Generalized $(\in,\in\vee q_{\bar{k}}^{\bar{\delta}})$-interval Valued fuzzy subnear-rings and $(\in,\in\vee q_{\bar{k}}^{\bar{\delta}})$-interval valued fuzzy ideals in Near-rings

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Abstract: *In this paper, we introduce the notion of* $(\in,\in\vee q_{k}^{\bar{\delta}})$ *interval valued fuzzy (in short,i-v fuzzy) bi-ideal and* $(\in,\in\vee q_{k}^{\bar{\delta}})$ *i-v fuzzy quasi-ideal in near-rings. We show that each* $(\in,\in\vee q_{k}^{\bar{\delta}})$ *i-v fuzzy quasi-ideal is in* $(\in,\in\vee q_{k}^{\bar{\delta}})$ *i-v fuzzy bi-ideal and each* $(\in,\in\vee q_{k}^{\bar{\delta}})$ *i-v fuzzy left (right) ideal is an* $(\iota n,\in\vee q_{k}^{\bar{\delta}})$ *i-v fuzzy quasi-ideal but the converses not true in general.*

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1. Introduction

In 1965 Zadeh [13] introduced the concept of fuzzy subsets and studied their properties on the lines parallel to set theory. In 1971, Rosenfeld [10] defined a fuzzy subgroup and gave some of its properties. The notions of fuzzy subnear-ring and ideal were first introduced by Abou-Zaid [1]in 1991. The concept of quasi-coincidence of a fuzzy point with a fuzzy subset was introduced by Pu Pao-Ming and Liu-Ying-Ming [7] in 1980. The idea of quasi-coincidence of a fuzzy point with a fuzzy set was introduced by Bhakat and Das [2] in 1992. Pu and Lia [7] introduced the notion of "belongs to" relation $(\in)$. In [8], Murali initiated the notion of belongingness (q) of a fuzzy point to a fuzzy subset under an expected equality on a fuzzy subset. These two notions played a vital role in generating some different types of fuzzy subgroups.

The concept of $(\in,\in\vee q)$-fuzzy subgroups is a possible generalization of Rosenfeld's fuzzy subgroups. The idea of $(\in,\in\vee q)$-fuzzy ideals are introduced in [3]. In [5], Davvaz introduced the concept of $(\in,\in\vee q)$-fuzzy ideals in a near-ring. The concept of $(\in,\in\vee q_k)$-fuzzy subsemigroup was initiated by Kang in [6]. In [11], Shabir et al, generalized the concept of $(\in,\in\vee q)$-fuzzy bi(interior, quasi)-ideal of semigroup and introduced the notion of an $(\in,\in\vee q_k)$-fuzzy bi-(interior, quasi)-ideal in a semigroup. Narayanan and Manikantan [9] have extended these results to near-rings. The notion of an $(\in,\in\vee q_k)$- fuzzy subnear-ring which is a generalization of an $(\in,\in\vee q)$-fuzzy subnear-ring.

As a generalization of fuzzy set Zadeh [13] in 1975 introduced a new notion of fuzzy subsets viz., interval valued (i-v) fuzzy subset, where the values of the membership function are closed intervals of numbers instead of a number. Thillaigovindan et.al.,[??0] introduced the notion of i-v fuzzy subnear-ring and i-v fuzzy left (right) ideal of near-ring and investigated some of their properties. In this paper, we introduce the concept of $(\in, \in\vee q_{\overline{k}}^{\overline{\delta}})$ i-v fuzzy (generalized) bi-ideal, $(\in, \in\vee q_{\overline{k}}^{\overline{\delta}})$-i-v fuzzy quasi-ideal in near-rings. We show that each $(\in, \in\vee q_{\overline{k}}^{\overline{line\delta}})$-i-v fuzzy bi-ideal and each $(\in, \in\vee q_{\overline{k}}^{\overline{\delta}})$-i-v fuzzy left (right) ideal is an $(\in, \in\vee q_{\overline{k}}^{\overline{\delta}})$-i-v fuzzy quasi-ideal but the converses are not true in general.

2. Preliminaries

We first recall some basic concepts for the sake of completeness. By a near-ring [10] we mean a non-empty set N with two binary operations `$+'$ and `$.'$ satisfying the following axioms:

(i) $(N, +)$ is a group,

(ii) $(N, .)$ is a semigroup,

(iii) $(x + y).z = x.z + y.z \; \forall \; x, y, z \in N$.

Precisely speaking, it is a right near-ring because it satisfies the right distributive law. We will use the word `` near-ring" to mean `` right near-ring". We denote xy instead of $x.y$. Note that $0x = 0$ but in general $x0 \neq 0$ for some $x \in N$.

If P and Q are two non-empty subsets of N we define

$PQ = \{ab/a \in P, \; b \in Q\}$

and

$P * Q = \{a(b + i) - ab/a, b \in P, i \in Q\}$.

A subgroup M of a near-ring N is called a subnear-ring of N if $MM \subseteq M$.

A near-ring N is called zero-symmetric if $x0 = 0 \; \forall x \in N$. A subset I of a near-ring N is called an ideal of N if

(i) $(I, +)$ is a normal subgroup of $(N, +)$,

(ii) $IN \subseteq I$,

(iii) $a(b + i) - ab \in I \; \forall \; a, b \in N$ and $i \in I$, that is, $N * I \subseteq I$.

A normal subgroup I of $(N, +)$ with (ii) is called a right ideal of N while a normal subgroup I of $(N, +)$ with (iii) is called a left ideal of N. A subgroup Q of $(N, +)$ is called a quasi-ideal of near-ring N if

$QN \cap NQ \cap N * Q \subseteq Q$.

We now review some fuzzy logic concepts.

Definition 2.1. *A fuzzy point x_t is said to belong to (resp. be quasi-coincident with) a fuzzy subset λ, written as $x_t \in \lambda$ (resp. $x_t q\lambda$) if $\lambda(x) \geq t$ (resp. $\lambda(x) + t > 1$). If $x_t \in \lambda$ or $x_t q\lambda$, then we write $x_t \in\vee \, q\lambda$.*

Definition 2.2. *An interval-valued fuzzy subset $\overline{A}$ of a set S of the form*

$$\overline{A}(y) = \begin{cases} \overline{t} \in D(0,1] & \text{if } y = x, \\ & \text{if } y \neq x. \end{cases}$$

is called interval-valued fuzzy point with support x at value $\overline{t}$ and is denoted by $x_{\overline{t}}$

For an interval-valued fuzzy subset $\overline{A}$ of a set S, we say that an interval-valued fuzzy point $x_{\overline{t}}$ is contained in $\overline{A}$, denoted by $x_{\overline{t}} \in \overline{A}$, if $\overline{A}(x) \geq \overline{t}$.

quasi-coincident with $\overline{A}$*, denoted by* $x_{\bar{t}}q\overline{A}$*, if* $\overline{A}(x)+\bar{t}>\bar{1}=[1,1]$.

For an interval-valued fuzzy point $x_{\bar{t}}$ *and an interval-valued fuzzy subset* $\overline{A}$ *of set* S*, we say that*

$x_{\bar{t}}\in\vee\, q\overline{A}$ *if* $x_{\bar{t}}\in\overline{A}$ *or* $x_{\bar{t}}q\overline{A}$.

$x_{\bar{t}}\overline{\alpha}\overline{A}$ *if* $x_{\bar{t}}\alpha\overline{A}$ *does not hold for* $\alpha\in\{\epsilon,q,\epsilon\vee q\}$.

Definition 2.3. *Let* A *be nonempty subset of* S*. We denote by* $\overline{\mu}_A$*, the interval valued characteristic function of* A*, that is the mapping of* S *into* $D[0,1]$ *defined by*

$$\overline{\mu}(x)=\begin{cases}[1,1] & \text{if } x\in A,\\ & \text{if } x\notin A.\end{cases}$$

Clearly $\overline{\mu}_A$ *is a fuzzy subset of* S.

