Linearization Algorithms of Non Differentiable Exact Penalty Function using different stepsize function

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Abstract: A non linear programming problem is a part of mathematical optimization. It is used in many problems but does not give exact solution of the problem. In this case, linearization plays an important role for solving this type of problems. The present study deals with Non-differentiable Exact Penalty method which requires only a single unconstrained problem. Here, three stepsizerules are used: (i) Minimization rule (ii) Limited Minimization rule (iii) Armijo rule. Stepsize of linearization method helps in the convergence of the function. This paper tells how a stepsize rule can be chosen for convergence of the function.

Keywords: Non-Linear Programming,Stepsize, Exact Penalty Method, Constrained and Unconstrained Problem, Linearization Algorithm.

Introduction: Firstly we will define the problem

(NLP) Minimize $f(x)$

Subject to $h(x)=0, \quad g(x)\le 0$

Where $f: R^n \to R,\ h: R^n \to R^m,\ g: R^n \to R^r$, and $m \le n$.

Special Cases of (NLP) are

(ECP) Minimize $f(x)$

Subject to $h(x)=0$

And

(ICP) Minimize $f(x)$

Subject to $g(x)\le 0$

Here $f,g,h \in C^1$ on R^n and the components of h and g are denoted by $h_1,\ldots,h_m$ and $g_1,\ldots,g_r$ respectively.

NOTE: A pair (triple) of vectors is said to be a Kuhn-Tucker (K-T) pair (triple) if it satisfies the first-order necessary optimality conditions. If (x^*,λ^*,μ^*) is a K-T triple for NLP, then it must satisfies

$$\nabla f(x^*) + \nabla h(x^*)\lambda^* + \nabla g(x^*)\mu^* = 0,$$

$$h(x^*) = 0, \qquad g(x^*) \le 0, \qquad \mu^* \ge 0, \qquad \mu_j^* g_j(x^*) = 0, \qquad j = 1,\ldots,r.$$

Non differentiable Exact Penalty Functions

In this, we will show that the solutions of (NLP) are related to the solutions of non differentiable unconstrained problem

$(NDP)_c$ Minimize $f(x) + cP(x)$

subject to $x \in R^n$

$P(x)$ is defined by

$P(x) = \max\{0, g_1(x), \ldots g_r(x), |h_1(x)|, \ldots, |h_m(x)|\}$ and $c > 0$.

Proposition 1: The vector x^* will be strict unconstrained local minimum of $f + cP$ if

$$c > \sum_{i=1}^{m} |\lambda_i^*| + \sum_{j=1}^{r} \mu_j^*$$

where the vector x^* will be strict local minimum of (NLP) satisfying assumptions, together with corresponding Lagrange multiplier vectors λ^* and μ^*.

Inequality Constrained Problems

The general problem is defined as

(ICP) Minimize $f(x)$

Subject to $g(x) \le 0,$

And for $c > 0$, the corresponding problem

$(NDP)_c$ Minimize $f(x) + cP(x)$

subject to $x \in R^n$

for convenience, we take $g_0(x) = 0 \qquad \forall x \in R^n$

so $P(x) = \max\{g_0(x), g_1(x), \ldots g_r(x)\}$

also we denote $J(x) = \{j \mid g_j(x) = P(x), j = 0,1,\ldots,r\}$

and $\theta_c(x;d) = \max\{[\nabla f(x) + c\nabla g_j(x)]' d \mid j \in J(x)\}$, for $x \in R^n, d \in R^n$ and $c > 0$.

Definition: A point x^* is said to be critical point of $f(x) + cP(x)$ if $\forall\, d \in R^n$ there exists

$\theta_c(x^*;d) \geq 0.$

We can find the descent direction of $f(x) + cP(x)$ only at noncritical points. These directions are obtained by the convex quadratic program given below in $(d,\xi) \in R^{n+1}$,

$$(QP)_c(x,H,J)\,\text{minimize}\,\nabla f(x)' d + \frac{1}{2} d' Hd + c\xi$$

subject to $g_j(x) + \nabla g_j(x)' d \leq \xi, \qquad j \in J,$

where $c > 0$

H = positive definite matrix

J = index set containing $J(x)$, which means

$0 < c, \quad 0 < H, \quad J(x) \subset J \subset \{0,1,\ldots,r\}$

The above quadratic program has unique optimal solution.

