\mathbf{S(1,W_{12})}$.	$\gamma(S(1,W_{12}))$

2.	$S(2,W_{12})$		Similarly,In $\mathbf{S(2,W_{12})}$, Here we have taken a subset $D = \left\{\begin{matrix} \boldsymbol{All\ \textbf{white coloured}} \\ \textbf{vertices}\ \boldsymbol{of}\ \mathbf{S(2,W_{12})} \end{matrix}\right\}$ of vertex set $\mathbf{V\{S(2,W_{12})\}}$. $\boldsymbol{D}$ has minimum number of cardinality which dominates all vertices of $\mathbf{S(2,W_{12})}$. Here each $\mathbf{S(1,W_{12})}$ contains **1 white coloured vertex**. Therefore.The total number of **white coloured vertices** of $\mathbf{S(2,W_{12})}$ will be **13** which is the domination number.	$\gamma(S(2,W_{12}))$

Since the figure of $\mathbf{S(3,W_{12})}$ is very large. But we can see in figure $\mathbf{S(2,W_{12})}$, it has **thirteen** $\mathbf{S(1,W_{12})}$ and each has **1** white coloured vertex. Therefore, total number of white coloured vertices of $\mathbf{S(2,W_{12})}$ will be **13**. we have taken a subset $\boldsymbol{D = \{All\ white\ coloured\ vertices\ of}\ \mathbf{S(2,W_{12})}\}$ of vertex set $\boldsymbol{V}\{\mathbf{S(2,W_{12})}\}$. Since $\boldsymbol{D}$ has minimum number of cardinality which dominates all vertices of $\mathbf{S(2,W_{12})}$, therefore the domination number of $\mathbf{S(2,W_{12})}$ will be $\mathbf{1 \times 13 = 13}$ = all white coloured vertices of $\mathbf{S(2,W_{12})}$. Similarly we can find the domination number of $\mathbf{S(3,W_{12})}$. In figure $\mathbf{S(3,W_{12})}$ there will be **thirteen** $\mathbf{S(2,W_{12})}$ and each has **13** white coloured vertices. Therefore the domination number of $\mathbf{S(3,W_{12})}$ will be $\mathbf{13 \times 13 = 169}$. Proceeding in this way the domination number of Sierpinski wheel graph of order **12 i.e.** $\mathbf{S(m,W_{12})}$ will be $\mathbf{(12+1)^{m-1}}$. The domination number of $\mathbf{S(1,W_{12})}$ **to** $\mathbf{S(m,W_{12})}$ has been tabulated in **Table- 4.2.**

4. RESULTS

From Table 3.1 to 3.10, we conclude that

Table-4.1

Domination Number	W_3	W_4	W_5	W_6	W_7
$\gamma(S\,(1,G))$	1	1	1	1	1
$\gamma(S\,(2,G))$	$(3+1)^{2-1}$	$(4+1)^{2-1}$	$(5+1)^{2-1}$	$(6+1)^{2-1}$	$(7+1)^{2-1}$
$\gamma(S\,(3,G))$	$(3+1)^{3-1}$	$(4+1)^{3-1}$	$(5+1)^{3-1}$	$(6+1)^{3-1}$	$(7+1)^{3-1}$
$\gamma(S\,(4,G))$	$(3+1)^{4-1}$	$(4+1)^{4-1}$	$(5+1)^{4-1}$	$(6+1)^{4-1}$	$(7+1)^{4-1}$
…	…	…	…	…	…
…	…	…	…	…	…
…	…	…	…	…	…
$\gamma(S\,(m,G))$	$(3+1)^{m-1}$	$(4+1)^{m-1}$	$(5+1)^{m-1}$	$(6+1)^{m-1}$	$(7+1)^{m-1}$

Table-4.2

Domination Number	W_8	W_9	W_{10}	W_{11}	W_{12}
$\gamma(S\,(1,G))$	1	1	1	1	1
$\gamma(S\,(2,G))$	$(8+1)^{2-1}$	$(9+1)^{2-1}$	$(10+1)^{2-1}$	$(11+1)^{2-1}$	$(12+1)^{2-1}$
$\gamma(S\,(3,G))$	$(8+1)^{3-1}$	$(9+1)^{3-1}$	$(10+1)^{3-1}$	$(11+1)^{3-1}$	$(12+1)^{3-1}$
$\gamma(S\,(4,G))$	$(8+1)^{4-1}$	$(9+1)^{4-1}$	$(10+1)^{4-1}$	$(11+1)^{4-1}$	$(12+1)^{4-1}$
…	…	…	…	…	…
…	…	…	…	…	…
…	…	…	…	…	…
$\gamma(S\,(m,G))$	$(8+1)^{m-1}$	$(9+1)^{m-1}$	$(10+1)^{m-1}$	$(11+1)^{m-1}$	$(12+1)^{m-1}$

Conclusion :

We have found domination number of Sierpinski wheel graph of order 3 to 12. Therefore we conclude that the domination Number of Sierpinski Wheel Graph of order $'n'$ is given by

- $\gamma(S(m, W_n)) = (n+1)^{m-1}, where\ n = 3,4,5,\cdots\ and\ m = 1,2,3,\cdots$

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