Definition 2.4. An interval number $\overline{a}$ on $[0,1]$ is a closed subinterval of $[0,1]$, that is, $\overline{a}=[a^-,a^+]$ such that $0\le a^-\le a^+\le 1$ where a^- and a^+ are the lower and upper end limits of $\overline{a}$ respectively. The set of all closed subintervals of $[0,1]$ is denoted by $D[0,1]$. We also identify the interval $[a,a]$ by the number $a\in[0,1]$. For any interval numbers

$\overline{a}_i=[a_i^-,a_i^+],\overline{b}_i=[b_i^-,b_i^+]\in D[0,1], i\in I,$ we define $max^i\{\overline{a}_i,\overline{b}_i\}=[max^i\{a_i^-,b_i^-\},max^i\{a_i^+,b_i^+\}]$,

$min^i\{\overline{a}_i,\overline{b}_i\}=[min^i\{a_i^-,b_i^-\},min^i\{a_i^+,b_i^+\}]$,

$inf^i\overline{a}_i=[\bigcap_{i\in I}a_i^-,\bigcap_{i\in I}a_i^+], sup^i\overline{a}_i=[\bigcup_{i\in I}a_i^-,\bigcup_{i\in I}a_i^+]$

In this notation $\bar{0}=[0,0]$ and $\bar{1}=[1,1]$. For any interval numbers $\overline{a}=[a^-,a^+]$ and $\overline{b}=[b^-,b^+]$ on $[0,1]$, define

(1) $\overline{a}\le\overline{b}$ if and only if $a^-\le b^-$ and $a^+\le b^+$.

(2) $\overline{a}=\overline{b}$ if and only if $a^-=b^-$ and $a^+=b^+$.

(3) $\overline{a}<\overline{b}$ if and only if $\overline{a}\le\overline{b}$ and $\overline{a}\ne\overline{b}$

(4) $k\overline{a}=[ka^-,ka^+]$, whenever $0\le k\le 1$.

Definition 2.5. Let X be any set. A mapping $\overline{A}:X\to D[0,1]$ is called an interval-valued fuzzy subset (briefly, i-v fuzzy subset) of X where $D[0,1]$ denotes the family of all closed subintervals of $[0,1]$ and $\overline{A}(x)=[A^-(x),\ A^+(x)]$ for all $x\in X$, where A^- and A^+ are fuzzy subsets of X such that $A^-(x)\le A^+(x)$ for all $x\in X$.

Note that $\overline{A}(x)$ is an interval (a closed subset of $[0,1]$) and not a number from the interval $[0,1]$ as in the case of fuzzy subset.

Let min^i and max^i be the interval min-norm and max-norm on $D[0,1]$ respectively. Then the following are true.

1. $min^i\{\overline{a},\overline{a}\}=\overline{a}$ and $max^i\{\overline{a},\overline{a}\}=\overline{a}$ for all $\overline{a}\in D[0,1]$.

2. $min^i\{\overline{a},\overline{b}\}=min^i\{\overline{b},\overline{a}\}$ and $max^i\{\overline{a},\overline{b}\}=max^i\{\overline{b},\overline{a}\}$ for all $\overline{a},\overline{b}\in D[0,1]$.

3. If $\overline{a}\ge\overline{b}\in D[0,1]$, then $min^i\{\overline{a},\overline{c}\}\ge min^i\{\overline{b},\overline{c}\}$ and $max^i\{\overline{a},\overline{c}\}\ge max^i\{\overline{b},\overline{c}\}$ for all $\overline{c}\in D[0,1]$. Let $\overline{A}$ and $\overline{B}$ be two i-v fuzzy subsets of semigroup X. We define the relation $\subseteq$ between $\overline{A}$ and $\overline{B}$, the intersection and product of $\overline{A}$ and $\overline{B}$, respectively as follows:

(i) $\overline{A}\subseteq\overline{B}$ if $\overline{A}(x)\le\overline{B}(x)\ \forall x\in X$,

(ii) $(\overline{A}\cap\overline{B})(x)=min^i\{\overline{A}(x),\ \overline{B}(x)\}\ \forall\ x\in X$,

(iii)

$$(\overline{A} \circ \overline{B})(x) = \begin{cases} sup^i_{x=yz}[min^i\{\overline{A}(y), \overline{B}(z)\}] & \text{if } x = yz, \text{ for } y, z \in X, \\ \overline{0} & \text{Otherwise} \end{cases}$$

(iv)

$$(\overline{A} * \overline{B})(x) = \begin{cases} sup^i_{x=a(b+c)-ab}[min^i\{\overline{A}(a), \overline{B}(c)\}] & \text{if } x = a(b+c) - ab, \text{ for } a, c \in X, \\ \overline{0} & \text{Otherwise} \end{cases}$$

It is easily verified that the `` product" of i-v fuzzy subsets is associative. Throughout this paper, N will denote a near-ring unless otherwise specified.

3. $(\in, \in\vee\, q^{\overline{\delta}}_{\overline{k}})$-interval Valued fuzzy subnear-rings and $(\in, \in\vee\, q^{\overline{\delta}}_{\overline{k}})$-interval valued fuzzy ideals in Near-rings

In this section, we introduce the notion of $\vee\, q^{\overline{\delta}}_{\overline{k}}$-fuzzy sets which are generalization of fuzzy sets.

Definition 3.1. *Let A be a non-empty subset of N. The characteristic function of A denoted by λ_A and is defined by the mapping from N into* $[0,1]$:

$$\lambda_A(a) = \begin{cases} 1 & \text{if } a \in A \\ 0 & \text{if } a \notin A\,. \end{cases}$$

is said to be a fuzzy point with support a and value t and is denoted by (a_t).

For a fuzzy subset $\lambda \in N$, a fuzzy point (a_t) is said to

- *be contained in λ, denoted by $(a_t) \in \lambda$, if $\lambda(a) \geq t$.*
- *be quasi-coincident with λ, denoted by $a_t q \lambda$, if $\lambda(a) + t > 1$.*

For a fuzzy subset λ and fuzzy point (a_t) in a set N, we say that

- *$(a_t \in\vee\, q\lambda)$ if $(a_t \in \lambda)$ or $(a_t)q\lambda$.*

generalized

Definition 3.2. *A fuzzy point $a_{\overline{t}}$ is said to belong to (resp., be k- quasi-coincident with) an i-v fuzzy subset $\overline{\lambda}$, written as $a_{\overline{t}} \in \overline{\lambda}$ (resp., $a_{\overline{t}} q_{\overline{k}} \overline{\lambda}$) if $\overline{b}da(a) \geq \overline{t}$ (resp., $\overline{\lambda} + \overline{t} > \overline{\delta} - \overline{k}$, where $\overline{k} \in D[0,1), \overline{\delta} \in D[0,1)$ and $\overline{k} < \overline{\delta}$).*

For any $\overline{t} \in D(0,1]$, $a_{\overline{t}} \in \overline{\lambda}$ or $a_{\overline{t}} q^{\overline{\delta}}_{\overline{k}} \overline{\lambda}$ will be denoted by $\dfrac{a_{\overline{t}} \vee q^{\overline{\delta}}_{\overline{k}} \overline{\lambda}.\ a_{\overline{t}} \in}{line\overline{\lambda},\ a_{\overline{t}} \in\vee q^{\overline{\delta}}_{\overline{k}} \overline{\lambda}}$ will respectively mean $a_{\overline{t}} \in \overline{\lambda}$ and $a_{\overline{t}} \in\vee\, q^{\overline{\delta}}_{\overline{k}} \overline{\lambda}$ do not hold.

arbitrary, but fixed.

Definition 3.3. *An i-v fuzzy subset $\overline{\lambda}$ is said to be an $(\in, \in\vee\, q^{\overline{\delta}}_{\overline{k}})$ i-v fuzzy subnear-ring of N if for all $a, b \in N$ and $\overline{t}, \overline{r} \in D(0,1]$ and $\overline{k} < \overline{\delta}$:*

(i) $a_{\overline{t}},\ b_{\overline{r}} \in \overline{\lambda}$ implies $(a+b)_{min\{\overline{t},\overline{r}\}} \in\vee\, q^{\overline{\delta}}_{\overline{k}} \overline{\lambda}$,

(ii) $a_{\overline{t}} \in \overline{\lambda}$ implies $(-a)_{\overline{t}} \in\vee\, q^{\overline{\delta}}_{\overline{k}} \overline{\lambda}$,

(iii) $a_{\overline{t}},\ b_{\overline{r}} \in \overline{\lambda}$ implies $(xy)_{min\{\overline{t},\overline{r}\}} \in\vee\, q^{\overline{\delta}}_{\overline{k}} \overline{\lambda}$.