Proposition 2: $f(x + \alpha d) + cP(x + \alpha d) - f(x) - cP(x) = \alpha\theta_c(x;d) + o(\alpha)\ \forall x \in R^n, d \in R^n$ and $\alpha > 0$ andalso $\lim_{\alpha \to 0^+} o(\alpha)/\alpha = 0$. Then $\overline{\alpha} > 0$ exists when $\theta_c(x;d) < 0$ such that

$$f(x + \alpha d) + cP(x + \alpha d) < f(x) + cP(x) \qquad \forall \alpha \in (0, \overline{\alpha}]$$

(b) $\theta_c(x;d) \leq d' Hd < 0$ when (d,ξ) will be the optimal solution of quadratic program $(QP)_c(x,H,J)$ with $d \neq 0$ where $x \in R^n$, $H > 0$ and $J(x) \subset J \subset \{0,1,\ldots,r\}$.

Proposition 3: The quadratic program $(QP)_c(x^*,H,J)$ possess $\{d = 0, \xi = P(x^*)\}$ as optimal solution for all J and H where $H > 0$ and $J(x^*) \subset J \subset \{0,1,\ldots,r\}$ when x^* will be critical the point of $f(x) + cP(x)$.

(b) x^* will be critical the point of $f(x) + cP(x)$ whenever $\{d = 0, \xi = P(x^*)\}$ will become the optimal solution of quadratic program $(QP)_c(x^*,H,J)$ where $H > 0$ and $J(x^*) \subset J \subset \{0,1,\ldots,r\}$.

Proposition 4: $\{x^*,(\mu_1^*,...,\mu_r^*)\}$ is a K-T pair of (ICP) then corresponding to this K-T pair, a $\mu_0^* \geq 0$ exists such that $\{d^* = 0, \{\mu_j^* | j \in J\}\}$ becomes a K-T pair for quadratic program $(QP)_0(x^*, H, J)$ for all J and H where $H > 0$ and $J(x^*) \subset J \subset \{0,1,...,r\}$.

This result also holds conversely.

i.e. If we have $\{d^* = 0, \{\mu_j^* | j \in J\}\}$ as a K-T pair for quadratic program $(QP)_0(x^*, H, J)$ for some J and H where $H > 0$ and $J(x^*) \subset J \subset \{0,1,...,r\}$ then $\{x^*,(\mu_1^*,...,\mu_r^*)\}$ will be the K-T pair for (ICP). Here also $\mu_0^* = 0 \forall j \notin J$.

Proposition 5: $\{d, \xi = 0, \{\bar{\mu}_j | j \in \bar{J}\}\}$ will become the K-T pair for $(QP)_c(x, H, J)$ if we have $\{d, \{\mu_j | j \in J\}\}$ as a K-T pair for $(QP)_0(x, H, J)$ with $c \geq \sum_{\substack{j \in J \\ j \neq 0}} \mu_j$ where also we define

$$\bar{J} = J \cup \{0\}, \qquad \bar{\mu}_j = \mu_j \quad \forall j \in \bar{J},\ j \neq 0, \quad \bar{\mu}_0 = c - \sum_{\substack{j \in J \\ j \neq 0}} \mu_j .$$

Proposition 6: x^* will be the critical point of $f + cP$, $\forall c$ if we have $\{x^*,(\mu_1^*,...,\mu_r^*)\}$ as a K-T pair of (ICP) where $c \geq \sum_{j=1}^{r} \mu_j^*$.

Proposition 7: If the set of gradients $\{\nabla g_j(x) | j \in J(x), j \neq 0\}$ is linear independent $,\forall x \in X$ where X is a compact set. Then $c^* \geq 0$ exists with $c > c^*$ such that:

(a) A $\mu^* \in R^r$ exists such that (x^*, μ^*) will become a K-T pair for (ICP) when x^* is a critical point for $f + cP$ where $x^* \in X$.

(b) x^* will become a critical point for $f + cP$ if (x^*, μ^*) becomes a K-T pair for (ICP) where $x^* \in X$.

Proposition 8: For every $x \in X$, where X is a compact set satisfying above conditions, a unique vector $\bar{\mu}(x) = [\bar{\mu}_1(x),...,\bar{\mu}_r(x)]$ exists, minimizing over $\mu = (\mu_1,...,\mu_r)$ the function

$$q_x(\mu) = \left| \nabla f(x) + \sum_{j=1}^{r} \mu_j \nabla g_j(x) \right|^2 + \sum_{j=1}^{r} [P(x) - g_j(x)]^2 \mu_j^2$$

Also $\bar{\mu}(x^*) = \mu^*$, If (x^*, μ^*) is a K-T pair for (ICP) with $x^* \in X$ where $\bar{\mu}(.)$ is continuous over X.

