Functionally Graded Thick Walled Hollow Cylindrical Vessel: A Review

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Abstract

Cylindrical structures consisted of functionally graded materials (FGMs) are frequently used in various engineering applications in the modern aerospace, marine, nuclear and military industries, automotive industries, etc. Functionally Graded Materials (FGMs) are the advanced materials in the field of composites, which can resist high temperatures and are proficient in reducing the thermal stresses. In recent decades, significant investigations are reported in the predicting the response of FGM plates subjected to thermal loads. A comprehensive review of various theories for the modeling and analysis of functionally graded plates and cylindrical shells is presented. The review is devoted to theoretical models which were developed to predict the global responses of functionally graded plates and shells under mechanical and thermal loadings. This review mainly focuses on the equivalent single layer theories including the classical plate theory, first order shear deformation theory, higher-order shear deformation theories, simplified theories and mixed theories since they were widely used in the modeling of functionally graded plates and shells.

Keywords: *Functionally Graded materials; Cylindrical shells; First Order Shear Deformation Theory; Higher Order Shear Deformation Theory*

1. Introduction

Functionally Graded Materials (FGMs) are the advanced materials in the group of composites, formed by continuous gradation of two or more constituent phases over a specified volume. FGMs can also be defined as a material which posses gradual variation in properties due to material heterogeneity. This gradation of properties can be tailored properly in order to achieve optimized characteristics of each components. The material property variation can be either unidirectional or multidirectional, and also it can be continuously or discontinuously graded from one surface to another. Commonly used FGMs are continuously graded in one particular direction. But in all types of FGMs smooth transition in thermo mechanical properties is ensured, thereby mitigating problems due to delamination and cracking.

FGMs were mainly designed and developed to resist high temperature gradients. Therefore in recent decades, thermal response of these structural members is extensively investigated by researchers.

Cylindrical structures consisted of functionally graded materials (FGMs) are frequently used in various engineering applications in the modern aerospace, marine, nuclear and military industries, automotive industries, etc. Axisymmetric hollow cylindrical shells are common structural elements in many engineering applications, including pressure vessels, submarine hulls, ship hulls, wings and fuselages of airplanes, containment structures of nuclear power plants, pipes, exteriors of rockets, missiles, automobile tires, concrete roofs, chimneys, cooling towers, liquid storage tanks, and many other structures. In order to optimize the weight, mechanical strength, displacement and stress distribution of a shell, one approach is to use shells with functionally graded materials.

1.1. Development of FGM

The concept of gradation in material composition was first proposed for composites and polymeric materials by Shen and Bever [1] in 1972. Most of these materials were used as coating materials, in order to improve the bonding strength and to reduce thermal stresses. The first practical application of FGMs was carried out at National Aerospace Laboratories of Japan in 1984 to create square shells for the base of fuselage and hemispherical bowls for nosecones of a space plane. In general, aerospace structures are supposed to perform at target service temperatures of 2100K and temperature gradient of 1600K across a cross section of less than 10mm [2]. There are no industrial materials available at present which can withstand such high-temperature gradients without losing their structural integrity.