Definition 3.4. *For any two interval numbers,* $\overline{\delta} = [\delta^-, \delta^+]$ *and* $\overline{k} = [k^-, k^+]$ *addition, subtraction, multiplication and division are defined as*

$$\overline{\delta} + \overline{k} = \begin{cases} [\delta^- + k^+, \delta^+ + k^+] & \text{if } [\delta^- + k^-, \delta^+ + k^+] \leq \overline{1} \\ & \text{if } [\delta^- + k^-, \delta^+ + k^+] > \overline{1}. \end{cases}$$

$$\overline{\delta} - \overline{k} = \begin{cases} [\delta^- - k^+, \delta^+ - k^-] & \text{if } [\delta^- - k^+, \delta^+ - k^-] \geq \overline{0} \\ & \text{if } [\delta^- - k^+, \delta^+ - k^-] < \overline{0}. \end{cases}$$

$$\overline{\delta} \cdot \overline{k} = \{[min\{\delta^- \cdot k^-, \delta^+ \cdot k^+\}, max\{\delta^- \cdot k^-, \delta^+ \cdot k^+\}].$$

$$\overline{\delta}/\overline{k} = \begin{cases} [min(\frac{\delta^-}{k^-}, \frac{\delta^+}{k^+}),\ max(\frac{\delta^-}{k^-}, \frac{\delta^+}{k^+})] & \text{if } \overline{\delta} \leq \overline{k} \neq \overline{0} \\ & \text{if } \overline{\delta} > \overline{k} \\ \text{not defined} & \text{if } \overline{\delta} = \overline{k} = \overline{0}. \end{cases}$$

Lemma 3.5. *Let* $\overline{\lambda}$ *be an i-v fuzzy subset of N and* $\overline{t}, \overline{r} \in (0,1]$ *and* $\overline{k} < \overline{\delta}$. *Then:*

(1) (a) $a_{\overline{t}},\ b_{\overline{r}} \in \overline{\lambda}$ *implies* $(a+b)_{min\{\overline{t},\overline{r}\}} \in \vee\, q_{\overline{k}}^{\overline{\delta}} \overline{\lambda}$, *and*

(b) $\overline{\lambda}(a+b) \geq min\{\overline{\lambda}(a),\ \overline{\lambda}(b),\ \frac{\overline{\delta}-\overline{k}}{2}\}$ *for all* $a, b \in N$ *are equivalent.*

(2) (c) $a_{\overline{t}} \in \overline{\lambda}$ *implies* $(-x)_{\overline{t}} \in \vee\, q_{\overline{k}}^{\overline{\delta}} \overline{\lambda}$, *and*

(d) $\overline{\lambda}(-a) \geq min^i\{\overline{\lambda}(a),\ \frac{\overline{\delta}-\overline{k}}{2}\}$ *for all* $a \in N$ *are equivalent.*

(3) (e) $a_{\overline{t}},\ b_{\overline{r}} \in \overline{\lambda}$ *implies* $(ab)_{min\{\overline{t},\overline{r}\}} \in \vee\, q_{\overline{k}}^{\overline{\delta}} \overline{\lambda}$, *and*

(f) $\overline{\lambda}(ab) \geq min\{\overline{\lambda}(a),\ \overline{\lambda}(b),\ \frac{\overline{\delta}-\overline{k}}{2}\}$ *for all* $a, b \in N$ *are equivalent.*

Proof. (1)(a) $\Rightarrow$ (b). Let $a, b \in N$ and $min^i\{\overline{\lambda}(a), \overline{\lambda}(b)\} \leq \frac{\overline{\delta}-\overline{k}}{2}$. Assume that $\overline{\lambda}(a+b) < min^i\{\overline{\lambda}(a), \overline{\lambda}(b)\}$. Choose t such that $\overline{\lambda}(a+b) < t < min^i\{\overline{\lambda}(a), \overline{\lambda}(b)\}$. This implies $a_{\overline{t}},\ b_{\overline{t}} \in \overline{\lambda}$ but $(a+b)_{\overline{t}}\ \overline{\in \vee\, q_{\overline{k}}^{\overline{\delta}} \overline{\lambda}}$,which contradicts (a). Next,let $min^i\{\overline{\lambda}(a), \overline{\lambda}(b)\} \geq \frac{\overline{\delta}-\overline{k}}{2}$. Assume that $\overline{\lambda}(a+b) < \frac{\overline{\delta}-\overline{k}}{2}$. Then $\frac{a_{e\delta-\overline{k}}}{2},\ \frac{b_{\overline{\delta}-\overline{k}}}{2} \in \overline{\lambda}$ but $(a+b)_{\frac{\overline{\delta}-\overline{k}}{2}}\ \overline{\in \vee\, q_{\overline{k}}^{\overline{\delta}} \overline{\lambda}}$, which contradicts (a). Thus $\overline{\lambda}(a+b) \geq min^i\{\overline{\lambda}(a), \overline{\lambda}(b), \frac{\overline{\delta}-\overline{k}}{2}\}$. $(b) \Rightarrow (a)$. Let $a_{\overline{t}}, b_{\overline{r}} \in \overline{\lambda}$. Then $\overline{\lambda}(a+b) \geq min^i\{\overline{\lambda}(a), \overline{\lambda}(b), \frac{\overline{\delta}-\overline{k}}{2}\} \geq min^i\{\overline{t}, er, \frac{\overline{\delta}-\overline{k}}{2}\}$. Thus $\overline{\lambda}(a+b) \geq min^i\{t, r\}$ if $\overline{t} < \frac{\overline{\delta}-\overline{k}}{2}$ and $\overline{\lambda}(a+b) \geq \frac{\overline{\delta}-\overline{k}}{2}$ if $t \geq \frac{\overline{\delta}-\overline{k}}{2}$ and $\overline{r} \geq \frac{\overline{\delta}-\overline{k}}{2}$ Hence $(a+b)_{min\{\overline{t},\overline{r}\}} \in \vee\, q_{\overline{k}}^{\overline{\delta}} \overline{\lambda}$. (d)$\Rightarrow$ (c). Let $a_{\overline{t}} \in \overline{\lambda}$. Then $\overline{\lambda}(a) \geq \overline{t}$. Now $\overline{\lambda}(-a) \geq min^i\{\overline{\lambda}(a), \frac{\overline{\delta}-\overline{k}}{2}\} \geq min^i\{\overline{t}, \hat{r}ac\overline{\delta} - \overline{k}2\}$. That is $\overline{\lambda}(-a) \geq \overline{t}$ or $\frac{\overline{\delta}-\overline{k}}{2}$ according as $\overline{t} \leq \frac{\overline{\delta}-\overline{k}}{2}$ or $\overline{t} > \frac{overline\delta-\overline{k}}{2}$. Hence $(-a)_{\overline{t}} \in \vee\, q_{\overline{k}}^{\overline{\delta}} \overline{\lambda}$.

(3) follows easily from (2). □

Theorem 3.6. *An i-v subset* $\overline{\lambda}$ *of* N *is an* $(\in, \in\vee\, q_{\overline{k}}^{\overline{\delta}})$ *i-v fuzzy subnear-ring of* N *if and only if* $\overline{\lambda}(a-b),\ \overline{\lambda}(ab) \geq min^{i}\{\overline{\lambda}(a), \overline{\lambda}(b), \frac{e\delta-\overline{k}}{2}\}$*, for all* $a, b \in N$.

Proof. It follows from Lemma 3.5.

Corollary 3.7. *An i-v fuzzy subset* $\overline{\lambda}$ *of* N *is an* $(\in, \in\vee\, q_{\overline{k}}^{\overline{\delta}})$ *i-v fuzzy subnear-ring of* N *if and only if* $\overline{\lambda}(a-b),\ \overline{\lambda}(ab) \geq min^{i}\{\overline{\lambda}(a), \overline{\lambda}(b), \frac{rline\delta-\overline{k}}{2}\}$*, for all* $a, b \in N$.

