Weighted Exponentiated Mukherjee-Islam Distribution

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Abstract: In this paper, we have proposed a new version of exponentiated Mukherjee-Islam distribution known as weighted exponentiated Mukherjee-Islam distribution. The distribution has four parameters (one scale and three shape). The different structural properties of the newly model have been studied. The maximum likelihood estimators of the parameters and the Fishers information matrix have been discussed. Further, a likelihood ratio test of the weighted model has been obtained.

Keywords: Weighted distribution, Exponentiated mukherjee-islam distribution, Reliability analysis, Maximum likelihood estimator, Order statistics, Likelihood ratio test.

Introduction

The concept of weighted distributions was given by Fisher (1934) to model the ascertainment bias. Later Rao (1965) developed this concept in a unified manner while modelling the statistical data when the standard distributions were not appropriate to record these observations with equal probabilities. As a result, weighted models were formulated in such situations to record the observations according to some weighted function. The weighted distribution reduces to length biased distribution when the weight function considers only the length of the units. The concept of length biased sampling was first introduced by Cox (1969) and Zelen (1974). More generally, when the sampling mechanism selects units with probability proportional to some measure of the unit size, resulting distribution is called size-biased. There are various good sources which provide the detailed description of weighted distributions. Different authors have reviewed and studied the various weighted probability models and illustrated their applications in different fields. Weighted distributions are applied in various research areas related to reliability, biomedicine, ecology and branching processes. Fatima and Ahmad (2017) proposed the weighted Inverse Rayleigh distribution and derived its structural properties. Shanker & Shukla (2018) discussed a new generalized size-biased, Poisson-Lindley distribution with its applications to model size distribution. Also Para and Jan (2018) introduced the weighted Pareto type-II distribution as a new model for handling medical science data and studied its statistical properties and applications. Rather *et al* (2018) obtained a new size biased Ailamujia distribution with applications in engineering and medical science which shows more flexibility than classical distributions. Recently, Rather and Subramanian (2018) discussed about the characterization and estimation of length biased weighted generalized uniform distribution.

Mukherjee-Islam distribution was introduced by Mukherjee and Islam (1983). It is finite range distribution, which is one of the most important properties of reliability analysis in recent time. Its mathematical form is simple and can be handled easily, that is why, it is preferred to use over more complex distribution such as normal, weibull, beta etc. Dar *et al*

(2018) discussed about weighted mukherjee-islam distribution and its applications. Recently, Rather and Subramanian (2018) obtained the exponentiated mukherjee-islam distribution and its various statistical properties.

The probability density function of exponentiated Mukherjee-Islam distribution is given by

$$f(x) = \frac{\alpha p x^{\alpha p - 1}}{\theta^{\alpha p}}, \quad 0 < x < \theta, \quad \alpha, p, \theta > 0 \tag{1}$$

And the corresponding cdf is given by

$$F(x) = \left(\left(\frac{x}{\theta}\right)^{p}\right)^{\alpha} \tag{2}$$

Weighted Exponentiated Mukherjee-Islam (WEMI) Distribution

Suppose X is a non-negative random variable with probability density function (pdf) $f(x)$. Let $w(x)$ be the non negative weight function, then the probability density function of the weighted random variable X_w is given by:

$$f_w(x) = \frac{w(x) f(x)}{E(w(x))}, \quad x > 0,$$

where $w(x)$ be a non-negative weight function and $E(w(x)) = \int w(x) f(x) dx < \infty$.

In this paper, we will consider the weight function as $w(x) = x^c$ to obtain the weighted exponentiated Mukherjee-Islam distribution . The probability density function of weighted exponentiated Mukherjee-Islam distribution is given as:

$$f_w(x) = \frac{x^c f(x)}{E(x^c)}, \quad x > 0$$

Substitute (1) in above equation, we will get the required pdf of weighted exponentiated Mukherjee-Islam distribution as

$$f_w(x) = \frac{(\alpha p + c) x^{\alpha p + c - 1}}{\theta^{\alpha p + c}}, \quad 0 < x < \theta, \quad \alpha, p, c, \theta > 0 \tag{3}$$