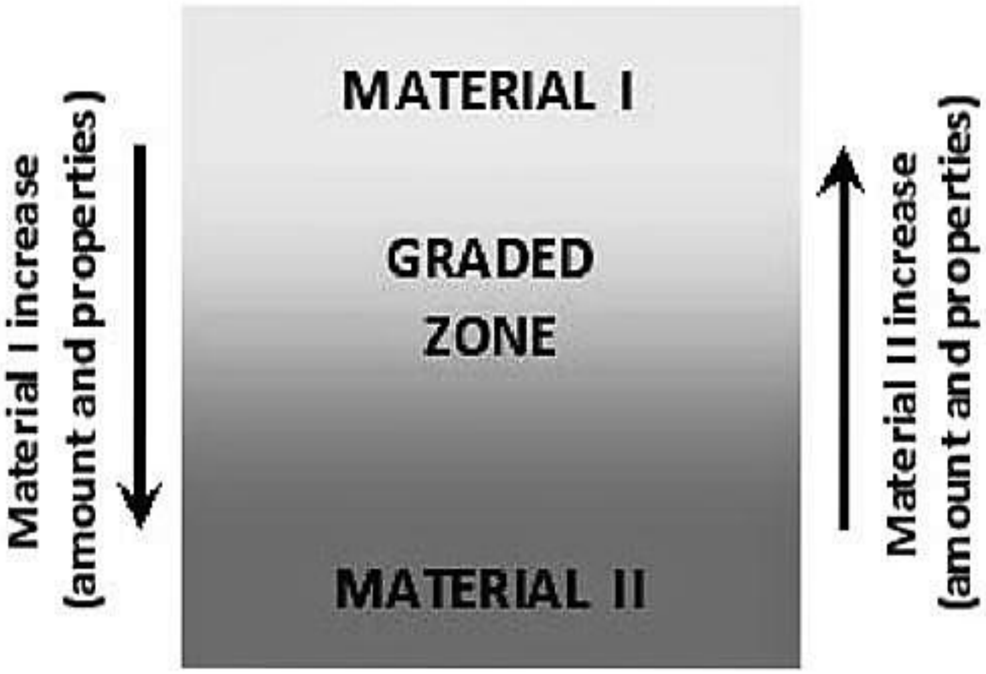

Figure- 1: Functionally Graded Materials

FGMs can be classified into two types based on the distribution of constituent phases, namely continuous or discontinuous (step-wise or layered) gradation of materials. Also, based on manufacturing techniques these can be further grouped as thin and bulk FGMs [3, 4]. Thin FGMs are manufactured by Physical Vapor Deposition (PVD), Self-propagating High temperature Synthesis (SHS) method, Chemical Vapor Deposition (CVD), etc., which are generally used as surface coating material. While bulk FGMs are consolidated to form a volumetric bulk material and are manufactured using Powder Metallurgical (PM) technique, Solid Free Foam (SFF) technique, Centrifugal Casting method, etc.

Types of FGM based on distribution of constituent phases:

- Continuous gradation of materials
- Discontinuous gradation of materials

Types of FGM based on manufacturing techniques:

- Thin FGM
- Bulk FGM

1.2. Application areas of FGM

Some of the applications of functionally graded materials are highlighted below:

- Space Applications : Rocket components, space plane frame
- Medical Application : Artificial bone, skin, dentistry
- Nuclear Reactor : Fusion pellets, plasma walls
- Energy : Solar cells, sensors
- Communication : Optical fibre, lenses, semi conductors
- Others : Building material, window glass, sports goods

1.3. Cylindrical Component

Cylindrical component (e.g. cylinder, cylindrical shell and cylindrical panel) is one of the main structural elements in engineering applications. Coupling problems of FGM cylindrical structures studied in the last decades, including thermo-elastic coupling, multi-fields coupling, fluid-structure coupling and structure-foundation coupling, are well organized in this paper, together with the theory of elasticity on cylindrical structures and typical FGM material models.

Hollow cylinder is a three-dimensional (3D) structure bounded by two parallel curved surfaces (an inner surface and an outer one), which are respectively formed by the points at a fixed distance from the axis of the cylinder. Hollow cylinders are axisymmetric. To analyse the mechanical behaviours of them, cylindrical coordinate (r, q, z) is always applied, where r, q and z denote the radial, circumferential and axial coordinates respectively.

1.4. Profile

Three typical 1D profiles of FGMs
Distribution of the compositions in FGMs can be designed to various spatial patterns in the light of engineering needs. Three typical gradations are adopted in the literature of 2005-2015. They are exponential law, power law and sigmoid law.

(1) Exponential law

The exponential law is commonly used in the fracture mechanics studies. According to this law, for a FGM structure with uniform thickness h, the effective material properties at any point from the reference surface (z = 0) can be given by:

$$P(z) = P_t e^{\lambda(1-\frac{2z}{h})} = P_t e^{(ln\frac{P_t}{P_b})(\frac{z}{h}+\frac{1}{2})} \quad \text{for } h/2 \leq z \leq h/2$$

Where, P_t and P_b represent the material properties of the top-most ($z = h/2$) and the bottom-most ($z = -h/2$) surfaces of the structure respectively, and λ is the material grading index in the exponential model.

