Comparative Study of Free and Forced vibration experimentally and Mathematical Modeling Using MAT Lab

Dr.B.Rajanesh Kumar
Associate Professor, Dept: of Mechanical Engineering B.M.S Evening College Of Engineering , Bangalore-19

ABSTRACT

Vibration studies have become increasingly important to mechanical engineers due to demands placed on performance of mechanical systems. Vibration often adversely affects performance due to noise and fatigue failures. Vibration studies are also important in positive applications such as vibratory conveyors. The main objective is to study the free and forced vibration experimentally using longitudinal spring-mass-damper system and compare the mathematical development through MATLAB software. Also to compare the experimental vibration signature with reference vibration signature, Time-Displacement curve is obtained theoretically using MATLAB programming Finally for few cases theoretical and practical vibration results are compared and the results from experiment correlated well with theoretical predictions.

Key words: Vibration, free and forced vibration, spring-mass-damper Time-Displacement curve and MAT Lab

INTRODUCTION

Vibration tends to have plenty of advantages and a lot more disadvantages, stirring up engineers worldwide to put a lot effort to use its advantages and curtail it disadvantages. “Vibration", a term that tends to induce the same in one’s mind when someone hears it. Invariably everything vibrates in the world, some vibrations are good and useful, some are really negligible, and some are tolerable, some annoying and the rest fall under the dangerous category.

In recent times, many investigations have been motivated by the engineering applications of vibration, such as the design of machines, foundations, structures, engines, turbines, and control systems. Most prime movers have vibrational problems due to the inherent unbalance in the engines. The unbalance may be due to faulty design or poor manufacture. Whenever the natural frequency of vibration in a machine or structure increases the **amplitude** increases in turn the Resonance also shoots up and finally leads to the failure of machine.

EXPERIMENTATION

In order to obtain the solution experimentally the graph-plotting mechanism which finally converts the vertical vibratory motion of the spring-mass-damper system into graphical representation on the TIME DOMAIN that is time-displacement curve. A constant initial displacement is given to system in each case in order to avoid complexity in the solution.

Experimental Procedure for Damped Free Vibration

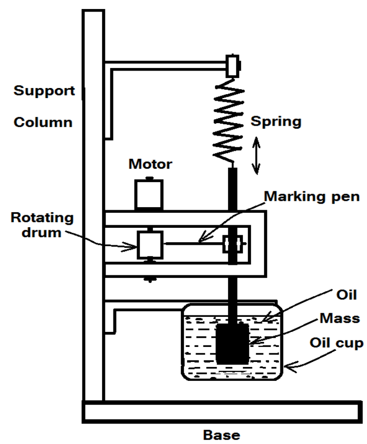


Fig1: Schematic Experimental Setup for Free Damped Vibration System

Procedure:

- Find stiffness of the spring (slope of load Vs deflection curve – undamped system).
- Fix the pen to the vibrating platform and graph sheet of the drum recorder.
- Adjust the drum recorder in vertical position perfectly perpendicular to the axis of the spring so that, the pen tip makes contact with the drum.
- Apply the initial force by pressing the platform downward manually.
- Start the drum recorder and release the platform.
- Form the graphs obtained measure the amplitude of the two successive cycles and hence calculate the logarithmic decrement, damping factor and other parameters

Observations

Radius of recording drum r = 70 mm
Speed of recording drum N = 6 rpm
Mass M = 11.6450 kg

Undamped free vibration:

Undamped free vibration experiment is conducted in order to find the stiffness of the spring.

Table 1 Undamped Natural Frequency

Sl. No.	Weight		Deflection mm	Stiffness	Undamped natural Frequency
	M		x	K	f_n
	Kg	N	m	N/m	(Hz)
1	0	0	0	4456.5	3.1151 (i.e 186.9 RPM)
2	11.5850	113.6489	0.0242		
3	15.9000	155.9790	0.0379		
4	20.2150	198.3092	0.0458		
5	24.5300	240.6393	0.0517		

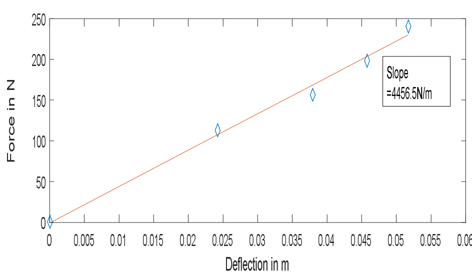


Fig: 2 Load Vs Deflection Curve for Spring

Specimen calculation:

