

## A Review on Tri-Directional Functionally Graded Beam with Various Boundary Condition

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### Abstract

*Functionally graded beam is a type of composite beam where the variation of material properties is created artificially as per a predefined function. This variation of material properties may be in any single dimensional direction or any two dimensional directions or in all the three dimensional direction. This governing function may be standard or may be customized as any author derives it. The structural behavior and the modal characteristic of a functionally graded beam depend upon the nature of governing function of the properties of the graded beam. In the present manuscript, forced vibration of tri-directional functionally graded material (TDFGM) beam due to a moving load is studied by using the energy approach. It is assumed that the material properties of the beam change exponentially in both axial and thickness directions. At the same time, free vibration frequencies are presented. The influences of the material gradation, moving load velocity, aspect ratio and boundary conditions on the dynamic responses of TDFGM beam is examined in detail.*

**Keywords:** Functionally graded beams, Natural frequency, Free vibration

### 1. Introduction

Pure metals are of little use in engineering applications because of the demand of conflicting property requirement. For example, an application may require a material that is hard as well as ductile, there is no such material existing in nature. To solve this problem, combination (in molten state) of one metal with other metals or non-metals is used. This combination of materials in the molten state is termed alloying (recently referred to as conventional alloying) that gives a property that is different from the parent materials. Bronze, alloy of copper and tin, was the first alloy that appears in human history. Since then, man has been experimenting with one form of alloy or the other with the sole reason of improving properties of material. When more quantity of the alloying material is desired, then the traditional alloying cannot be used. Another limitation of conventional alloying is when alloying two dissimilar materials with wide apart melting temperatures; it becomes prohibitive to combine these materials through this process.

Composite material are a class of advanced material, made up of one or more materials combined in solid states with distinct physical and chemical properties. Composite material offers an excellent combination of properties which are different from the individual parent materials and are also lighter in weight. Composite materials will fail under extreme working conditions through a process called delamination (separation of fibers from the matrix). This can happen for example, in high temperature application where two metals with different coefficient of expansion are used. To solve this problem, the FGM concept originated by researchers in Japan in 1984 during the space-plane project, in the form of a proposed thermal barrier material capable of withstanding a surface temperature of 2000 K and a temperature gradient of 1000 K across a cross-section <10 mm, came up with a novel material called Functionally Graded Material (FGM).

Functionally Graded Material (FGM), a revolutionary material, belongs to a class of advanced materials with varying properties over a changing dimension. The main advantages of FGMs over the classical composites are that cracking and delamination phenomenon, stress concentrations and residual stresses can be avoided, and thus structural integrity can be maintained to a desirable level. Due to the wide applications of FGMs in engineering structures, static, buckling, free and forced vibration behavior of FG structures have been examined extensively by several researchers. On the other hand, since the temperature field in advanced machines such as modern aerospace shuttles and craft develops in two or three directions, conventional FG materials may not be useful in the design of such structures. As a result of this demand, the material properties of FGMs are required to be graded in two or three directions.

## 2.Literature Review

In this recent year many works have been done on the analysis of Functionally Graded Beam regarding the variation of their mechanical properties as per a governing equation controlling the properties spatially and temporally. E. Amalet al [1], presented the dynamic characteristics of functionally graded beam with material gradation in axially or transversally through the thickness based on the power law. The present model is more effective for replacing the non-uniform geometrical beam with axially or transversally uniform geometrical graded beam. The system of equations of motion is derived by using the principle of virtual work under the assumptions of the Euler–Bernoulli beam theory. The finite element method is employed to discretize the model and obtain a numerical approximation of the motion equation. The model has been verified with the previously published works and found a good agreement with them. Numerical results are presented in both tabular and graphical forms to figure out the effects of different material distribution, slenderness ratios, and boundary conditions on the dynamic characteristics of the beam. The above mention effects play very important role on the dynamic behavior of the beam. JinChunhua et al [2], Based on the classical beam theory (CBT) and differential quadrature (DQ) rule, an N-node novel weak form quadrature functionally graded (FG) beam element has been established in this paper. Both Young's modulus and mass density of the beam materials have been made varied exponentially through the thickness. The element node points are different from the integration points. Either Gauss–Lobatto–Legendre (GLL) quadrature or Gauss quadrature has been used to obtain the element stiffness matrix and mass matrix. Detailed formulations are given. Convergence study has been performed. For verification, results have been compared with available solutions in literature. It has been shown that the proposed thin beam element can yield very accurate frequencies with relatively small number of nodal points. MesutSimsek[3], studied nonlinear free vibration of axially functionally graded (AFG) Euler–Bernoulli micro-beams with immovable ends by using the modified couple stress theory. The nonlinearity of the problem stems from the von-Kármán's nonlinear strain–displacement relationships. Elasticity modulus and mass density of the micro-beam vary continuously in the axial direction according to a simple power-law form. The nonlinear governing partial differential equation and the associated boundary conditions are derived by Hamilton's principle. By using Galerkin's approach, the nonlinear governing partial differential equation is reduced to a nonlinear ordinary differential equation. He's variational method is utilized to obtain the approximate closed form solution of the nonlinear ordinary governing equation. Pinned–pinned and clamped–clamped boundary conditions are considered. The influences of the length scale parameters, material variation, vibration amplitude, and boundary conditions on vibration responses are examined in detail. FarzadEbrahimi[4], investigated for the first time the large-amplitude nonlinear vibration characteristics of functionally graded (FG) Timoshenko beams made of porous material. Material properties of FG porous beam are supposed to vary continuously along the thickness