where $E(x^c) = \dfrac{\alpha p \theta^c}{\alpha p + c}$

and the corresponding cdf of weighted exponentiated Mukherjee-Islam distribution is obtained as

$$F_w(x) = \int_0^x f_w(x) dx$$

$$= \int_0^x \frac{(\alpha p + c) x^{\alpha p + c - 1}}{\theta^{\alpha p + c}} dx$$

$$\Rightarrow F_w(x) = \left(\frac{x}{\theta} \right)^{\alpha p + c} \quad (4)$$

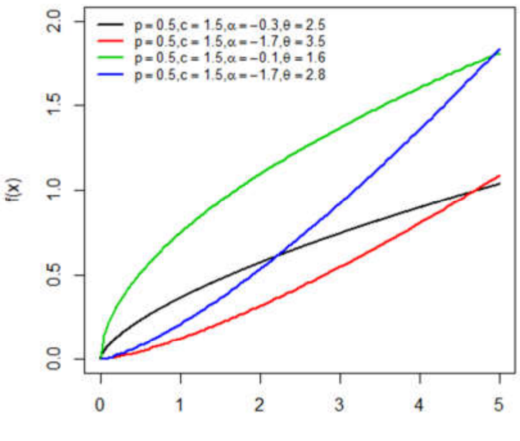

Fig.1:Pdf plot of WE Mukherjee-Islam distribution

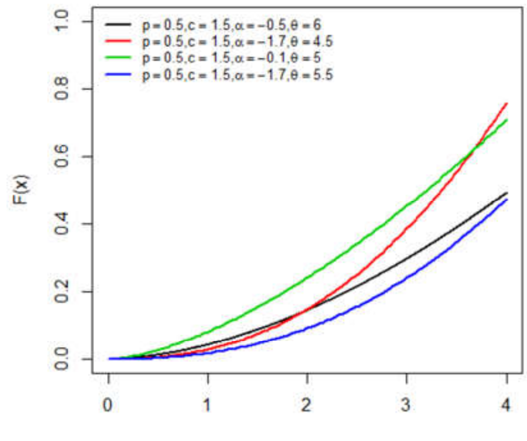

Fig.2:Cdf plot of WE Mukherjee-Islam distribution

Reliability Analysis

In this section, we have obtained the survival function, failure rate, reverse hazard rate and Mills ratio of the weighted exponentiated Mukherjee-Islam distribution.

The survival function or the reliability function of the weighted exponentiated Mukherjee-Islam distribution is

$$S(x) = 1 - F(x)$$

$$\Rightarrow S(x) = 1 - \left(\frac{x}{\theta} \right)^{\alpha p + c}$$

The hazard function or failure rate is given by

$$h(x) = \frac{f(x)}{S(x)}$$

$$\Rightarrow h(x) = \frac{(\alpha p + c)\, x^{\alpha p + c - 1}}{\left(\theta^{\alpha p + c} - x^{\alpha p + c} \right)}$$

The reverse hazard rate function is given by

$$h_r(x) = \frac{f_w(x)}{F_w(x)}$$

$$\Rightarrow h_r(x) = \left(\frac{\alpha p + c}{x} \right)$$

and the Mills ratio of the weighted exponentiated Mukherjee-Islam distribution is