Undamped free vibration:

i) Stiffness of spring, K = Slope of load deflection curve = 4456.5 N/m

ii) Natural frequency with a damped mass m, $f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$ Hz

=3.1151 Hz

Nn =186.9 Rpm

Damped free vibration

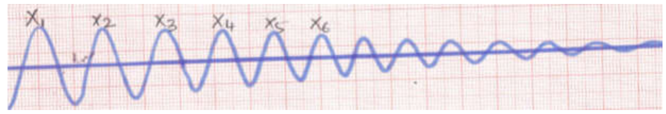


Fig 3 Free Damped Vibration Amplitude Graph

Table2 Damped Natural Frequency

Sl. No.	From graph Amplitudes	Amplitude ratio	Average Amplitude ratio	Logarithmic decrement	Damping ratio	Critical Damping Co efficient	Damping Co efficient	Damped frequency	
	x_i	(x_i/x_{i+1})	(x_i/x_{i+1}) Avg	δ	ξ	Cc	C	F_{nd}	ω_{nd}
						Ns/m	Ns/m	Hz	rad/s
1 2 3 4 5 6	x_1=11.5 x_2=10.5 x_3=9.5 x_4=8.5 x_5=7.5 x_6=6.6	x_1/x_2= 1.1048 x_2/x_3 =1.1053 x_3/x_4 =1.1176 x_4/x_5= 1.1333 x_5/x_6= 1.1364	1.1195	0.1129	0.018	455.6136	8.1824	3.11	19.55

Average of successive amplitude 1.1195
Logarithmic decrement (from graph)

$$\delta = \ln\left(\frac{x_n}{x_{n+1}}\right) = 0.1129$$

Damping ratio (ξ) $\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}}$ i.e

$$\xi = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}} = 0.0180$$

Undamped natural frequency = $\omega_n = \sqrt{k/m}$
= 19.5626 rad. / sec

Critical damping coefficient C_c= $2m\omega_n$
=455.6136 Ns/m

Actual damping coefficient C = ξ C_c
= 8.1284 Ns/m

Damped natural frequency $\omega_d = \omega_n\sqrt{1-\xi^2}$
= 19.5595 rad/sec

$$f_d = f_n\sqrt{1-\xi^2} = 3.1151 \text{ Hz}$$

Experimental Procedure for Damped Forced Vibration System:

Fig4:

Experimental Setup for Forced Damped Vibration System

Procedure:

- Find stiffness of the spring (slope of load Vs deflection curve – undamped system).
- Find damping coefficient of the damper by conducting free damped vibration experiment as discussed earlier
- Note down the vibrating mass' M'(Includes the vibrating platform and disturbing masses 'm')
- Fix the pen to the vibrating platform and graph sheet of the drum recorder.
- Subject the mass to sinusoidal excitation at a particular frequency ω
- Adjust the drum recorder in vertical position perfectly perpendicular to the axis of the spring so that, the pen tip makes contact with the drum.
- From the graph obtained measure the amplitude of vibration at steady state.
- Repeat steps from 5 for various values of ω

Observations

Exciting Mass m = 0.240 kg

Vibrating Mass M = 11.645 kg

Stiffness of the Spring K = 4456.5 N/m

Damping coefficient C = 8.1824 N S/m

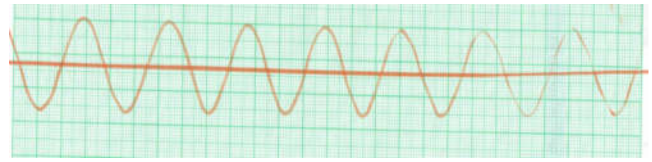
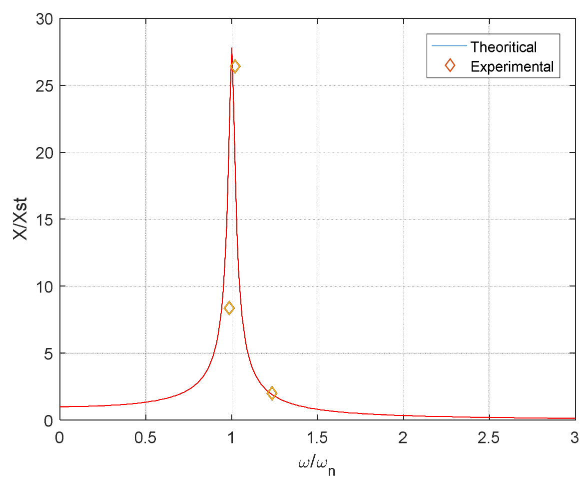