according to the rule of mixture which modified to approximate material properties with porosity phases. The governing equations are derived based on Timoshenko beam theory through Hamilton's principle and they are solved utilizing both Galerkin's method and the method of multiple scales. According to the numerical results, it is revealed that the proposed modeling can provide accurate frequency results of the FG porous beams as compared to the literature. The detailed mathematical derivations are presented and numerical investigations are performed while the emphasis is placed on investigating the effect of the several parameters such as material distribution profile, porosity volume fraction, aspect ratio and mode number on the normalized natural frequencies of the FG porous beams in detail. It is explicitly shown that the vibration behavior of a FG beams is significantly influenced by these effects. Numerical results are presented to serve as benchmarks for future analyses of FG porous beams. Kyungho Yoon et al [5], presented a geometrically nonlinear finite element formulation for analysis of functionally graded 3D beams. The proposed formulation employs the continuum mechanics based beam element with the warping displacement to model complex modes of deformation. The novelty of the developed method is that the warping function can be accurately calculated for any beam with arbitrary cross-sections and material grading patterns. Superb performances of the proposed beam element are demonstrated through several representative examples. Mesut Simsek [6], investigated free and forced vibration of bi-directional functionally graded (BDFG) Timoshenko beam under the action of a moving load. The material properties of the beam vary exponentially in both axial and thickness directions. The equations of motion are derived by means of Lagrange equations based on Timoshenko beam theory (TBT) as well as Euler–Bernoulli beam (EBBT) theory. In order to obtain free and forced vibration responses, the trial functions for axial, transverse deflections and rotation of the cross-sections are expressed in polynomial forms. Various boundary conditions considered in the study are satisfied by adding auxiliary functions to the displacement functions. The resulting time dependent equations of motion are solved with the help of the implicit time integration Newmark-b method. At the same time, free vibration frequencies are presented. Some numerical results are provided in tabular form and in figures to examine the effects of the material gradation, moving load velocity, aspect ratio and different boundary conditions on the dynamic responses of BDFG beam. X.L. Jia et al [7], investigated the size effect on the free vibration of functionally graded micro-beams under the combined electrostatic force, temperature change and Casimir force based on Euler–Bernoulli beam theory and von Kármán geometric nonlinearity. Taking into consideration the temperature-dependency of the effective material properties, material properties of the functionally graded materials (FGMs) are assumed to be graded in the thickness direction according to the Voigt model and exponential distribution model. The principle of minimum total potential energy is used to derive the nonlinear governing differential equation which is then solved using the differential quadrature method (DQM). A parametric study is conducted to show the significant combined effects of the size effect, material gradient, temperature change, geometric parameters. C.M.C. Roquet et al [8], used Differential evolution optimization to find the volume fraction that maximizes the first natural frequency for a functionally graded beam. A formulation using three parameters is used to describe volume fraction. Beams with different ratios of material properties are considered. Two methods are used to compute the natural frequencies, analytical and meshless numerical method. Results show that differential evolution is capable to find distributions for volume fraction that increase the natural frequency of beams. It was also found that the RBF numerical method can be used with differential evolution to solve problems related to maximization of natural frequencies in functionally graded beams. L.W. Zhang et al [9], employed the element-free improved moving least-squares Ritz (IMLS-Ritz) method for the analysis of the problem of the free vibration of functionally graded carbon nanotube (FG-CNT) reinforced composite moderately thick rectangular plates with edges elastically

restrained against transverse displacements and rotation of the plate cross section. In their work they used the first-order shear deformation theory (FSDT), to account for transverse shear strains and rotary inertia. The numerical results which they got were validated through comparison and convergence studies. Zhi-haiWanget al [10], presented a theoretical investigation in free vibration of a functionally graded beam which has variable material properties along the beam length and thickness. It is assumed that material properties vary through the length according to a simple power law distribution with an arbitrary power index and have an exponential gradation along the beam thickness. The characteristic equations are derived in closed form. The governing equation can analytically reduce to the classical forms of Euler–Bernoulli beams if the gradient index disappears. Analytical solutions of the natural frequencies are obtained for graded beams with clamped-free and hinged–hinged end supports. Results show that the variations of material properties in the beam length and thickness have a strong influence on the natural frequencies. It is also shown that there exists a critical frequency depending on the gradient parameter. The natural frequencies have an abrupt jump when across its critical frequencies. The derived results can be useful for designing non-homogeneous beams which may be required to vibrate with a particular frequency.

### 3. Conclusion

However, the vast majority of the previous works deal with FG structures whose material properties vary in only thickness or axial direction. Very few numbers of studies on beams and plates made of bi-directional functionally graded material (BDFGM) or two-dimensional functionally graded material (2D-FGM) have been studied. Thus, the number of studies on beams and plates made of tri-directional functionally graded material (TDFGM) i.e. 3-dimensional functionally graded material (3D-FGM) is still very limited.

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