Proposition 9: Suppose that $g_1,...,g_r$ are convex over R^n and a vector $\bar{x}$ exists such that

$g_j(\bar{x}) < 0 \qquad \forall\, j = 1,\dots,r.$

And for every compact set X, $c^* \geq 0$ exists such that for all $c > c^*$:

(a) When $x^* \in X$ and x^* is a critical point of $f + cP$ then $\mu^* \in R^r$ exists such that (x^*, μ^*) is a K-T pair for (ICP).
(b) When $x^* \in X$ and (x^*, μ^*) is a K-T pair for (ICP), then x^* is also a critical point of $f + cP$.

The above result will be proved with the help of following Lemma:

Lemma 10: Let X be a subset of R^n i.e. $X \subset R^n$ such that at least one solution of system of inequalities in d

$$g_j(x) + \nabla g_j(x)' d \leq 0, \qquad j \in J(x)$$

exists for each $x \in X$. By fixing $H > 0$ and suppose that $c^* \geq 0$, exists with the following properties:

For each $x \in X$, a set of Lagrange multipliers exists for $(QP)_0(x, H, J(x))$

$$\{\mu_j(x) | j \in J(x)\}$$

satisfying $c^* \geq \sum_{j \in J(x)} \mu_j(x)$

then $\forall\, c > c^*$:

(a) When $x^* \in X$ is a critical point of $f + cP$ then $\mu^* \in R^r$ exists such that (x^*, μ^*) is a K-T pair for (ICP).
(b) When $x^* \in X$ and (x^*, μ^*) is a K-T pair for (ICP), then x^* is a critical point of $f + cP$.

Proposition 11: Suppose that the functions $f, g_1, \dots, g_r$ are convex over R^n and at least one Lagrange multiplier vector $\mu^* = (\mu_1^*, \dots, \mu_r^*)$ for (ICP) exists such that $\mu_j^* \geq 0, j = 1,\dots,r,$ and

$$\inf_{x \in R^n} \{f(x) + \mu^{*'} g(x)\} = \inf_{g(x) \leq 0} f(x)$$

Then a vector x^* is a global minimum for (ICP) iff x^* is a global minimum of $f + cP$, for every $c > \sum_{j=1}^{r} \mu_j^*$.

Linearization Algorithms Based on Nondifferentiable Exact Penalty Functions

(1) Algorithms for Minimax Problems:

Firstly we will consider the algorithm which will help us for finding critical points of $f + cP$ where $c > 0$,

$P(x) = \max\{g_0(x), g_1(x), \ldots, g_r(x)\} \qquad \forall x \in R^n$

$g_0(x) = 0 \qquad \forall x \in R^n$

and $f, g_j \in C^1, j = 1, \ldots, r.$ Then we will go further and concentrate on algorithm and check the convergence analysis for (ICP).

Linearization Algorithm: firstly we will choose a vector $x_0 \in R^n$ and the kth iteration of algorithm takes a form

$$x_{k+1} = x_k + \alpha_k d_k, \qquad (1)$$

$\alpha_k =$ nonnegative scalar stepsize

$d_k =$ direction obtained by quadratic program in (d, ξ)

$(QP)_c(x_k, H_k, J_k)$ minimize $\nabla f(x_k)' d + \frac{1}{2} d' H_k d + c\xi$

subject to $g_j(x_k) + \nabla g_j(x_k)' d \leq \xi, \qquad j \in J_k$.

In this δ is some positive scalar which will be fixed throughout the algorithm and H_k and J_k must satisfy

$0 < H_k, \qquad J_\delta(x_k) \subset J_k \subset \{0, 1, \ldots, r\},$

where $J_\delta(x_k) = \{ j \mid g_j(x_k) \geq P(x_k) - \delta, j = 0, 1, \ldots, r\},$

The stepsize α_k can be chosen by any of stepsizes given below:

(a) Minimization rule: In this, α_k is chosen so that
$f(x_k + \alpha_k d_k) + cP(x_k + \alpha_k d_k) = \min_{\alpha \geq 0} \{ f(x_k + \alpha d_k) + cP(x_k + \alpha d_k) \}$