Fig 5 Forced Damped Vibration Amplitude Graph for 190 rpm

Table 3 Magnification factor for Damped Forced Vibration
Fig 6 Magnification factor

Sl. No.	Frequency of Excitation	Angular velocity of Excitation	Ratio	Deflection	Static Deflection	Magnification Factor	
	N rpm	ω rad/s	ω/ω_n	X	X_{ST}	MF_{ex}	MF_{th}
1	0	0	0	0	0	0	0
2	184	19.2684	0.9850	0.0040	0.4799	8.3356	21.5746
3	190	19.8968	1.0171	0.0135	0.5117	26.3840	19.8908
4	230	24.0855	1.2312	0.0015	0.7498	2.0006	1.9314



Theoretical v/s Experimental

Error in percentage

1.61.3637 %

2. -30.0967 %

3. -0.3205 %

>> Since point 1 & 2 are very close to natural frequency , percentage of error more.

Calculations

ω=2*3.14 *N/60 = 19.2684 rad/sec
Amplitude of Excitation Force F = mω^2 = 89.10 N

Static deflection $X_{st} = \frac{F_0}{k}$ = 0.01999 m (4.2.9)

Where X_{st} may be defined as zero frequency deflection of the spring-mass system under a steady force F_0 It may not be confused with Δ_{st} the static deflection of the spring-mass system under the supporting load

$$Magnification\ Factor\ theritical\ \ Mf = \frac{X_{st}}{\sqrt{[1-(\omega/\omega_n)^2]^2+[2\zeta(\omega/\omega_n)^2]}} = 21.5746$$

$$Magnification\ Factor\ experimental\ \ Mf = \frac{X}{X_{st}} = 8.3356$$

APPLICATION OF MATLAB

MATLAB is produced by Math Works, and is one of a number of commercially available software packages for numerical computing and programming. MATLAB provides an interactive environment for algorithm development, data visualization, data analysis, and numerical computation. MATLAB, which derives its name from MATrix LABoratory, excels at matrix operations and graphics. In order to solve the system analytically MATLAB programming is used to obtain theoretical time-displacement curve in this study.

Damped Free Vibration

a. Program for plotting solution

```
wn=3.1151; zy=[0.018 0.1,0.4];
x0=11.5; v0=0;
t=[0:0.1:10]
xlim([0 10])
ylim([0 15])
for i=1:length(zy)
    wd=wn*sqrt(1-zy(i)^2);

c=sqrt(x0^2+((zy(i)*wn*x0+v0)/wd)^
2);

phi=atan((zy(i)*wn*x0+v0)/(wd*x0))
;
    x=c.*exp(-
zy(i)*wn.*t).*cos(wd.*t-phi);
    plot(t,x);
    hold on;
end
legend('zy=0.018','zy=0.1','zy=0.4
')
xlabel('Time t in Secs')
ylabel('Amplitude X in mms')
```