Proof. The result follows easily from Lemma 3.5 if we take $k = 0$. □

Corollary 3.8. $\overline{\lambda}$ *is an* $(\in, \in\vee\, q_{\overline{k}}^{\overline{\delta}})$ *i-v fuzzy subring if and only if* $\overline{\lambda}(a-b),\ \overline{\lambda}(ab) \geq min^{i}\{\overline{\lambda}(a),\ \overline{\lambda}(b), \frac{\overline{\delta}-\overline{k}}{2}\}$*, for all* $a, b \in N$.

Remark 3.9.. *Every i-v fuzzy subnear-ring and* $(\in, \in\vee\, q_{\overline{k}})$ *i-v fuzzy subnear-ring of* N *is an* $(\in, \in\vee\, q_{\overline{k}}^{\overline{\delta}})$ *i-v fuzzy subnear-ring of* N*, but, as the following example shows, the converse is not necessarily true.*

Example 3.10. Let $N = \{0, a, b, c\}$ be the near-ring with $(N, +)$ as the Klein's four group and $(N, .)$ as defined below

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

·	0	a	b	c
0	0	0	0	0
a	A	a	a	a
b	0	0	0	b
c	A	a	a	c

Consider the near-ring $(N, +, \cdot)$. Define an i-v fuzzy subset $\overline{\lambda}: N \to [0,1]$ by $\overline{\lambda}(0) = [0.42, 0.43]$ $\overline{\lambda}(a) = \overline{\lambda}(c) = [0.4, 0.41]$ $\overline{\lambda}(b) = [0.44, 045]$. Then $\overline{\lambda}$ is an $(\in, \in\vee\, q_{\overline{0.1}}^{\overline{0.3}})$ i-v fuzzy subnear-ring of N. But, since $\overline{\lambda}(0) = \overline{\lambda}(b-b) \ngeq min^{i}\{\overline{\lambda}(b), \overline{\lambda}(b)\}$ and $\overline{\lambda}(0) = \overline{\lambda}(b-b) \ngeq min^{i}\{\overline{\lambda}(b), \overline{\lambda}(b), \frac{\overline{\delta}-\overline{k}}{2}\}$, $\overline{\lambda}$ is neither a fuzzy subnear-ring of N nor an $(\in, \in\vee\, q_{k})$ i-v fuzzy subnear-ring of N.

Now we generalize the notions of i-v fuzzy ideals of N defined by Zaid [1] and $(\in, \in\vee\, q)$ i-v fuzzy ideals of N defined by Narayanan and Manikantan [9].

Definition 3.11. *An i-v fuzzy subset* $\overline{\lambda}$ *of* N *is said to be an* $(\in, \in\vee\, q_{\overline{k}}^{\overline{\delta}})$ *i-v fuzzy ideal of* N *if for all* $a, b, c \in N$ *and for all* $\overline{r}, \overline{t} \in (0,1]$ *and* $\overline{k} < \overline{\delta}$.

(1) $a_{\overline{r}}, b_{\overline{t}} \in \overline{\lambda}$ *implies* $(a-b)_{min\{\overline{r},\overline{t}\}} \in\vee q_{\overline{k}}^{\overline{\delta}}\overline{\lambda}$.

(2) $a_{\overline{t}} \in \overline{\lambda}$ *and* $b \in N$ *implies* $(b+a-b)_{\overline{t}} \in\vee q_{\overline{k}}^{\overline{\delta}}\overline{\lambda}$,

(3) $a_{\overline{t}} \in \overline{\lambda}$ *and* $b \in N$ *implies* $(ab)_{\overline{t}} \in\vee q_{\overline{k}}^{\overline{\delta}}\overline{\lambda}$,

(4) $c_{\overline{t}} \in \overline{\lambda}$ *and* $a, b \in N$ *implies* $(a(b+c)-ab)_{\overline{t}} \in\vee q_{\overline{k}}^{\overline{\delta}}\overline{\lambda}$.

An i-v fuzzy subset $\overline{\lambda}$ *with conditions (1),(2) and (3) is called an* $(\in, \in\vee q_{\overline{k}}^{\overline{\delta}})$ *i-v fuzzy right ideal of* N. *If* $\overline{\lambda}$ *satisfies (1), (2) and (4), then it is called an* $(\in, \in\vee q_{\overline{k}}^{overline\delta})$ *i-v fuzzy left ideal of* N.

Lemma 3.12. Let $\overline{\lambda}$ be an i-v fuzzy subset of N. Then:

(1) (a) $a_{\overline{r}}, b_{\overline{t}} \in \overline{\lambda}$ implies $(a-b)_{min\{\overline{r},\overline{t}\}} \in\vee q_{\overline{k}}^{\overline{\delta}}\overline{\lambda}$ and

(b) $\overline{\lambda}(a-b) \geq min^i\{\overline{\lambda}(a), \overline{\lambda}(b), \frac{\overline{\delta}-\overline{k}}{2}\}$ for all $a, b \in N$ are equivalent.

(2) (c) $a_{\overline{t}} \in \overline{\lambda}$ and $b \in N$ implies $(b+a-b)_{\overline{t}} \in\vee q_{\overline{k}}^{\overline{\delta}}\overline{\lambda}$ and

(d) $\overline{\lambda}(b+a-b) \geq min^i\{\overline{\lambda}(a), \frac{\overline{\delta}-\overline{k}}{2}\}$ for all $a, b \in N$ are equivalent.

(3) (e) $a_{\overline{t}} \in \overline{\lambda}$ and $b \in N$ implies $(ab)_{\overline{t}} \in\vee q_{\overline{k}}^{\overline{\delta}}\overline{\lambda}$ and

(f) $\overline{\lambda}(ab) \geq min^i\{\overline{\lambda}(a), \frac{\overline{\delta}-\overline{k}}{2}\}$ for all $a, b \in N$ are equivalent.

(4) (g) $c_{\overline{t}} \in \overline{\lambda}$ and $a, b \in N$ implies $(a(b+c)-ab)_t \in\vee q_{\overline{k}}^{\overline{\delta}}\overline{\lambda}$ and

(h) $\overline{\lambda}(a(b+c)-ab) \geq min^i\{\overline{\lambda}(c), \frac{\overline{\delta}-\overline{k}}{2}\}$ for all $a, b, c \in N$ are equivalent.

Proof. (a) ⇔ (b). It follows from Lemma 3.5.

(c) ⇒ (d). Let $a, b \in N$ and $\overline{\lambda}(b+a-b) < \frac{\overline{\delta}-\overline{k}}{2}$. Assume that $\overline{\lambda}(b+a-b) < \overline{\lambda}(a)$. Choose $\overline{t}$ such that $\overline{\lambda}(b+a-b) < \overline{t} < \overline{\lambda}(a)$.

Then $a_{\overline{t}} \in \overline{\lambda}$ and $(b+a-b)_{\overline{t}} \overline{\in\vee q_{\overline{k}}^{\overline{\delta}}}\overline{\lambda}$, Which contradicts (c). Thus $\overline{\lambda}(b+a-b) \geq \overline{\lambda}(a)$. Next let $\overline{\lambda}(a) geq \frac{\overline{\delta}-\overline{k}}{2}$. Assume that $\overline{\lambda}(b+a-b) < \frac{\overline{\delta}-\overline{k}}{2}$. Then $\frac{a_1-\overline{k}}{2} \in \overline{\lambda}$ and $b+a-b \in \overline{\vee q^{\overline{\delta}} \overline{k}\overline{\lambda}}$, which contradicts (c). Hence (d) holds.