The exponential model of FGMs has other form which analyse fracture mechanics of functionally graded layered structures, in which the material properties were assumed as

$$P(z) = P_0 e^{\delta z} \quad \text{for } 0 \leq z \leq h$$

Where, P_0 and δ were known constants. Considered a linear distribution pattern to study the thermal stress of a thin FGM cylindrical shell under thermal shock, in which the linear function was:

$$P(z) = P_1 + P_1(\frac{P_2}{P_1} - 1)(\frac{z}{h}) \quad \text{for } 0 \leq z \leq h$$

It can be easily seen that the linear function is a special case of the exponential for $\delta = [\ln(P_2/P_1)]/h$. studied the free vibration of three-layer circular cylindrical shells with FGM middle layer, the two cases of which with Yong's modulus of the FGM layer varying either following a linear function or a parabolic one were considered.

(2) Power law

The power law is more generally adopted to describe the distribution of the volume fractions of the compositions in the recent 10 year's literature. Here, the volume fraction of the matrix V_1 follows a power law formulation:

$$V_1 = (\frac{z}{h} + \frac{1}{2})^n \text{ or } V_1 = (\frac{1}{2} - \frac{z}{h})^n$$

Where, n denotes the material grading index in the power model. Ranges of the material grading indexes depend on engineering requirements. Substituting this function into results of the corresponding micromechanics schemes, the effective material properties at any location can be obtained.

For FGM hollow cylinders and cylindrical shells, some other forms of power law functions have also been used. The one is

$$P(r) = P_0 r^n \text{ or } P(r) = P_0 (\frac{r}{n})^n$$

Where, P_0 is a known constant. The range $-2 \leq n \leq 2$ was usually used in literature. Another formulation is

$$V_2 = (\frac{r - r_i}{r_0 - r_i})^n$$

And the material properties $P(r)$ then can be expressed by:

$$P(z) = P_1(1 - V_2) + P_2 V_2 = P_1 + (P_2 - P_1)(\frac{r - r_i}{r_0 - r_i})^n$$

Where, r denotes the radial coordinate of the FGM structure.

In addition, a generalization of power-law distribution with four-parameter, in terms of volume fraction of the constituents, has been used to describe the profile of FGMs in cylindrical structures. Various material profiles through the thickness of a FGM cylindrical shell can be illustrated by varying the four parameters of power-law distributions. For example, the matrix phase volume fraction 1 V follows the following simple four-parameter power-law distribution:

$$V_1 = (1 - a\left(\frac{1}{2} + \frac{z}{h}\right) + b(\frac{1}{2} + \frac{z}{h})^c)^m$$

$$V_1 = (1 - a\left(\frac{1}{2} - \frac{z}{h}\right) + b(\frac{1}{2} - \frac{z}{h})^c)^m$$

Where, the volume fraction index m (0 £ m £ ¥) and the parameters a, b and c govern the material variation profile through the thickness.

2. Theory of elasticity on cylindrical structures

2.1. Theory of elasticity on hollow cylinders

Hollow cylinder is a three-dimensional (3D) structure bounded by two parallel curved surfaces (an inner surface and an outer one), which are respectively formed by the points at a fixed distance from the axis of the cylinder. Hollow cylinders are axisymmetric. To analyze the mechanical behaviors of them, cylindrical coordinate (r, q, z) is always applied, where r, q and z denote the radial, circumferential and axial coordinates respectively. A summary of equations for hollow cylinders is made in this section, and the geometric equations, constitutive equations and equilibrium equations are mainly described. These equations associated with boundary conditions (and initial conditions) make up a complete set of equations of hollow cylinder theories.

