FOURIER SERIES OF THE PERIODIC –EULER'S FUNCTIONS

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ABSTRACT:--

Euler's formula is an important concept in Fourier series. We discuss the Euler's formulae using Periodic functions.

INTRODUCTION

Fourier series is an infinite series representation of a periodic function in terms of sines and cosines of an angle and its multiples. Fourier series is possible for continuous, periodic functions and functions discontinuous in their values and derivatives. Fourier series is useful to solve ODE and PDE particularly with periodic functions appearing as non-homogeneous. The numbers and polynomials of Euler are very useful in complex analysis of mathematics.

KEYWORDS:--Fourier coefficients, fourier series ,period,sine and cosine functions.

PERIODIC FUNCTION :A function $f(\theta)$ is said to be periodic if $f(\theta+p) = f(\theta)$ for all θ and for some positive number p , p is known as the period of $f(\theta)$

Example,

i. $\text{Sin}(\theta+2\pi) = \sin\theta$

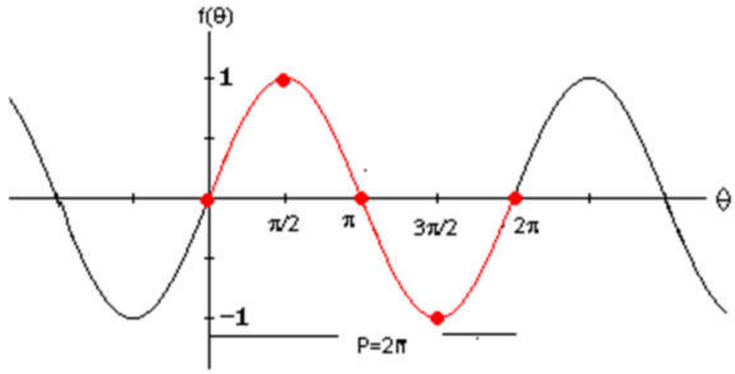
ii. $\cos(\theta+2\pi) = \cos\theta$

iii. $\tan(\theta+\pi) = \tan\theta$

iv. $\cot(\theta+\pi) = \cot\theta$

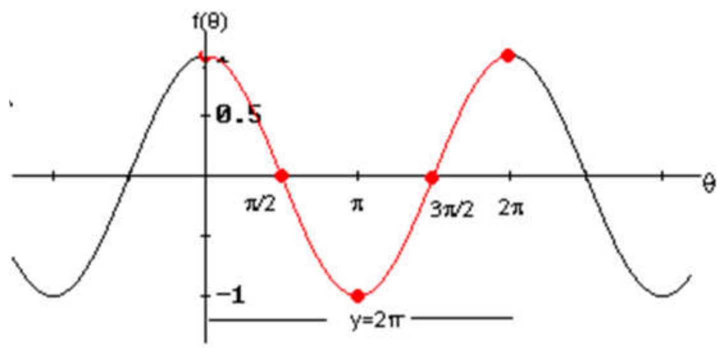
So, that $\sin\theta$, $\cos\theta$ are periodic functions with period 2π and $\tan\theta$, $\cot\theta$ are periodic functions with period π.

Graph of Y=sin θ:-



Therefore the period of sin θ is 2π

Graph of Y=cos θ:-



There fore the period of cos θ is 2π

Suppose that a given function f(θ) defined in the interval $[-\pi, \pi]$ or $[0, 2\pi]$ can be expressed as

$$f(\theta) = \frac{x_0}{2} + x_1 \cos\theta + x_2 \cos 2\theta + x_3 \cos 3\theta + \ldots + x_n \cos n\theta + \ldots +$$

$$y_1 \sin\theta + y_2 \sin 2\theta + y_3 \sin 3\theta + \ldots + y_n \sin n\theta + \ldots$$

$$f(\theta) = \frac{x_0}{2} + (x_1 \cos\theta + y_1 \sin\theta) + (x_2 \cos 2\theta + y_2 \sin 2\theta) + \ldots + (x_n \cos n\theta + y_n \sin n\theta) + \ldots$$

$$f(\theta) = \frac{x_0}{2} + \sum_{n=1}^{\infty}(x_n \cos n\theta + y_n \sin n\theta) \text{-----------------------------------}(1)$$

where x's and y's are constants with in a desired range of values of the variable θ .

Such series is known as the Fourier series for $f(\theta)$ and constants $x_0, x_1, x_2, x_3, \ldots x_n$, $y_1, y_2, y_3, \ldots y_n$ are called Fourier coefficients of $f(\theta)$

EULER'S FORMULAE:-

The fourier series for the function $f(\theta)$ in the interval $T \leq \theta \leq T+2\pi$ is given by

$$f(\theta) = \frac{x_0}{2} + \sum_{n=1}^{\infty}(x_n \cos n\theta + y_n \sin n\theta) \text{----------------------------------} (2)$$

where

$$x_0 = \frac{1}{\pi}\int_T^{T+2\pi} f(\theta)\, d\theta,$$

$$x_n = \frac{1}{\pi}\int_T^{T+2\pi} f(\theta) \cos n\theta\, d\theta \text{ and}$$

$$y_n = \frac{1}{\pi}\int_T^{T+2\pi} f(\theta) \sin n\theta\, d\theta.$$

These values of x_0, x_n, y_n are known as Euler's formulae.