$$\text{Mills Ratio} = \frac{1}{h_r(x)} = \left(\frac{x}{\alpha p + c}\right)$$

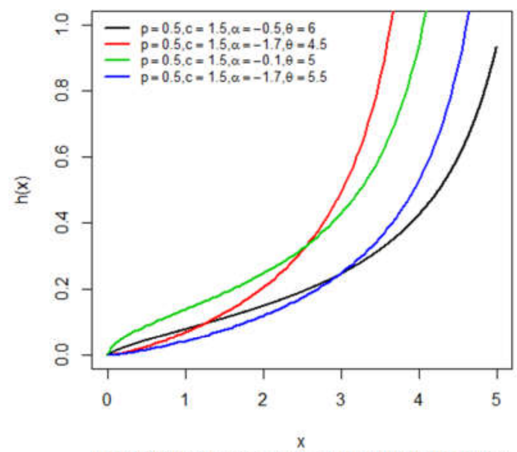

Fig.3:Showing hazard function plot of WEMI distribution

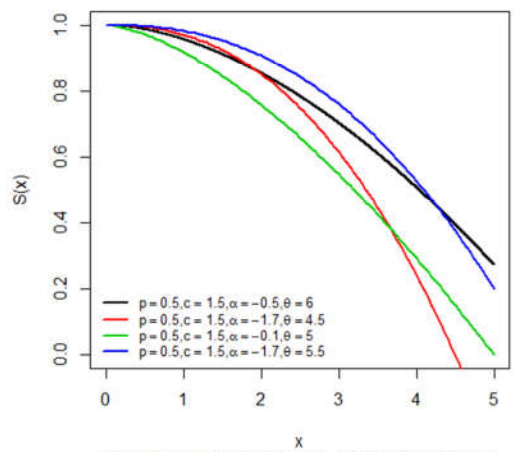

Fig.4:Showing Survival function plot WEMI distribution

Moments and Relative measures

Let X denotes the random variable of weighted exponentiated Mukherjee-Islam distribution with parameters α, p, c and θ, then the r-th order moment $E(X^r)$ of weighted exponentiated Mukherjee-Islam distribution can be obtained as

$$E(X^r) = \mu_r' = \int_0^\theta x^r f_w(x)dx$$

$$= \int_0^\theta \frac{(\alpha p + c)\, x^{\alpha p + c + r - 1}}{\theta^{\alpha p + c}} dx$$

$$= \frac{(\alpha p + c)\,\theta^{\alpha p + c + r}}{\theta^{\alpha p + c}(\alpha p + c + r)}$$

$$\Rightarrow E(X^r) = \mu_r' = \frac{(\alpha p + c)\,\theta^r}{(\alpha p + c + r)} \quad (5)$$

Put r =1 in (5), we will get mean of weighted exponentiated Mukherjee-Islam distribution which is given by

$$\textbf{Mean} = \mu_1' = \frac{(\alpha p + c)\,\theta}{(\alpha p + c + 1)}$$

If we put r =2 in (5), we have

$$\mu_2' = \frac{(\alpha p + c)\,\theta^2}{(\alpha p + c + 2)}$$

Thus the variance of weighted exponentiated Mukherjee-Islam distribution is obtained as

$$\textbf{Variance} = \mu_2 = \frac{(\alpha p + c)\theta^2}{(\alpha p + c + 2)(\alpha p + c + 1)^2}$$

$$\text{Standard deviation} = \sigma = \frac{\theta}{(\alpha p + c + 1)}\sqrt{\frac{(\alpha p + c)}{(\alpha p + c + 2)}}$$

$$\text{Coefficient of variation} = C.V = \frac{\sigma}{\mu_1{}'} = \frac{1}{(\alpha p + c)}\sqrt{\frac{(\alpha p + c)}{(\alpha p + c + 2)}}$$

$$\text{Coefficient of dispersion } (\gamma) = \frac{\sigma^2}{\mu_1{}'} = \frac{\theta}{(\alpha p + c + 2)(\alpha p + c + 1)}$$

Harmonic mean

The Harmonic mean of the proposed model can be obtained as

$$H.M = E\left(\frac{1}{x}\right) = \int_0^\theta \frac{1}{x} f_w(x)\,dx$$

$$= \int_0^\theta \frac{1}{x}\frac{(\alpha p + c)\,x^{\alpha p + c - 1}}{\theta^{\alpha p + c}}\,dx$$

$$= \frac{(\alpha p + c)}{\theta^{\alpha p + c}}\int_0^\theta x^{\alpha p + c - 2}\,dx$$

$$= \frac{(\alpha p + c)}{\theta^{\alpha p + c}}\left(\frac{\theta^{\alpha p + c - 1}}{\alpha p + c - 1}\right)$$

$$\Rightarrow H.M = \frac{(\alpha p + c)}{\theta(\alpha p + c - 1)}$$

Moment generating function and Characteristic function

Let X have a weighted exponentiated Mukherjee-Islam distribution, then the MGF of X is obtained as