(d)⇒ (c). Let $a_{\overline{t}} \in \overline{\lambda}$ and $b \in N$. Then $\overline{\lambda}(a) \geq \overline{t}$ and, by (d), $\overline{\lambda}(b+a-b) \geq min^i\{\overline{\lambda}(a), \frac{\overline{\delta}-\overline{k}}{2}\}$. This implies $\overline{\lambda}(b+a-b) \geq min^i\{\overline{t}, \frac{\overline{\delta}-\overline{k}}{2}\}$. Then $\overline{\lambda}(b+a-b) \geq \overline{t}$ if $\overline{t} \leq \frac{\overline{\delta}-\overline{k}}{2}$ and $\overline{\lambda}(b+a-b) \geq c\delta - k\overline{2}$ if $\frac{\overline{\delta}-\overline{k}}{2} < \overline{t}$. Thus $b+a-b \in\vee q_{\overline{k}}^{\overline{\delta}}\overline{\lambda}$. Hence (c) holds.

(e)⇒ (f). Let $a, b \in N$. Let $\overline{\lambda}(a) < \frac{\overline{\delta}-\overline{k}}{2}$. Assume that $\overline{\lambda}(ab) < \overline{\lambda}(a)$. Choose $\overline{t}$ such that $\overline{\lambda}(ab) < \overline{t} < \overline{\lambda}(a)$. Then $a_{\overline{t}} \in \overline{\lambda}$ and $(ab)_{\overline{t}} \overline{\in\vee q_{rlinek}^{\overline{\delta}}}\overline{\lambda}$ which contradicts (e). So $\overline{\lambda}(ab) \geq \overline{\lambda}$. Next let $\overline{\lambda}(a) \geq \frac{\delta-k}{2}$. Assume that $\overline{\lambda}(ab) < \frac{\overline{\delta}-\overline{k}}{2}$. Then $\hat{r}aca_1 - k\overline{2} \in \overline{\lambda}$ and $(ab)_{\frac{\overline{\delta}-\overline{k}}{2}} \overline{\in\vee q_k^{\delta}\overline{\lambda}}$, which contradicts (e). Hence (f) holds.

(g)⇒ (h). Let $a, b, c \in N$ and $\overline{\lambda}(c) < \frac{\overline{\delta}-\overline{k}}{2}$. Assume that $\overline{\lambda}(a(b+c)-ab) < \overline{\lambda}(c)$. Then there exists $\overline{t}$ such that $\overline{\lambda}(a(b+c)-ab) < net < \overline{\lambda}(c)$. This implies that $c_{\overline{t}} \in \overline{\lambda}$ and $\overline{\lambda}(b+c) - ab \; \overline{\in\vee\, q_{\overline{k}}^{\overline{\delta}}} \overline{\lambda}$, which contradicts (g). Assume that $\overline{\lambda}(c) \geq \frac{erline\delta - \overline{k}}{2}$. Then $\frac{c_1 - k}{\overline{2}} \in \overline{\lambda}$. Suppose $\overline{\lambda}(a(b+c)-ab) < \frac{\overline{\delta}-\overline{k}}{2}$. Then $(a(b+c)-ab)_{\frac{\overline{\delta}-\overline{k}}{2}} \overline{\in\vee\, q_{\overline{k}}^{\overline{\delta}}} \overline{\lambda}$, which contradicts (h). Thus $\overline{\lambda}(a(b+c)-ab) \geq min^i\{\overline{\lambda}(c), \frac{\overline{\delta}-\overline{k}}{2}\}$ for all $a, b, c \in N$. Therefore (h) holds.

(h) ⇒ (g). Let $c_{\overline{t}} \in \overline{\lambda}$ and $a, b \in N$. Then, by (h), $\overline{\lambda}(a(b+c)-ab) \geq min^i\{\overline{\lambda}(c), \frac{\overline{\delta}-k}{\overline{2}} \geq min^i\{t, \frac{\overline{\delta}-\overline{k}}{2}\}\}$. Thus $\overline{\lambda}(a(b+c)-ab) \geq \overline{t}$ if $\overline{t} \leq \frac{\overline{\delta}-\overline{k}}{2}$ or $\overline{\lambda}(a(b+c)-ab) \geq \frac{\overline{\delta}-\overline{k}}{2}$ if $\overline{t} > \frac{\overline{\delta}-\overline{k}}{2}$. Therefore $(a(b+c)-ab)_{\overline{t}} \in\vee\, q_{\overline{k}}^{\overline{\delta}} \overline{\lambda}$. Hence (g) holds. □

Example 3.13. Consider the near-ring $(N, +, \cdot)$. Define an i-v fuzzy subset $\overline{\lambda}: N \to D[0,1]$ by $\overline{\lambda}(0) = [0.42, 0.43]$, $\overline{\lambda}(a) = \overline{\lambda}(c) = [0.4, 0.5]$, $\overline{\lambda}(b) = [0.44, 0.45]$. Then $\overline{\lambda}$ is an $(\in, \in\vee\, q_{\overline{0.2}}^{\overline{0.3}})$ i-v fuzzy ideal of N. But, since $\overline{\lambda}(b0) = \overline{\lambda}(0) \ngeq \overline{\lambda}(b)$ and $\overline{\lambda}(b0) = \overline{\lambda}(0) \ngeq min^i \overline{\lambda}(b), \frac{\overline{\delta}-\overline{k}}{2}\}$, $\overline{\lambda}$ is neither an i-v fuzzy ideal nor an $(\in, \in\vee\, q)$ i-v fuzzy ideal of N.

Theorem 3.14. Let $\{\overline{\lambda}_i\}_{i=1}^n$ be a family of $(\in, \in\vee\, q_{\overline{k}}^{\overline{\delta}})$ i-v fuzzy subnear-rings (ideals) of N. Then $\overline{\lambda} = \cap\, \overline{\lambda}_i$, is an $(\in, \in\vee\, q_{\overline{k}}^{\overline{\delta}})$ i-v fuzzy subnear-ring (ideal) of N.

Proof. Let $a, b \in \overline{\lambda}$.

$\overline{\lambda}(a+b) = \cap_{i=1}^n \overline{\lambda}_i(a+b)$

$\geq min^i_{1 \leq i \leq n}\{min\{\overline{\lambda}_i(a), \overline{\lambda}_i(b), \frac{\overline{\delta}-\overline{k}}{2}\}\}$

$\geq min\{min^i_{1 \leq i \leq n}\{\overline{\lambda}_i(a)\}, min_{1 \leq i \leq n}\{\overline{\lambda}_i(b)\}, \frac{\overline{\delta}-\overline{k}}{2}\}$

$= min\{(\cap_{i=1}^n \overline{\lambda}_i)(a), (\cap_{i=1}^n \overline{\lambda}_i)(b), \frac{\overline{\delta}-\overline{k}}{2}\}$

$= min\{\overline{\lambda}(a), \overline{\lambda}(b), \frac{\overline{\delta}-\overline{k}}{2}\}$.

$\overline{\lambda}(-a) = \cap_{i=1}^n \overline{\lambda}_i(-a)$

$\geq min^i_{1 \leq i \leq n}\{min\{\overline{\lambda}_i(a), \frac{\overline{\delta}-\overline{k}}{2}\}\}$

$= min\{min^i_{1 \leq i \leq n}\{\overline{\lambda}_i(a), \frac{\overline{\delta}-\overline{k}}{2}\}\}$

$= min\{(\cap_{i=1}^n \overline{\lambda}_i)(a), \frac{\overline{\delta}-\overline{k}}{2}\}$

$= min\{\overline{\lambda}(a), \frac{\overline{\delta}-\overline{k}}{2}\}$.

$\overline{\lambda}(ab) = \cap_{i=1}^n \overline{\lambda}_i(ab) \geq min^i_{1 \leq i \leq n}\{min\{\overline{\lambda}_i(a), \overline{\lambda}_i(b), \frac{\overline{\delta}-\overline{k}}{2}\}\}$

$\geq min\{min^i_{1 \leq i \leq n}\{\overline{\lambda}_i(a)\}, min_{1 \leq i \leq n}\{\overline{\lambda}(b)\}, \frac{\overline{\delta}-\overline{k}}{2}\}$

$= min\{(\cap_{i=1}^n \overline{\lambda}_i)(a), (\cap_{i=1}^n \overline{\lambda}_i)(b), \frac{\overline{\delta}-\overline{k}}{2}\}$

$= min\{\overline{\lambda}(a), \overline{\lambda}(b), \frac{\overline{\delta}-\overline{k}}{2}\}$.