2.1.1 3D generalized equations

Based on the theory of elasticity, generalized governing equations of hollow cylinders in cylindrical coordinate (r, θ, z) can be expressed as:

Geometric equations:

$$\varepsilon_r = \frac{\partial u}{\partial r} \qquad \varepsilon_\theta = \frac{u}{r} + \frac{\partial v}{r\partial\theta} \qquad \varepsilon_z = \frac{\partial w}{\partial z}\ ,$$

$$\varepsilon_{rz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \qquad \varepsilon_{z\theta} = \frac{\partial v}{\partial z} + \frac{\partial w}{r\partial\theta} \qquad \varepsilon_{r\theta} = \frac{\partial u}{r\partial\theta} + \frac{\partial v}{\partial r} - \frac{v}{r}$$

Where *u*, *v* and *w* denote the radial, circumferential and axial displacements respectively, ε_r, ε_θ and ε_z are the principle strain in the radial, circumferential and axial directions respectively, and ε_{ij} (i, j = r, θ, z) is the shear strain between the coordinate *i* and the coordinate *j* .

Constitutive equations:

$$\sigma = C\colon\varepsilon$$

Where $\sigma = [\sigma_r, \sigma_\theta, \sigma_z, \tau_{z\theta}, \tau_{rz}, \tau_{r\theta}]^T$ and $\varepsilon = [\varepsilon_r, \varepsilon_\theta, \varepsilon_z, \varepsilon_{z\theta}, \varepsilon_{rz}, \varepsilon_{r\theta}]$ are stress and strain tensors, respectively, and C is the elastic constant tensor, components of which are determined by the material properties.

2.1.2 2D profiles of FGMs

Variations of the material properties of FGMs are 1D and vary continuously from one surface to the other with a prescribed function. Such conventional FGMs may not be so effective in some harsher working environment since all the outer or inner surfaces of them are of the same composition distribution, but the temperature distribution in these advanced components may changes in two or three directions. Therefore, the FGM with 2D dependent material property is introduced, in which the material properties vary continuously in two directions. Meanwhile, significant advances in manufacturing and processing techniques have made it possible to produce FGMs with 2D or 3D gradients.

3. ESL theories (Equivalent-single-layer)

3.1. CPT model (Classical Plate Theory)

The CPT model is based on the Kirchhoff-Love hypothesis that the straight lines remain straight and perpendicular to the mid plane after deformation. These assumptions imply the vanishing of the shear and normal strains, and consequently, neglecting the shear and normal deformation effects. The CPT is the simplest ESL model and it is only suitable for thin FG plates/shells where the shear and normal deformation effects are inconsiderable. Feldman and Aboudi [10] studied the elastic buckling of FG plates under uniaxial compressive loading using a combination of micromechanical and structural approaches. Governing equations derived from the CPT were analytically solved for the buckling load of FG plates with various boundary conditions.

3.2. FSDT model (First-order Shear Deformation Theory)

The FSDT developed by Mindlin [5] accounts for the shear deformation effect by the way of a linear variation of the in-plane displacements through the thickness. It is noted that the theory developed by Reissner [6,7] also accounts for the shear deformation effect. However, the Reissner theory is not similar with the Mindlin theory like erroneous perception of many researchers through the use of misleading descriptions such as "Reissner-Mindlin plates" and "FSDT of Reissner". The major difference between two theories was established by Wang et al. [8] by deriving the bending relationships between Mindlin and Reissner quantities for a general plate problem. Since the Reissner theory was based on the assumption of a linear bending stress distribution and a parabolic shear stress distribution, its formulation will inevitably lead to the displacement variation being not necessarily linear across the plate thickness.

3.3. TSDT model (Third-order Shear Deformation Theory)

The TSDT developed by Reddy [9] for laminated composite plate accounts for the transverse shear deformation effect and satisfies the zero-traction boundary conditions on the top and bottom surfaces of a plate. A shear correction factor is therefore not required. It is worth noting that the displacement field of Reddy theory is identical with that of Levinson theory [11]. However, the equations of motion of two theories are different each other. This is due to the fact that Levinson [11] used the equilibrium equations of the FSDT which are variationally inconsistent with those derived from the variational approach by Reddy [9].