$$M_X(t) = E(e^{tx}) = \int_0^\theta e^{tx} f_w(x)dx$$

Using Taylor's series

$$M_X(t) = E(e^{tx}) = \int_0^\theta \left(1 + tx + \frac{(tx)^2}{2!} + \cdots\right) f_w(x)dx$$

$$= \int_0^\theta \sum_{j=0}^{\infty} \frac{t^j}{j!} x^j f_w(x) dx$$

$$= \sum_{j=0}^{\infty} \frac{t^j}{j!} \mu_j'$$

$$= \sum_{j=0}^{\infty} \frac{t^j}{j!} \frac{(\alpha p + c)\theta^j}{(\alpha p + c + j)}$$

$$\Rightarrow M_x(t) = (\alpha p + c) \sum_{j=0}^{\infty} \frac{t^j}{j!} \frac{\theta^j}{(\alpha p + c + j)}$$

Similarly, the characteristic function of WEMID can be obtained

$$\varphi_x(t) = M_x(it)$$

$$\Rightarrow \varphi_x(t) = (\alpha p + c) \sum_{j=0}^{\infty} \frac{(it)^j}{j!} \frac{\theta^j}{(\alpha p + c + j)}$$

Shannon's Entropy of weighted exponentiated Mukherjee-Islam distribution

Claude Shannon (1964) defined a formal measure of entropy, called Shannon Entropy. The Shannon's entropy of a random variable *X* is a measure of the uncertainty and is given by *E*{*-logf(x)*}, where *f(x)* is the probability function of the random variable *X.* Thus, the Shannon's entropy of the weighted exponentiated Mukherjee-Islam distribution can be obtained as

$$H(x) = -E\left(\log f_w(x)\right)$$

$$= -E\left(\log\left(\frac{(\alpha p + c)\, x^{\alpha p + c - 1}}{\theta^{\alpha p + c}}\right)\right)$$

$$\Rightarrow H(x) = -\log(\alpha p + c) + (\alpha p + c)\log\theta - (\alpha p + c - 1)E\left(\log x\right) \qquad (6)$$

Now, $E(\log x) = \int_0^\theta \log x\, f(x) dx$

$$= \int_0^\theta \log x \left(\frac{(\alpha p + c)\, x^{\alpha p + c - 1}}{\theta^{\alpha p + c}}\right) dx$$

$$= \frac{(\alpha p + c)}{\theta^{\alpha p + c}} \int_0^\theta \log(x)\, x^{\alpha p + c - 1}\, dx$$

$$= \frac{(\alpha p + c)}{\theta^{\alpha p + c}} \left(\frac{(\alpha p + c)\theta^{\alpha p + c}\{(\alpha p + c)\log\theta - 1\}}{(\alpha p + c)^3}\right)$$

$$= \frac{(\alpha p + c)\log\theta - 1}{(\alpha p + c)}$$

$$\Rightarrow E(\log x) = \log\theta - \frac{1}{(\alpha p + c)} \quad (7)$$

Substitute equation (7) in (6), we get

$$H(x) = -\log(\alpha p + c) + (\alpha p + c)\log\theta - (\alpha p + c - 1)\left(\log\theta - \frac{1}{(\alpha p + c)}\right)$$

Renyi Entropy

Entropies quantify the diversity, uncertainty, or randomness of a system. The Renyi entropy is named after Alfred Renyi in the context of fractual dimension estimation, the Renyi entropy forms the basis of the concept of generalized dimensions. The Renyi entropy is important in ecology and statistics as index of diversity. The Renyi entropy is also important in quantum information, where it can be used as a measure of entanglement. For a given probability distribution, Renyi entropy is given by

$$e(\beta) = \frac{1}{1-\beta}\log\left(\int f^{\beta}(x)dx\right)$$

where, $\beta > 0$ and $\beta \neq 1$

$$e(\beta) = \frac{1}{1-\beta}\log\left[\int_0^{\theta}\left(\frac{(\alpha p + c)\,x^{\alpha p + c - 1}}{\theta^{\alpha p + c}}\right)^{\beta} dx\right]$$