Thus $\overline{\lambda}$ is an $(\in, \in \vee q_{\overline{k}}^{\overline{\delta}})$ i-v fuzzy subnear ring of N. □

Theorem 3.15. A non-empty subset A of N is an ideal (subnear-ring) of N if and only if $\overline{\lambda}_A$ is an $(\in, \in \vee q_{\overline{k}}^{\overline{\delta}})$ i-v fuzzy ideal (subnear-ring) of N.

Proof. Let A be an ideal of N. Then $\overline{\lambda}_A$ is an $(\in, \in \vee q_{\overline{k}}^{\overline{\delta}})$ i-v fuzzy ideal of N.

Conversely, let $\overline{\lambda}_A$ be an $(\in, \in \vee q_{\overline{k}}^{\overline{\delta}})$ i-v fuzzy ideal of N. For any $a, b \in A$, we have $\overline{\lambda}_A(a - b) \geq min^i\{\overline{\lambda}_A(a), \{\overline{\lambda}_A(b), \frac{\overline{\delta}-\overline{k}}{2}\} = min^i\{1,1, \frac{\overline{\delta}-\overline{k}}{2}\}$. Since $k, \delta \in [0,1)$, $a - b \in A$.

Let $a \in A$ and $b \in N$. Then $\overline{\lambda}(b + a - b) \geq min^i\{\overline{\lambda}_A(a), \frac{\overline{\delta}-\overline{k}}{2}\} = \frac{\overline{\delta}-\overline{k}}{2} \neq 0$.

This implies that $b + a - b \in A$. Now let $a \in N$ and $a \in A$. Then $\overline{\lambda}_A(ax) \geq min^i\{\overline{\lambda}_A(a), \frac{\overline{\delta}-\overline{k}}{2}\} = \frac{\overline{\delta}-\overline{k}}{2}$. This implies that $ax \in A$. Let $a, b \in N$ and $c \in A$, $\overline{\lambda}_A(a(b + c) - ab) \geq min^i\{\overline{\lambda}_A(c), \frac{\overline{\delta}-\overline{k}}{2}\} = \frac{\overline{\delta}-\overline{k}}{2}$. This implies that $a(b + c) - ab \in A$. Thus A is an ideal in N. □

Theorem 3.16. An i-v fuzzy subset $\overline{\lambda}$ of N is an $(\in, \in \vee q_{\overline{k}}^{\overline{\delta}})$ i-v fuzzy ideal (subnear-ring) of N if and only if the level subset $\overline{\lambda}_{\overline{t}}$ is an ideal(subnear-ring) of N, for all $rline0 < \overline{t} \leq \frac{\overline{\delta}-\overline{k}}{2}$ and $\overline{k} \in D[0,1)$, and $\overline{\delta} \in D[0,1)$.

Proof. Let $\overline{\lambda}$ be an $(\in, \in q_{\overline{k}}^{\overline{\delta}})$ i-v fuzzy ideal of N. Let $\overline{0} < \overline{t} \leq \frac{\overline{\delta}-\overline{k}}{2}$ and $a, b, c \in \overline{\lambda}_{\overline{t}}$.

Then $\overline{\lambda}(a - b) \geq min^i\{\overline{\lambda}(a), \overline{\lambda}(b), \frac{\overline{\delta}-\overline{k}}{2}\} \geq min\{\overline{t}, \frac{\overline{\delta}-\overline{k}}{2}\} = t$ and hence $a - b \in \overline{\lambda}_{\overline{t}}$.

Now $\overline{\lambda}(ab) \geq min\{\overline{\lambda}(a), \frac{\overline{\delta}-\overline{k}}{2}\} = \overline{t}$.

Thus $ab \in \overline{\lambda}_{\overline{t}}$. $\overline{\lambda}(a + b - a) \geq min\{\overline{\lambda}(b), \frac{\overline{\delta}-\overline{k}}{2}\} = \overline{t}$ implies $a + b - a \in \overline{\lambda}_{\overline{t}}$. Hence $\overline{\lambda}(a(b + c) - ab) \geq min\{e\lambda(c), \frac{\overline{\delta}-\overline{k}}{2}\} = t$ for every $a, b, c \in N$. This implies that $a(b + c) - ab \in \overline{\lambda}_{\overline{t}}$. So $\overline{\lambda}_{\overline{t}}$ is an ideal of N.

Conversely, Let $\overline{\lambda}_{\overline{t}}$ be an ideal of N for all $\overline{0} < \overline{t} \leq \frac{\overline{\delta}-\overline{k}}{2}$. Let $a, b \in N$. Suppose $\overline{\lambda}(a - b) < min^i\{\overline{\lambda}(a), \overline{\lambda}(b), c\overline{\delta} - \overline{k2}\}$. Choose $\overline{t}$ such that $\overline{\lambda}(a - b) < \overline{t} < min^i\{\overline{\lambda}(a), \overline{\lambda}(b), \frac{\overline{\delta}-\overline{k}}{2}\}$. This implies $a, b \in \overline{\lambda}_{overlinet}$. Then $a - b \in \overline{\lambda}_{\overline{t}}$, since $\overline{\lambda}_{\overline{t}}$ is an ideal of N. This implies $\overline{\lambda}(a - b) \geq \overline{t}$, a contradiction. Thus $\overline{\lambda}(a - b) \geq min^i\{\overline{\lambda}(a), \not{o}verline\lambda(b), \frac{\overline{\delta}-\overline{k}}{2}\}$. Suppose $\overline{\lambda}(a + b - a) < min^i\{\overline{\lambda}(a), \frac{\overline{\delta}-\overline{k}}{2}\}$. Choose $\overline{t}$ such that $\overline{\lambda}(a + b - a) < \overline{t} < min^i\{\overline{\lambda}, \frac{\overline{\delta}-\overline{k}}{2}\}$. Then $b \in \overline{\lambda}_{\overline{t}}$. Since $\overline{\lambda}_{\overline{t}}$ is an ideal of N, $a + b - a \in \overline{\lambda}_{\overline{t}}$ and $\overline{\lambda}(a + b - a) \geq \overline{t}$, a contradiction. Thus $\overline{\lambda}(a + b - a) \geq min^i\{\overline{\lambda}(a), \frac{\overline{\delta}-\overline{k}}{2}\}$. Similarly, it can be shown that $\overline{\lambda}(a(b + c) - ab) \geq min^i\{\overline{\lambda}(c), \frac{rline\delta-\overline{k}}{2}\}$ for all $a, b \in N$. Thus $\overline{\lambda}$ is an $(\in, \in \vee q_{\overline{k}}^{\overline{\delta}})$ i-v fuzzy ideal of N. □

Remark 3.17. Let $\overline{\lambda}$ be an $(\in, \in \vee q_{\overline{k}}^{\overline{\delta}})$ i-v fuzzy subnear-ring (ideal) of N. Then the level subset $\overline{\lambda}_{\overline{t}}$ is not necessarily a subnear-ring (ideal) in N. In Example 3.10, if we take $\overline{t} = [0.43,0.435]$ then $\overline{\lambda}_{\overline{t}} = \{b\}$ which is not subnear-ring in N, because $\overline{t} \notin \overline{0}$ to $\overline{0.4}$.

4. $(\in, \in\vee\, q_{\bar{k}}^{\bar{\delta}})$ **i-v fuzzy quasi-ideal and** $(\in, \in\vee\, q_{\bar{k}}^{\bar{\delta}})$ **i-v fuzzy bi-ideal**

In this section, we introduce the notions of $(\in, \in\vee\, q_{\bar{k}}^{\bar{\delta}})$ i-v fuzzy quasi-ideals and $(\in, \in\vee\, q_{\bar{k}}^{\bar{\delta}})$ i-v fuzzy bi-ideals of N which, respectively are generalizations of fuzzy quasi-ideals and bi-ideals of N.