3.4. HSDT models (Higher-order Shear Deformation Theory)

The HSDTs account for higher-order variations of the in-plane displacements or both in-plane and transverse displacements (i.e. quasi-3D theory) through the thickness, and consequently, capturing the effects of shear deformation or both shear and normal deformations. The HSDTs can be developed using polynomial shape functions or nonpolynomial shape functions.

3.4.1. Polynomial function based-models

Qian et al. [12-13] and Gilhooley et al. [14] employed a meshless local Petrov- Galerkin method and the quasi-3D of Batra and Vidoli [15] to study the bending and vibration of FG plates. This quasi-3D theory was also used by Sheikholeslami and Saidi [16] to study the vibration of FG plates resting on an elastic foundation using Navier solution. Qian and Batra [17] extended their previous work to the transient problems of FG plates under thermal and mechanical loadings. Patel et al. [18] studied the free vibration characteristics of FG elliptical cylindrical shells using a quasi-3D theory and the finite element method.

3.4.2. Non-polynomial function based-models

The non-polynomial function was first used by Levy [29] with a sinusoidal function to develop a refined theory for thick isotropic plates. The sinusoidal function was later adopted by Stein [19] and Touratier [20] to develop a five-unknown sinusoidal shear deformation theory (SSDT) for isotropic and laminated composite plates, respectively.

3.5. Simplified theories

It is well known that the HSDTs and quasi-3D theories developed by expanding the displacements in power series of the thickness coordinate are more computationally expensive since each additional power of the thickness coordinate will induce an additional unknown to the theory. Therefore, there is a need to simplify the existing HSDTs and quasi-3D theories or to develop simple theories with fewer unknowns.

Senthilnathan et al. [24] simplified the TSDT by dividing the transverse displacement into the bending and shear components and making further assumptions to the TSDT. Therefore, the number of unknowns is reduced by one. In fact, the idea of partitioning the transverse displacement into the bending and shear parts was first proposed by Huffington [25] and later adopted by Krishna Murty [26].

3.6. Mixed theories

The above-mentioned ESL models are developed based on the principle of virtual displacements (PVD) where the displacement components are regarded as the primary variables and the stress components are calculated from the displacement components using the strain-displacement and constitutive relationships. An alternative variational approach, namely the Reissner mixed variational theorem (RMVT), was proposed by Reissner [27-28] by assuming two independent fields for the displacements and transverse stresses. The advantage of the RMVT over the PVD is that the compatibility of displacements and the equilibrium between two adjacent layers can be ‘‘naturally” satisfied. This is the main reason why RMVT can be considered as a powerful tool for the analysis of multilayered plates.

4. Identified Problem

In pressure vessels or hollow cylindrical shells reduction of weight is an important criteria with retaining or even enhancing its stress bearing capacity. By implying FGM the weight can be reduced, it will not change the working of cylindrical shell. Different combinations of metal - metal and metal - ceramic will be carried out. Exponential law is selected to solve the problem through APDL - ANSYS Parametric Design Language, since modulus of elasticity varies exponentially therefore the outcome will be more precise.

Conclusion

It is expected that the hollow cylindrical vessel prepared using FGM will have greater sustainability over conventionally prepared pressure vessel, and different combination of metal - metal and metal – ceramic can be used.

References

[1] Shen M, Bever MB. Gradients in polymeric materials. J Mater Sci 1972;7:741–6.
[2] Koizumi M. FGM activities in Japan. Compos Part B Eng 1997;28:1–4.
[3] Makwana AB, Panchal KC. A review of stress analysis of functionally graded material plate with cut-out. Int J Eng Res Technol 2014;3:2020–5.
[4] Bohidar SK, Sharma R, Mishra PR. Functionally graded materials: A critical review. Int J Res 2014;1:289–301.
[5] Mindlin RD. Influence of rotatory inertia and shear on flexural motions of isotropic, elastic plates. Journal of Applied Mechanics 1951;18(1):31-38.