$$= \frac{1}{1-\beta}\log\left[\left(\frac{\alpha p + c}{\theta^{\alpha p + c}}\right)^{\beta}\int_0^{\theta}\left(x^{\alpha p + c - 1}\right)^{\beta} dx\right]$$

$$= \frac{1}{1-\beta}\log\left[\left(\frac{\alpha p + c}{\theta^{\alpha p + c}}\right)^{\beta}\left(\frac{\theta^{\beta(\alpha p + c - 1) + 1}}{\beta(\alpha p + c - 1) + 1}\right)\right]$$

$$\Rightarrow e(\beta) = \frac{1}{1-\beta}\log\left[\left(\frac{(\alpha p + c)^{\beta}\,\theta^{1-\beta}}{\beta(\alpha p + c - 1) + 1}\right)\right]$$

Tsallis Entropy

In recent years, a generalization of Boltzmann-Gibbs (B-G) statistical mechanics initiated by Tsallis has focussed a great deal to attention. This generalization of B-G statistics was proposed firstly by introducing the mathematical expression of Tsallis entropy (Tsallis, 1988) for a continuous random variable is defined as follows

$$S_\lambda = \frac{1}{\lambda-1}\left(1-\int_0^\theta f^\lambda(x)dx\right)$$

$$= \frac{1}{\lambda-1}\left[1-\int_0^\theta\left(\frac{(\alpha p+c)\,x^{\alpha p+c-1}}{\theta^{\alpha p+c}}\right)^\lambda dx\right]$$

$$= \frac{1}{\lambda-1}\left[1-\left(\frac{\alpha p+c}{\theta^{\alpha p+c}}\right)^\lambda\int_0^\theta\left(x^{\alpha p+c-1}\right)^\lambda dx\right]$$

$$= \frac{1}{\lambda-1}\left[1-\left(\frac{\alpha p+c}{\theta^{\alpha p+c}}\right)^\lambda\left(\frac{\theta^{\lambda(\alpha p+c-1)+1}}{\lambda(\alpha p+c-1)+1}\right)\right]$$

$$\Rightarrow S_\lambda = \frac{1}{\lambda-1}\left[1-\left(\frac{(\alpha p+c)^\lambda\,\theta^{1-\lambda}}{\lambda(\alpha p+c-1)+1}\right)\right]$$

Order Statistics

Let $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$ be the order statistics of a random sample $X_1, X_2, \ldots, X_n$ drawn from the continuous population with probability density function $f_x(x)$ and cumulative density function with $F_x(x)$, then the pdf of r^{th} order statistics $X_{(r)}$ can be written as

$$f_{X(r)}(x) = \frac{n!}{(r-1)!(n-r)!} f_X(x)\left[F_X(x)\right]^{r-1}\left[1-F_X(x)\right]^{n-r} \tag{8}$$

Substitute the values of (3) and (4) in equation (8), we will get the pdf of r^{th} order statistics $X_{(r)}$ for weighted exponentiated Mukherjee-Islam distribution and is given by

$$f_{X(r)}(x) = \frac{n!}{(r-1)!(n-r)!}\frac{(\alpha p+c)\,x^{\alpha p+c-1}}{\theta^{\alpha p+c}}\left[\left(\frac{x}{\theta}\right)^{\alpha p+c}\right]^{r-1}\left[1-\left(\frac{x}{\theta}\right)^{\alpha p+c}\right]^{n-r} \tag{9}$$

From equation (9), the probability density function of higher order statistics $X_{(n)}$ can be obtained as

$$f_{X(n)}(x) = \frac{n\,(\alpha p+c)\,x^{\alpha p+c-1}}{\theta^{\alpha p+c}}\left[\left(\frac{x}{\theta}\right)^{\alpha p+c}\right]^{n-1}$$

Similarly, the pdf of 1^{st} order statistic $X_{(1)}$ can be obtained as

$$f_{X(1)}(x) = \frac{n\,(\alpha p+c)\,x^{\alpha p+c-1}}{\theta^{\alpha p+c}}\left[1-\left(\frac{x}{\theta}\right)^{\alpha p+c}\right]^{n-1}$$