(b) Limited minimization rule: In this a fixed scalar $s > 0$ will be selected and α_k will be chosen so that
$f(x_k + \alpha_k d_k) + cP(x_k + \alpha_k d_k) = \min_{\alpha \in [0,s]} \{ f(x_k + \alpha d_k) + cP(x_k + \alpha d_k) \}$

(c) Armijo rule: In this we select fixed scalars $s, \beta,$ and $\sigma,$ with $s > 0, \beta \in (0,1)$ and $\sigma \in \left(0, \frac{1}{2}\right),$ and by taking $\alpha_k = \beta^{m_k} s,$ where m_k is the first nonnegative integer for which
$f(x_k) + cP(x_k) - f(x_k + \beta^m s d_k) - cP(x_k + \beta^m s d_k) \geq \sigma \beta^m s d_k' H_k d_k$ (2)

If $d_k \neq 0$, then Armijo rule gives a stepsize after few steps. We can see this also as we also have $\forall \alpha > 0,$

$$f(x_k)+cP(x_k)-f(x_k+\alpha\, d_k)-cP(x_k+\alpha\, d_k)=-\alpha\theta_c(x_k;d_k)+o(\alpha) \tag{3}$$

$$\geq \alpha\, d_k^{'} H_k d_k + o(\alpha)$$

Hence if $\alpha \in (0,\overline{\alpha}]$ and $\overline{\alpha} > 0$, then we have $(1-\sigma)\alpha\, d_k^{'} H_k d_k + o(\alpha) \geq 0$, using (3) we have

$$f(x_k)+cP(x_k)-f(x_k+\alpha\, d_k)-cP(x_k+\alpha\, d_k)=\sigma\alpha\, d_k^{'} H_k d_k \qquad \forall \alpha \in (0,\overline{\alpha}]$$

Also we have an integer m such that (2) is satisfied.

When we are implementing algorithm, it is convenient to solve a dual problem instead of solving $(QP)_c(x_k, H_k, J_k)$. Dual problem which involves maximization with respect to Lagrange multipliers $\mu_j, j \in J_k$, is given by

$$\text{Maximize } -\frac{1}{2}\left[\nabla f(x_k)+\sum_{j\in J_k}\mu_j \nabla g_j(x_k)\right]^{'} H_k^{-1}\left[\nabla f(x_k)+\sum_{j\in J_k}\mu_j \nabla g_j(x_k)\right]+\sum_{j\in J_k}\mu_j g_j(x_k)$$

$$\text{Subject to } \sum_{j\in J_k}\mu_j = c, \qquad \mu_j \geq 0 \quad \textit{for all } j \in J_k$$

It is easy to solve dual problem as it contains smaller number of variables than $(QP)_c(x_k, H_k, J_k)$ and also it has a simpler constraints set.

CONVERGENCE RESULT:

Proposition 12: When the sequence $\{x_k\}$ is generated by the linearization algorithm and α_k, the stepsize of iteration is chosen by any of the three rules given above then γ and Γ, two positive scalars exists such that

$$\gamma\left|z^2\right| \leq z^{'} H_k z \leq \Gamma\left|z^2\right| \qquad \forall z \in R^n, \quad k = 0,1,... \tag{4}$$

Then every limit point of the sequence $\{x_k\}$ is a critical point of $f + cP$.

If x is not a critical point of $f + cP$ then (4) can be replaced by the condition given below:

$$\gamma\left|w(x_k)\right|^{q_1}\left|z^2\right| \leq z^{'} H_k z \leq \Gamma\left|w(x_k)\right|^{q_2}\left|z^2\right| \qquad \forall z \in R^n, \quad k = 0,1,...$$

In this $w(\,.\,)$ is a continuous function with $w(x) \neq 0$ and q_1, q_2 are two nonnegative scalars.