To Evaluate x_0 :-

Integrating on both sides of equation (2) from $\theta = T$ to $\theta = T+2\pi$, We get

$$\int_T^{T+2\pi} f(\theta) d\theta = \frac{x_0}{2}\int_T^{T+2\pi} d\theta + \sum_{n=1}^{\infty}\left(x_n \int_T^{T+2\pi} \cos n\theta\, d\theta + y_n \int_T^{T+2\pi} \sin n\theta\, d\theta\right)$$

$$= \frac{x_0}{2}[\theta]_T^{T+2\pi} + \sum_{n=1}^{\infty}\left[x_n\left(\frac{\sin n\theta}{n}\right)_T^{T+2\pi} + y_n\left(\frac{-\cos n\theta}{n}\right)_T^{T+2\pi}\right]$$

$$= \frac{x_0}{2}[T+2\pi - T] + \sum_{n=1}^{\infty}\left[x_n\left\{\frac{\sin n(T+2\pi)}{n} - \frac{\sin nT}{n}\right\} + y_n\left\{-\frac{\cos n(T+2\pi)}{n} + \frac{\cos nT}{n}\right\}\right]$$

$$= 2\pi\frac{x_0}{2} + \sum_{n=1}^{\infty}\left[x_n\left\{\frac{\sin nT}{n} - \frac{\sin nT}{n}\right\} + y_n\left\{-\frac{\cos nT}{n} + \frac{\cos nT}{n}\right\}\right]$$

$$= \pi x_0 + \sum_{n=1}^{\infty}(0)$$

$$= \pi x_0$$

Therefore,

$$X_0 = \frac{1}{\pi}\int_T^{T+2\pi} f(\theta) d\theta.$$

To Evaluate x_n:--

Multiply on both sides of equation (2) by cos mθ and apply integrating from θ=T to θ = T+2π,We get

$$\int_T^{T+2\pi} f(\theta)\cos m\theta d\theta = \frac{x_0}{2}\int_T^{T+2\pi} \cos m\theta\, d\theta + \sum_{n=1}^{\infty}(x_n \int_T^{T+2\pi} \cos n\theta \cos m\theta d\theta + y_n \int_T^{T+2\pi} \sin n\theta \cos m\theta d\theta)$$

The first and third integrals on right hand side are always equal to zero , but the second integral is equal to π when m=n . Otherwise it also disclose when m$\neq$ n, so that

$$\int_T^{T+2\pi} f(\theta)\cos n\theta d\theta = \sum_{n=1}^{\infty}(x_n \int_T^{T+2\pi} \cos n\theta \cos n\theta d\theta) \quad \text{(since m=n)}$$

$$=\sum_{n=1}^{\infty}(x_n \int_T^{T+2\pi} \cos^2 n\theta d\theta)$$

$$=\sum_{n=1}^{\infty} x_n \int_T^{T+2\pi} \frac{1+\cos 2n\theta}{2} d\theta$$

$$=\frac{1}{2}\sum_{n=1}^{\infty} x_n \int_T^{T+2\pi} (1+\cos 2n\theta) d\theta$$

$$=\frac{1}{2}\sum_{n=1}^{\infty} x_n \left[\theta + \frac{\sin 2n\theta}{2n}\right]_T^{T+2\pi}$$

$$=\frac{1}{2}\sum_{n=1}^{\infty} x_n \left[T + 2\pi + \frac{\sin 2n(T+2\pi)}{2n} - \left(T+\frac{\sin 2nT}{2n}\right)\right]$$

$$=\frac{1}{2}\sum_{n=1}^{\infty} x_n \left[2\pi + \frac{\sin 2n(T+2\pi)}{2n} - \frac{\sin 2nT}{2n}\right]$$

$$=\frac{1}{2}\sum_{n=1}^{\infty} x_n \left[2\pi + \frac{\sin 2nT}{2n} - \frac{\sin 2nT}{2n}\right] \quad \text{(since sine period is } 2\pi \text{)}$$

$$=\frac{1}{2}\sum_{n=1}^{\infty} x_n [2\pi]$$

$$=\pi x_n$$

Therefore,

$$X_n = \frac{1}{\pi}\int_T^{T+2\pi} f(\theta) \cos n\theta d\theta$$

To Evaluate y_n : --

Multiply on both sides of equation (2) by sin mθ and apply integrating from θ=T to θ = T+2π,We get

$$\int_T^{T+2\pi} f(\theta)\sin m\theta d\theta = \frac{x_0}{2}\int_T^{T+2\pi} \sin m\theta\, d\theta + \sum_{n=1}^{\infty}(x_n \int_T^{T+2\pi} \cos n\theta \sin m\theta d\theta + y_n \int_T^{T+2\pi} \sin n\theta \sin m\theta d\theta)$$

The first two integrals on right hand side are always equal to zero. But third integral is equal to π

When m=n . Otherwise it also disclose when m $\neq$ n , so that,

$$\int_T^{T+2\pi} f(\theta) sin\, n\theta d\theta = \frac{x_0}{2}(0)+x_n(0)+y_n\int_T^{T+2\pi} sin\, n\theta sin\, n\theta\, d\theta \quad (\text{ since m=n})$$

$$=y_n\int_T^{T+2\pi} sin^2\, n\theta\, d\theta$$

$$=y_n\int_T^{T+2\pi} \frac{1-cos2\, n\theta}{2}\, d\theta$$

$$=\frac{y_n}{2}\int_T^{T+2\pi} [1-cos2\, n\theta]\, d\theta$$

$$=\frac{y_n}{2}[\theta-\frac{sin2n\theta}{2n}]_T^{T+2\pi}$$

$$=\frac{y_n}{2}[T+2\pi-\frac{sin\, 2n(T+2\pi)}{2n}-(T-\frac{sin\, 2nT}{2n})]$$

$$=\frac{y_n}{2}[2\pi-\frac{sin\, 2nT}{2n}+\frac{sin\, 2nT}{2n}] \quad (\text{since sine period is } 2\pi)$$

$$=\pi y_n$$

Therefore ,

$$y_n = \frac{1}{\pi}\int_T^{T+2\pi} f(\theta)\, sin\, n\theta d\theta$$

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