Maximum Likelihood Estimators and Fisher's Information Matrix

In this section, we will discuss the maximum likelihood estimators of the parameters of weighted exponentiated Mukherjee-Islam distribution. Consider $X_1, X_2, \ldots, X_n$ be the random sample of size n from the weighted exponentiated Mukherjee-Islam distribution, then the likelihood function can be written as

$$L(x) = \prod_{i=1}^{n}\left(\frac{(\alpha p + c)\, x^{\alpha p + c - 1}}{\theta^{\alpha p + c}}\right)$$

The log likelihood function is

$$\log L(x) = n\log(\alpha p + c) + (\alpha p + c - 1)\sum_{i=1}^{n}\log x_i - n(\alpha p + c)\log\theta \qquad (9)$$

The maximum likelihood estimates of α, p, c, θ can be obtained by differentiating equation (9) with respect to α, p, c, θ and must satisfy the normal equation

$$\frac{\partial \log L}{\partial \alpha} = \frac{np}{\alpha p + c} + p\sum_{i=1}^{n}\log x_i - np\log\theta = 0 \qquad (10)$$

$$\frac{\partial \log L}{\partial p} = \frac{n\alpha}{\alpha p + c} + \alpha\sum_{i=1}^{n}\log x_i - n\alpha\log\theta = 0 \qquad (11)$$

$$\frac{\partial \log L}{\partial c} = \frac{n}{\alpha p + c} + \sum_{i=1}^{n}\log x_i - n\log\theta = 0 \qquad (12)$$

$$\frac{\partial \log L}{\partial \theta} = -\frac{n(\alpha p + c)}{\theta} = 0 \qquad (13)$$

From equations (10), (11), (12) and (13) we obtain the MLE of α, p, c and θ given by

$$\hat{\alpha} = \frac{1}{p}\left(\frac{n}{n\log\theta - \sum_{i=1}^{n}\log x_i} - c\right)$$

$$\hat{p} = \frac{1}{\alpha}\left(\frac{n}{n\log\theta - \sum_{i=1}^{n}\log x_i} - c\right)$$

$$\hat{c} = \frac{\alpha p\left(n\log\theta - \sum_{i=1}^{n}\log x_i\right) - n}{\left(\sum_{i=1}^{n}\log x_i - n\log\theta\right)}$$

$\hat{\theta} = \infty$, which is an absured result.

Now we apply inspection method. Let us consider n-ordered samples $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$, then

$$0 \le X_{(1)} \le X_{(2)} \le \ldots \le X_{(n)} \le \theta$$

$$\Rightarrow \quad \theta \ge X_{(n)}$$

Therefore, MLE of $\theta = X_{(n)}$ = the largest sample observation.

To obtain confidence interval we use the asymptotic normality results. We have that, if $\hat{\lambda} = (\hat{\alpha}, \hat{p}, \hat{c}, \hat{\theta})'$ denotes the MLE of $\hat{\lambda} = (\alpha, p, c, \theta)'$, then

$$\sqrt{n}(\hat{\lambda} - \lambda) \to N(0, I^{-1}(\lambda))$$

Where $I^{-1}(\lambda)$ is Fisher's Information Matrix given by

$$I(\lambda) = -\frac{1}{n}\begin{pmatrix} E\left(\frac{\partial^2 \log L}{\partial \alpha^2}\right) & E\left(\frac{\partial^2 \log L}{\partial \alpha\,\partial p}\right) & E\left(\frac{\partial^2 \log L}{\partial \alpha\,\partial \theta}\right) & E\left(\frac{\partial^2 \log L}{\partial \alpha\,\partial c}\right) \\ E\left(\frac{\partial^2 \log L}{\partial \alpha\,\partial p}\right) & E\left(\frac{\partial^2 \log L}{\partial p^2}\right) & E\left(\frac{\partial^2 \log L}{\partial p\,\partial \theta}\right) & E\left(\frac{\partial^2 \log L}{\partial p\,\partial c}\right) \\ E\left(\frac{\partial^2 \log L}{\partial \alpha\,\partial \theta}\right) & E\left(\frac{\partial^2 \log L}{\partial p\,\partial \theta}\right) & E\left(\frac{\partial^2 \log L}{\partial \theta^2}\right) & E\left(\frac{\partial^2 \log L}{\partial \theta\,\partial c}\right) \\ E\left(\frac{\partial^2 \log L}{\partial \alpha\,\partial c}\right) & E\left(\frac{\partial^2 \log L}{\partial p\,\partial c}\right) & E\left(\frac{\partial^2 \log L}{\partial \theta\,\partial c}\right) & E\left(\frac{\partial^2 \log L}{\partial c^2}\right) \end{pmatrix}$$