Definition 4.1. An $(\in, \in\vee\, q_{\bar{k}})$ i-v fuzzy subgroup $\bar{\lambda}$ of N is called an $(\in, \in\vee\, q_{\bar{k}})$ i-v fuzzy quasi-ideal of N if for all $a \in N$, $\bar{\lambda}(a) \geq min^{i}\{(\bar{\lambda}N) \cap (N\bar{\lambda})\rho p(N * \bar{\lambda})(a), \frac{\bar{1}-\bar{k}}{2}.\}$

An $(\in, \in\vee\, q_{k}^{\delta})$ i-v fuzzy subgroup $\bar{\lambda}$ of N is called an $(\in, \in\vee\, q_{\bar{k}})$ i-v fuzzy bi-ideal of N if for all $a \in N$, $\bar{\lambda}(a) \geq min^{i}\{((\bar{\lambda}N\bar{\lambda}) \cap (\bar{\lambda}N * \bar{\lambda}))(a), \frac{\bar{1}-\bar{k}}{2}\}$.

Definition 4.2. An $(\in, \in\vee\, q_{\bar{k}}^{\bar{\delta}})$ i-v fuzzy subgroup of $\bar{\lambda}$ of N is called an $(\in, \in\vee\, q_{\bar{k}}^{\bar{\delta}})$ i-v fuzzy quasi-ideal of N if for all $a \in N$, $\bar{\lambda}(a) \geq min^{i}\{(rline\lambda N) \cap (N\bar{\lambda}) \cap (N * \bar{\lambda})(a), \frac{\bar{\delta}-\bar{k}}{2},\}$ where $\bar{k} \in D[0,1)$ and $\bar{\delta} \in D[0,1)$.

An $((\in, \in\vee\, q_{\bar{k}}^{\bar{\delta}}))$ i-v fuzzy subgroup $\bar{\lambda}$ of N is called an $(\in, \in\vee\, q_{\bar{k}}^{\bar{\delta}})$ i-v fuzzy bi-ideal of N if for all $a \in N$, $\bar{\lambda}(a) \geq min^{i}\{((ne\lambda N\bar{\lambda}) \cap (\bar{\lambda}N * \bar{\lambda}))(a), \frac{\bar{\delta}-\bar{k}}{2}\}$, where $\bar{k} \in D[0,1)$ and $\bar{\delta} \in D[0,1)$.

Remark 4.3. Every i-v fuzzy quasi-ideal, $(\in, \in\vee\, q$ i-v quasi-ideal and $(\in, \in\vee\, q_{\bar{k}})$i-v fuzzy quasi-ideal of N is an $(\in, \in\vee\, q_{\bar{k}}^{\bar{\delta}})$ i-v fuzzy quasi-ideal of N. Also every i-v fuzzy bi-ideal,$(\in, \in \check{e}eq$ i-v bi-ideal and $(\in, \in\vee\, q_{\bar{k}})$ i-v fuzzy bi-ideal of N is an $(\in, \in\vee\, q_{\bar{k}}^{\bar{\delta}})$ i-v fuzzy bi-ideal of N. However, as the following example shows, the converse is not necessarily true.

Example 4.4. Let $N = \{0,1,2,3\}$ be the group under addition modulo 4. Define multiplication as follows:

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

·	0	1	2	3
0	0	0	0	0
1	0	3	0	1
2	0	2	0	2
3	0	1	0	3

Then $(N, +, \cdot)$ is a near-ring (see [P.407] scheme 7). Let $\overline{\lambda}: N \to D[0,1]$ be an i-v fuzzy subset of N such that $\overline{\lambda}(0) = [0.4, 0.41]$, $\overline{\lambda}(1) = \overline{\lambda}(3) = [0.3, 0.32]$, $\overline{\lambda}(2) = [0.42, 0.43]$. Then $\overline{\lambda}$ is an $(\in, \in\vee\, q_{\overline{0.1}}^{\overline{0.2}})$ i-v fuzzy quasi-ideal of N. Since $\overline{\lambda}(0) = [0.4, 0.41] \ngeq min^i\{(\overline{\lambda}N)(0), (N\overline{\lambda})(0), (N * \overline{\lambda})(0)\} = [0.42, 0.43]$ and $\overline{\lambda}(0) = [0.4, 0.41] \ngeq min^i\{(\overline{\lambda}N)(0), (N\overline{\lambda})(0), (N * \overline{\lambda})(0), \frac{\overline{1}-\overline{k}}{2}\}\} = [0.42, 0.43]$, $\overline{\lambda}$ is neither an i-v fuzzy quasi-ideal of N nor an $(\iota n, \in\vee\, q)$ i-v fuzzy quasi-ideal of N. Also $\overline{\lambda}$ is an $(\in\in\vee\, q_{0.2}^{0.3})$ fuzzy bi-ideal of N. Since $\overline{\lambda}(0) \ngeq min^i\{(\overline{\lambda}N\overline{\lambda})(0), (\overline{\lambda}N * \overline{\lambda})(0)\}$ and $\overline{\lambda}(0) \ngeq min^i\{(\overline{\lambda}N\overline{\lambda})(0), (\overline{\lambda}N * \overline{\lambda})(0), \frac{\overline{1}-\overline{k}}{2}\}\}$, $\overline{\lambda}$ is neither an i-v fuzzy bi-ideal of N nor an $(\in, \in\vee\, q_{ek})$ i-v fuzzy bi-ideal of N.

Lemma 4.15. Let A be any nonempty subset of N. Then

(1) A is a quasi-ideal of N if and only if $\overline{\lambda}_A$ is an $(\in, \in\vee\, q_{\overline{k}}^{\overline{\delta}})$ i-v fuzzy quasi-ideal of N.

(2) A is a bi-ideal of N if and only if $\overline{\lambda}_A$ is an $(\in, \in\vee\, q_{\overline{k}}^{\overline{\delta}})$ fuzzy bi-ideal of N.

Proof. (1) Let A be a quasi-ideal of N. $\overline{\lambda}_A$ is an i-v fuzzy quasi-ideal of N and by Remark 4.3, $\overline{\lambda}_A$ is an $(\in, \in\vee\, q_{\overline{k}}^{\overline{\delta}})$ i-v fuzzy quasi-ideal of N.

Conversely, let $\overline{\lambda}_A$ be an $(\in, \in\vee\, q_{\overline{k}}^{\overline{\delta}})$ i-v fuzzy quasi-ideal of N. Let a be any element of $AN \cap N \cap N * A$. Then we have,

$\overline{\lambda}_A(a) \geq min^i\{(\overline{\lambda}_A\overline{\lambda}_N\overline{\lambda}_A \cap \overline{\lambda}_N\overline{\lambda}_A \cap \overline{\lambda}_N * \overline{\lambda}_A)(a), \frac{\overline{\delta}-\overline{k}}{2}\}$

$= min\{\overline{\lambda}_{(AN\cap NA\cap N*A)}(a), \frac{\overline{\delta}-\overline{k}}{2}\}$

$= min\{\overline{1}, \frac{\overline{\delta}-\overline{k}}{2}\}$

$= \frac{\overline{\delta}-\overline{k}}{2}$.

This implies that $a \in A$ and so $AN \cap NA \cap N * A \subseteq A$. This means that A is a quasi-ideal of N.

(2) Let A be a bi-ideal of N. $\overline{\lambda}_A$ is an i-v fuzzy bi-ideal of N and by Remark 4.3, $\overline{\lambda}_A$ is an $(\in, \in\vee\, q_{\overline{k}}^{\overline{\delta}})$ i-v fuzzy bi-ideal of N.

Conversely, let $\overline{\lambda}_A$ be an $(\in, \in\vee\, q_{\overline{k}}^{\overline{\delta}})$ i-v fuzzy bi-ideal of N. Let b be an element of $ANA \cap AN * A$. Then we have

$\overline{\lambda}_A(b) \geq min^i\{(\overline{\lambda}_A\overline{\lambda}_N \cap \overline{\lambda}_A\overline{\lambda}_N * \overline{\lambda}_A)(b), \frac{\overline{\delta}-\overline{k}}{2}\}$

$= min\{\overline{\lambda}_{(ANA\cap AN*A)}(b), \frac{\overline{\delta}-\overline{k}}{2}\}$

$= min\{\overline{1}, \frac{\overline{\delta}-\overline{k}}{2}\}$

$= \frac{\overline{\delta}-\overline{k}}{2}$. This implies that $b \in A$ and so $ANA \cap AN * A \subseteq A$. This means that A is a bi-ideal of N. □

Lemma 4.6. Any $(\in, \in\vee q_{\bar{k}}^{\bar{\delta}})$ i-v fuzzy quasi-ideal of N is an $(\in, \in\vee q_{\bar{k}}^{\bar{\delta}})$ i-v fuzzy bi-ideal of N.