The result of above proposition is also holds if Armijo rule takes the form

$$f(x_k)+cP(x_k)-f(x_k+\beta^{m_k} s\, d_k)-cP(x_k+\beta^{m_k} s\, d_k) \geq -\sigma\xi_c(x_k;\beta^{m_k} s\, d_k),$$

where ξ_c is given by

$$\xi_c(x;d)=\nabla f(x)' d + c\max\{g_j(x)+\nabla g_j(x)' d \mid j=0,1,\ldots,r\}-cP(x) \quad (5)$$

Algorithms for Constrained Optimization Problems

The inequality constrained problem is given by

(ICP) minimize $f(x)$

subject to $g_j(x)\le 0, \quad j=1,\ldots,r$

As we already know that when (x^*,μ^*) will become a K-T pair of (ICP) then there also exists a critical point of $f+cP$ provided $c\ge\sum_{j=1}^{r}\mu_j^*$. For finding the critical values of $f+cP$, we can apply linearization algorithm. But the difficulty with this method is only that we may not know a threshold value for c. In this situation, we choose an initial value for c and increase it until we can't find adequate value c_k for the algorithm. And suitable value of c_k is $\sum_{\substack{j\in J_k\\ j\ne 0}}\mu_j^k$ with $\{\mu_j^k \mid j\in J_k\}$, where $\{\mu_j^k \mid j\in J_k\}$ are Lagrange multipliers which are obtained by solving $(QP)_0(x_k,H_k,J_k)$. Also we know that if

$$c_k \ge \sum_{\substack{j\in J_k\\ j\ne 0}}\mu_j^k$$

then $(QP)_0(x_k,H_k,J_k)$ and $(QP)_{c_k}(x_k,H_k,J_k\cup\{0\})$ are equivalent with d_k, as a optimal solution of the former iff $(d_k,0)$ is the optimal solution of the latter. Hence by solving $(QP)_0(x_k,H_k,J_k)$, we can solve $(QP)_{c_k}(x_k,H_k,J_k\cup\{0\})$ and also we can obtain a suitable value of c_k.

Modified Linearization Algorithm: in this, firstly we select a vector $x_0\in R^n$ and a penalty parameter $c_0>0$ and the kth iteration of the algorithm takes the form

$$x_{k+1}=x_k+\alpha_k d_k, \qquad c_{k+1}=\bar{c}_k,$$

where α_k is a stepsize parameter, chosen by any one of stepsize rules given above and here c is replaced by $\bar{c}_k$ which means that here minimization rules takes the form

$$f(x_k+\alpha_k d_k)+\bar{c}_k P(x_k+\alpha_k d_k)=\min_{\alpha\ge 0}\{f(x_k+\alpha d_k)+\bar{c}_k P(x_k+\alpha d_k)\}$$

Here the vector d_k and the scalar $\bar{c}_k$ are depends upon x_k, c_k, a matrix H_k and an index set J_k satisfying

$$0<H_k, \qquad J_\delta(x_k)\subset J_k\subset\{0,1,\ldots,r\},$$

where $J_\delta(x_k)=\{j \mid g_j(x_k)\ge P(x_k)-\delta,\ j=0,1,\ldots,r\}$

where the scalar $\delta>0$ is fixed throughout the algorithm.

Now we will discuss two cases:

Case 1: When $d \in R^n$ exists and satisfying

$$g_j(x_k) + \nabla g_j(x_k)' d \leq 0 \qquad \textit{for all } j \in J_k \tag{6}$$

In this we take

$$(QP)_0(x_k, H_k, J_k) \text{ minimize } \nabla f(x_k)' d + \frac{1}{2} d' H_k d$$

subject to $g_j(x_k) + \nabla g_j(x_k)' d \leq 0 \qquad \textit{for all } j \in J_k$

and d_k is the unique solution of $(QP)_0(x_k, H_k, J_k)$.

Also $\bar{c}_k$ is defined by

$$\bar{c}_k = \begin{cases} \sum\limits_{\substack{j \in J_k \\ j \neq 0}} \mu_j^k + \varepsilon & \textit{if } \sum\limits_{\substack{j \in J_k \\ j \neq 0}} \mu_j^k \geq c_k \\ c_k & \textit{otherwise} \end{cases}$$

where $\{\mu_j^k \mid j \in J_k\}$ = set of Lagrange multipliers for $(QP)_0(x_k, H_k, J_k)$

and $\varepsilon > 0$ = scalar, which is fixed throughout the algorithm.