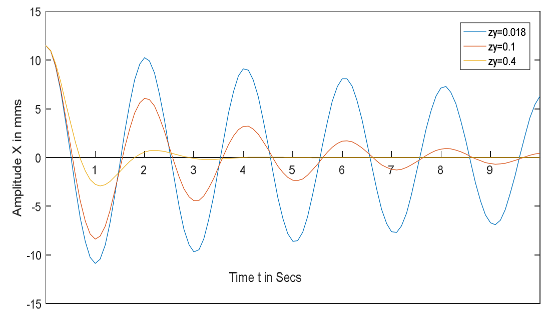


Fig 7 Amplitude v/s Time for Different Values of Zy

b. Program for solving ODE and plotting solution

```
m=11.645; wn=3.1151; k=wn^2*m;
zy=0.018;
x0=11.5; v0=0.;
[t,x]=ode45('vibration',[0 10],[0
0.6]);
plot(t,x(:,1),'*');
hold on;
 t=[0:0.1:10];
wd=wn*sqrt(1-zy^2);
c=sqrt(x0^2+((zy*wn*x0+v0)/wd)^2);
phi=atan((zy*wn*x0+v0)/(wd*x0));
x=c.*exp(-zy*wn.*t).*cos(wd.*t-
phi);
plot(t,x);

legend('response_ODE45','theoretic
al');
Tp=2*pi/wd*5
tspan=[0 4];
y0=[0.02;0];
[t,y]=ode45('unforced1',tspan,y0);
plot(t,y(:,1));
grid on
xlabel('time')
ylabel('Displacement')
title('Displacement Vs Time')
function dx=vibration(t,x)
    m=1; wn=5; zy=0.2; k=wn^2*m;
    c=zy*2*sqrt(k*m);
    x1=x(1);
    dx=zeros(2,1);
    dx(1)=x(2);
%first derivative of x%
    dx(2)=((-c/m)*x(2)-(k/m)*x1);
%second derivative of x%
end
```

Damped Forced Vibration

```
zeta=[0;0.018;0.05; 0.1; 0.15;
0.25; 0.5; 1; 1.25; 1.5];    %
Damping factors
r = [0:0.01: 3];  % Frequency
ratios
xlim([0 3])
ylim([0 10])
for k =1:length(zeta),
    MF = 1./sqrt ((1-
r.^2).^2+(2*zeta(k)*r).^2);
    plot (r, MF)
    hold on
end
xlabel('\omega/\omega_n')
ylabel('X/Xst')
legend('ZETA=0','0.018','0.05','0.
1','0.15','0.25','0.5','1',1)
grid
```

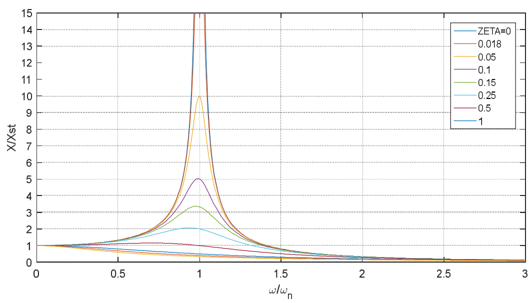


Figure 8 shows the output of the program Variation of magnification factor with ω/ω_n for different values of damping ratios

CONCLUSION

Vibration of a physical structure often is thought of in terms of model consisting of a mass and a spring. The vibration of such a model as a system may be "FREE" or "FORCED". Free vibrations arise once the external excitation dies down. Free vibrations are thus the transient vibrations after the external disturbance is removed but before the system comes to a halt. Free or natural vibration behavior of the system enables us to gain significant insight into the system behavior. From the transient response, we will actually able to measure the system damping. Knowledge of these helps us to predict how the system will respond to an external disturbance. Thus it is equally important to study free or natural vibration. "Forced Vibration", in contrast to free vibration, continuous under "Steady State" conditions because energy is supplied to the system continuously to compensate for that dissipated in the system. It is important to understand the response of the system under forced vibration as in reality many systems will be under forced vibration.

In this study Experiment was conducted for a particular spring stiffness and damping medium under free and forced conditions. The results from the experiment correlated well with theoretical predictions which were obtained from MATLAB coding.

REFERENCES:

[1] Dr.Paul Young, Free Vibration of Single Degree of Freedom System, School of Mechanical and Manufacturing Engineering, September 2001

[2] Andrew K. Costain, J MichaelRobichaud, Practical Methods for Vibration Control of Industrial Equipment, B.Sc Engg. Bretech Engineering Ltd.

[3] Uchenna H.Diala, Gloria N.Ezeh, Nonlinear damping for vibration isolation and control using semi active methods, Journal ISSN-L: 2223-9553, ISSN:2223-9944 vol.3, No.3, November 2012

[4] Ranjit G. Todkar, Shridhar G. Joshi, The Effect of Mass Ratio and Air Damper Characteristics on the Resonant Response of an Air Damped Dynamic Vibration Absorber, Journal of Scientific Research Modern Mechanical Engineering 2011.

[5] P.B.Ravi Kumar, Study of Vibration Applications in Mechanical Engineering Undergraduate Education Using Mathematics Software, ASEE North Midwest Sectional Conference 2008.

[6] V P Singh, Mechanical Vibration, Dhanpatrai & Co Ltd.

[7] G. K. Grover, S. P. Nigam, Mechanical Vibration, Nem Chand & Bros, 2003

[8] S. S. Rao, Mechanical Vibration, Nem Chand & Bros, 2010