where, $E\left(\frac{\partial^2 \log L}{\partial \alpha^2}\right) = -\frac{np^2}{(\alpha p + c)^2}$ $\qquad E\left(\frac{\partial^2 \log L}{\partial p^2}\right) = -\frac{n\alpha^2}{(\alpha p + c)^2}$

$$E\left(\frac{\partial^2 \log L}{\partial \theta^2}\right) = -\frac{n(\alpha p + c)}{\theta^2} \qquad E\left(\frac{\partial^2 \log L}{\partial c^2}\right) = -\frac{n}{(\alpha p + c)^2}$$

Also, $E\left(\frac{\partial^2 \log L}{\partial \alpha\,\partial p}\right) = -\frac{nc}{(\alpha p + c)^2} + \sum_{i=1}^{n}\log x_i - n\log\theta$

$$E\left(\frac{\partial^2 \log L}{\partial \alpha\,\partial \theta}\right) = -\frac{np}{\theta}, \qquad E\left(\frac{\partial^2 \log L}{\partial \alpha\,\partial c}\right) = -\frac{np}{(np + c)^2}$$

$$E\left(\frac{\partial^2 \log L}{\partial p\,\partial\theta}\right) = -\frac{n\alpha}{\theta}, \qquad E\left(\frac{\partial^2 \log L}{\partial p\,\partial c}\right) = -\frac{n\alpha}{(\alpha p+c)^2}$$

$$E\left(\frac{\partial^2 \log L}{\partial\theta\,\partial c}\right) = -\frac{n}{\theta}$$

λ being unknown, we estimate $I^{-1}(\lambda)$ by $I^{-1}(\hat{\lambda})$ and can use this to obtain asymptotic confidence intervals for α, p, c and θ.

Likelihood Ratio Test

Let $X_1, X_2, \ldots, X_n$ be a random sample from the weighted exponentiated Mukherjee-Islam distribution. To test the hypothesis

$$H_o : f(x) = f(x;\alpha, p, \theta) \quad \text{against} \quad H_1 : f(x) = f_w(x;\alpha, p, c, \theta)$$

For testing whether the random sample of size n comes from the exponentiated Mukherjee-Islam distribution or weighted exponentiated Mukherjee-Islam distribution, the following test statistic is used

$$\Delta = \frac{L_1}{L_0} = \prod_{i=1}^{n} \frac{f_w(x;\alpha, p, c, \theta)}{f(x;\alpha, p, \theta)}$$

$$= \prod_{i=1}^{n}\left[\frac{(\alpha p + c)\, x_i^{\alpha p + c - 1}}{\theta^{\alpha p + c}} \times \frac{\theta^{\alpha p}}{\alpha p\, x_i^{\alpha p - 1}}\right]$$

$$= \prod_{i=1}^{n}\left[\frac{(\alpha p + c)}{\alpha p\,\theta^c} x_i^{c}\right]$$

$$= \left[\frac{(\alpha P + c)}{\alpha p\,\theta^c}\right]^n \prod_{i=1}^{n} x_i^{c}$$

We reject the null hypothesis if

$$\Delta = \left[\frac{(\alpha p + c)}{\alpha p\,\theta^c}\right]^n \prod_{i=1}^{n} x_i^{c} > k$$

Equivalently we reject the null hypothesis where

$$\Delta^* = \prod_{i=1}^{n} x_i^{c} > k^*, \text{ where } k^* = k\left(\frac{\alpha p\,\theta^c}{\alpha p + c}\right)^n > 0$$

For large sample size n, $2\log\Delta$ is distributed as chi-square distribution with one degree of freedom and also p-value is obtained from the chi-square distribution. Thus we reject the null hypothesis, when the probability value is given by

$$p\left(\Delta^* > \beta^*\right), \text{ where } \beta^* = \prod_{i=1}^{n} x_i^{c} \text{ is less than a specified level of significance and } \prod_{i=1}^{n} x_i^{c} \text{ is}$$

the observed value of the statistic Δ^*.