NOTES: (1) If we have equality constraints of the form $h_i(x) = 0$, then we can convert this into inequality constraints of the form $h_i(x) \leq 0$, and $-h_i(x) \leq 0$. In this case, we take the quadratic program of the form

$$\text{minimize } \nabla f(x_k)' d + \frac{1}{2} d' H_k d$$

subject to $g_j(x_k) + \nabla g_j(x_k)' d \leq 0 \qquad \textit{for all } j \in J_k$

$$h_i(x_k) + \nabla h_i(x_k)' d = 0 \qquad \textit{for all } i \in I_k$$

where I_k = an index set containing $\{i \,|\, |h_i(x_k)| \geq P(x_k) - \delta\}$. Now $\bar{c}_k$ becomes

$$\bar{c}_k = \begin{cases} \sum\limits_{\substack{j \in J_k \\ j \neq 0}} \mu_j^k + \sum\limits_{i \in I_k} |\lambda_i^k| + \varepsilon & \textit{if } \sum\limits_{\substack{j \in J_k \\ j \neq 0}} \mu_j^k + \sum\limits_{i \in I_k} |\lambda_i^k| \geq c_k \\ c_k & \textit{otherwise} \end{cases}$$

where $\{\mu_j^k, \lambda_i^k \mid j \in J_k, i \in I_k\}$ = set of Lagrange multipliers for quadratic program.

(2) We can solve dual problem in $\mu_j, j \in J_k$ instead of solving $(QP)_0(x_k, H_k, J_k)$, which is given by

$$\text{maximize} -\frac{1}{2}\left[\nabla f(x_k)+\sum_{j\in J_k}\mu_j \nabla g_j(x_k)\right]' H_k^{-1}\left[\nabla f(x_k)+\sum_{j\in J_k}\mu_j \nabla g_j(x_k)\right]+\sum_{j\in J_k}\mu_j g_j(x_k)$$

subject to $\mu_j \geq 0, \quad j \in J_k$

Case 2: When $d \in R^n$ does not exists which satisfies (6). In this case, d_k and $\xi_k > 0$ are the two unique solutions of

$$(QP)_{c_k}(x_k, H_k, J_k) \text{minimize} \nabla f(x_k)' d_k + \frac{1}{2} d_k' H_k d_k + c_k \xi$$

subject to $g_j(x_k) + \nabla g_j(x_k)' d \leq \xi \qquad j \in J_k$

here $\bar{c}_k = c_k$.

By this, we see that if for the sequence $\{x_k\}$, the system of equation (6) is feasible for an infinite number of indices k with $\sum_{j\in J_k, j\neq 0}\mu_j^k \geq c_k$ then the sequence $\{c_k\}$ generated by the above algorithm will be unbounded. Otherwise we will get $c_k = \bar{c}$ for some $\bar{c} > 0$ and then the above algorithm will be equivalent to the linearization algorithm.

Proposition 13: Suppose that a sequence $\{x_k\}$ generated by the modified linearization algorithm where the stepsize α_k can be chosen any of the manner either by minimization rule or limited minimization rule or the Armijo rule. Suppose that two positive scalars γ and Γ exists such that

$$\gamma|z|^2 \leq z' H_k z \leq \Gamma |z|^2 \qquad \textit{for all } z \in R^n, \qquad k = 0,1,...$$

(a) When $\bar{k}$ and $\bar{c}$ exists such that

$$c_k = \bar{c} \qquad \forall k \geq \bar{k}, \tag{7}$$

Then every limit point of the sequence $\{x_k\}$ is a critical point of $f + \bar{c}P$. Furthermore if the system of inequalities

$$g_j(x_k) + \nabla g_j(x_k)' d \leq 0 \qquad \forall j \in J_k \tag{8}$$

have solution for an infinite set of indices K, every limit point of the sequence $\{x_k, \mu^k\}_K$ is a K-T pair of (ICP), where we have for $k \in K$

$\mu^k = (\mu_1^k, ..., \mu_r^k)$,

where $(QP)_0(x_k, H_k, J_k)$ has $\{\mu_j^k \mid j \in J_k\}$ as a set of Lagrange multipliers and $\mu_j^k = 0$ for $j \notin J_k$.

(b) When the functions $g_1, ..., g_r$ are convex and a vector $\bar{x} \in R^n$ exists such that

$$g_j(\bar{x}) < 0 \qquad \forall j = 1, ..., r$$

and $\{x_k\}$ is bounded, then also every limit point of $\{x_k, \mu^k\}$ is a K-T pair of (ICP).

Conclusion: Stepsize has vital role in Linearization algorithm. In this paper, with Non-Differentiable Exact Penalty function, different stepsize functions are discussed. Kuhn-Tucker (K-T) pair and critical points are discussed under certain conditions. Convergence result is also discussed. Linearization Algorithms and Modified Linearization Algorithms based on Non-Differentiable Exact Penalty functions are discussed.

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