[6] Reissner E. On bending of elastic plates. Quarterly of Applied Mathematics 1947;5(1):55-68.
[7] Reissner E. The effect of transverse shear deformation on the bending of elastic plates. Journal of Applied Mechanics 1945;12(2):69-72.
[8] Wang CM, Lim GT, Reddy JN, Lee KH. Relationships between bending solutions of Reissner and Mindlin plate theories. Engineering Structures 2001;23(7):838-849.
[9] Reddy JN. A simple higher-order theory for laminated composite plates. Journal of Applied Mechanics 1984;51:745-752.
[10] Feldman E, Aboudi J. Buckling analysis of functionally graded plates subjected to uniaxial loading. Composite Structures 1997;38(1):29-36.
[11] Levinson M. An accurate, simple theory of the statics and dynamics of elastic plates. Mechanics Research Communications 1980;7(6):343-350.
[12] Qian LF, Batra RC, Chen LM. Static and dynamic deformations of thick functionally graded elastic plates by using higher-order shear and normal deformable plate theory and meshless local Petrov–Galerkin method. Composites Part B: Engineering 2004;35(6):685-697.
[13] Qian LF, Batra RC, Chen LM. Free and forced vibrations of thick rectangular plates using higher-order shear and normal deformable plate theory and meshless Petrov-Galerkin (MLPG) method. Computer Modeling in Engineering and Sciences 2003;4(5):519-534.
[14] Gilhooley DF, Batra RC, Xiao JR, McCarthy MA, Gillespie Jr JW. Analysis of thick functionally graded plates by using higher-order shear and normal deformable plate theory and MLPG method with radial basis functions. Composite Structures 2007;80(4):539-552.
[15] Batra RC, Vidoli S. Higher-order piezoelectric plate theory derived from a three-dimensional variational principle. AIAA Journal 2002;40(1):91-104.
[16] Sheikholeslami SA, Saidi AR. Vibration analysis of functionally graded rectangular plates resting on elastic foundation using higher-order shear and normal deformable plate theory. Composite Structures 2013;106:350-361.
[17] Qian LF, Batra RC. Transient thermoelastic deformations of a thick functionally graded plate. Journal of Thermal Stresses 2004;27(8):705-740.
[18] Patel BP, Gupta SS, Loknath MS, Kadu CP. Free vibration analysis of functionally graded elliptical cylindrical shells using higher-order theory.
[19] Stein M. Nonlinear theory for plates and shells including the effects of transverse shearing. AIAA Journal 1986;24(9):1537-1544.
[20] Touratier M. An efficient standard plate theory. International Journal of Engineering Science 1991;29(8):901-916.
[21] Zenkour AM. Thermal effects on the bending response of fiber-reinforced viscoelastic composite plates using a sinusoidal shear deformation theory. Acta Mechanica 2004;171(3-4):171-187.
[22] Ferreira A, Roque C, Jorge R. Analysis of composite plates by trigonometric shear deformation theory and multiquadrics. Computers & Structures 2005;83(27):2225-2237.
[23] Zenkour AM. The refined sinusoidal theory for FGM plates on elastic foundations. International Journal of Mechanical Sciences 2009;51(11–12):869- 880.
[24] Senthilnathan NR, Chow ST, Lee KH, Lim SP. Buckling of shear-deformable plates. AIAA Journal 1987;25(9):1268-1271.
[25] Huffington NJ. Response of elastic columns to axial pulse loading. AIAA Journal 1963;1(9):2099-2104.
[26] Krishna Murty AV. Flexure of composite plates. Composite Structures 1987;7(3):161-177.
[27] Reissner E. On a mixed variational theorem and on shear deformable plate theory. International Journal for Numerical Methods in Engineering 1986;23(2):193-198.
[28] Reissner E. On a certain mixed variational theorem and a proposed application. International Journal for Numerical Methods in Engineering 1984;20(7):1366-1368.
[29] Levy M. Memoire sur la theorie des plaques elastiques planes. Journal de Mathematiques Pures et Appliquees 1877:219-306.