Conclusion

In this paper, we have proposed a new version of exponentiated Mukherjee-Islam distribution known as weighted exponentiated Mukherjee-Islam distribution. The distribution has a scale parameter and three shape parameters. The survival function, hazard rate, reverse hazard rate, moments have been obtained. The maximum likelihood estimators of the parameters and the Fishers information matrix have been discussed. The order statistics and the entropies like Shannon's, Renyi and Tsallis have been obtained. Finally, a likelihood ratio test of the weighted model has been obtained.

References

[1] A. A. Rather and C. Subramanian, (2018), Characterization and Estimation of length biased weighted generalized uniform distribution, *International Journal of Scientific Research in Mathematical and Statistical Sciences, vol.5, issue.5, pp 72-76.*

[2] A. A. Rather and C. Subramanian, (2018), Exponentiated Mukherjee-Islam Distribution, *Journal of Statistics Applications & Probability,* 7(2), 357-361.

[3] A. A. Rather and C. Subramanian, (2018), Transmuted Mukherjee-Islam failure model, *Journal of Statistics Applications & Probability,* 7(2), 343-347.

[4] A. A. Rather, C. Subramanian, S. Shafi, K. A. Malik, P. J. Ahmad, B. A. Para, and T. R. Jan, (2018). "*A new Size Biased Distribution with applications in Engineering and Medical Science*", International Journal of Scientific Research in Mathematical and Statistical Sciences, Vol.5, Issue.4, pp.75-85.

[5] B. A. Para & T. R. Jan, (2018), On Three Parameter Weighted Pareto Type II Distribution: Properties and Applications in Medical Sciences. *Applied Mathematics & Information Sciences Letters*, *6*(1), 13-26.

[6] C. R. Rao (1965), "On discrete distributions arising out of method of ascertainment, in classical and Contagious Discrete", G.P. Patiled; *Pergamum Press and Statistical publishing Society, Calcutta.* 320-332.

[7] C. Shannon and W. Weaver, (1964), The Mathematical Theory of Communication. University of Illinois Press, Urbana, Ill.

[8] C. Tsallis (1988), Possible generalization of boltzmann-gibbs statistics. *Journal of Statistical Physics, 52*, 479-487. https://doi.org/10.1007/BF01016429.

[9] D. R. Cox (1969), Some sampling problems in technology, In New Development in Survey Sampling, Johnson, N. L. and Smith, H., Jr .(eds.) *New York Wiley- Interscience*, 506-527.

[10] Javid Gani dar, Uzma Manzoor, Essey Kebede Muluneh, (2018), Weighted Mukherjee-Islam distribution and its applications, *International Journal of Statistics &Economics,* vol-19, No.2, p 108-122-

[11] K. Fatima & S.P. Ahmad, (2017), Weighted Inverse Rayleigh Distribution. *International Journal of Statistics and System.* 12 (1), 119-137.

[12] M. Zelen (1974), Problems in cell kinetic and the early detection of disease, in Reliability and Biometry, F. Proschan & R. J. Sering, eds, *SIAM, Philadelphia*, 701-706.

[13] R. A. Fisher (1934), The effects of methods of ascertainment upon the estimation of frequencies. *Annals of Eugenics*, 6, 13-25.

[14] R. Shanker & K. K. Shukla, (2018), A Generalized Size-Biased Poisson-Lindley Distribution and Its Applications to Model Size Distribution of Freely-Forming Small Group, *Journal of Scientific Research*, *10*(2), 145.

[15] S. P. Mukherjee and A. Islam, (1983), "A finite-range distribution of finite times", *Naval Research Logistics Quarterly:* pp. 487-491.