Proof. Let $\bar{\lambda}$ be an $(\in, \in\vee q_{\bar{k}}^{\bar{\delta}})$ i-v fuzzy quasi-ideal of N. Then we have

$\bar{\lambda}N\bar{\lambda} \leq \bar{\lambda}(NN) \leq \bar{\lambda}N$

$\bar{\lambda}N\bar{\lambda} \leq (NN)\bar{\lambda} \leq N\bar{\lambda}$

$\bar{\lambda}N * \bar{\lambda} \leq (NN) * \bar{\lambda} \leq N * \bar{\lambda}$

Hence $\bar{\lambda}N\bar{\lambda} \cap \bar{\lambda}N * \bar{\lambda} \leq \bar{\lambda}N \cap N\bar{\lambda} \cap N * \bar{\lambda} \leq \bar{\lambda}$.

Hence $\bar{\lambda}N\bar{\lambda} \cap \bar{\lambda}N * \bar{\lambda} \leq \bar{\lambda}N \cap N\bar{\lambda} \cap N * \bar{\lambda} \leq \bar{\lambda}$.

Let $a \in N$. Now

$min^i\{(\bar{\lambda}N\bar{\lambda} \cap \bar{\lambda}N * \bar{\lambda})(a), (\frac{\bar{\delta}-\bar{k}}{2}) \leq \bar{\lambda}(a)\}$.

It follows that $\bar{\lambda}$ is an $(\in, \in\vee q_{\bar{k}}^{\bar{\delta}})$ i-v fuzzy bi-ideal of N.

However, as the following example shows, the converse of the Lemma 4.6 is not necessarily true. □

Example 4.7. Consider the near-ring $(N, +, \cdot)$.

Define an i-v fuzzy subset $\bar{\lambda}: N \to D[0,1]$ by:

$$\bar{\lambda}(a) = \begin{cases} [0.3, 0.35] & if \ \ a = 0, x(1) \\ & if \ \ otherwise. \end{cases} \quad (2)$$

Let $\bar{k} = \bar{0}.2$. For all $\bar{t} \in (\bar{0}, \frac{\bar{\delta}-\bar{k}}{2}]$, $\bar{\lambda}_{\bar{t}}$ is the bi-ideal of N. Hence $\bar{\lambda}$ is an $(\in, \in\vee q_{\overline{0.2}}^{\overline{0.9}})$ i-v fuzzy bi-ideal of N. For $\bar{t} = \overline{0.24}$, $\bar{\lambda}_{\bar{t}} = \{0, a\}$ and $N\bar{\lambda}_{\bar{t}} \cap \bar{\lambda}_{\bar{t}}N \cap N * \bar{\lambda}_{\bar{t}} = \{0, b\} \not\subseteq \{0, a\}$. Thus $\bar{l}ambda_{\bar{t}}$ is not a quasi-ideal of N. Hence $\bar{\lambda}$ is not an $(\in, \in\vee q_{\overline{0.2}}^{\overline{0.3}})$ i-v fuzzy quasi-ideal of N.

Theorem 4.8. Let $\bar{\lambda}$ be an $(\in, \in\vee q_{\bar{k}}^{\bar{\delta}})$ i-v fuzzy subset of N. If $\bar{\lambda}$ is an $(\in, \in\vee q_{\bar{k}}^{\bar{\delta}})$ i-v fuzzy left ideal (right ideal, N-subgroup, subnear-ring) of N,then $\bar{\lambda}$ is an $(\in, \in\vee q_{\bar{k}}^{\bar{\delta}})$ i-v fuzzy quasi ideal of N.

Proof. Let $\bar{\lambda}$ be an $(\in, \in\vee q_{\bar{k}}^{\bar{\delta}})$ i-v fuzzy left ideal of N. Let $a \in N$. Suppose $a = xy = n_1(n_2 + z) - n_1 n_2$, where x, y, n_1, n_2 and z are in N. We have

$(\bar{\lambda}N \cap N\bar{\lambda} \cap N * \bar{\lambda})(a) = min^i\{(\bar{\lambda}N)(a), (N\bar{\lambda})(a), (N * \bar{\lambda})(a)\}$

$= min\{sup_{a=xy}\bar{\lambda}(x), sup_{a=xy}\bar{\lambda}(y), sup_{a=n_1(n_2+z)-n_1n_2}\bar{\lambda}(z)\}$.

Now

$min\{(\bar{\lambda}N \cap N\bar{\lambda} \cap N * \bar{\lambda})(a), \frac{\bar{\delta}-\bar{k}}{2}\} = min^i\{min\{sup_{a=xy}\bar{\lambda}(x), sup_{a=xy}\bar{\lambda}(b),$

$sup_{x=n_1((n_2+z)-n_1n_2)}\bar{\lambda}(z), \frac{\bar{\delta}-\bar{k}}{2}\}\} \leq \overset{i}{\min}\{min\{1,1, sup_{a=n_1(n_2+z)-n_1n_2}\bar{\lambda}(z), \frac{\bar{\delta}-\bar{k}}{2}\}\}$

$= min\{sup_{a=(n_1(n_2+z)-n_1n_2)}\bar{\lambda}(z), \frac{\bar{\delta}-\bar{k}}{2}\}$

[since $\overline{\lambda}$ is an $(\in, \in\vee q_{\overline{k}}^{\overline{\delta}})$ i-v fuzzy left ideal, $\overline{\lambda}(n_1(n_2+z)-n_1n_2) \geq min^i\{\overline{\lambda}(c), \frac{\overline{\delta}-\overline{k}}{2}\}$]

$\leq \overline{\lambda}(n_1(n_2+z)-n_1n_2) = \overline{\lambda}(a)$.

We remark that if a is not expressed as $a = xy = n_1(n_2+z)-n_1n_2$, then $(\overline{\lambda}N \cap N\overline{\lambda} \cap N * \overline{\lambda})(a) = 0 \leq \overline{\lambda}(a)$ and $min^i\{(\overline{\lambda}N \cap N\overline{\lambda} \cap N * \overline{l}ambda)(a), \frac{\overline{\delta}-\overline{k}}{2} = 0 \leq \overline{\lambda}(a)\}$.

Thus $\overline{\lambda}(a) \geq min^i\{(\overline{\lambda}N \cap N\overline{\lambda} \cap N * \overline{\lambda})(a), \frac{\overline{\delta}-\overline{k}}{2}\}$, for all $a \in N$. Hence $\overline{\lambda}$ is an $(\in, \in\vee q_{\overline{k}}^{\overline{\delta}})$ i-v fuzzy quasi-ideal of N. □

Theorem 4.9. Let $\overline{\lambda}$ be an $(\in, \in\vee q_{\overline{k}}^{\overline{\delta}})$ i-v fuzzy subset of N. If $\overline{\lambda}$ is an $(\in, \in\vee q_{\overline{k}}^{\overline{\delta}})$ i-v fuzzy left ideal (right ideal, N-subgroup, subnear-ring) of N, then$\overline{\lambda}$ is an $(\in, \in\vee q_{\overline{k}}^{\overline{\delta}})$ i-v fuzzy bi-ideal of N.

Proof. By Theorem 4.8, Every $(\in, \in\vee q_{\overline{k}}^{\overline{\delta}})$ i-v fuzzy left ideal of N is an $(\in, \in\vee q_{\overline{k}}^{\overline{\delta}})$ i-v fuzzy quasi-ideal of N. and by Lemma ?, it is an $(\in, \in\vee q_{\overline{k}}^{ne\delta})$ i-v fuzzy bi-ideal of N. □

Theorem 4.10. Let $\overline{\lambda}$ be an $(\in, \in\vee q_{\overline{k}}^{\overline{\delta}})$ i-v fuzzy subset of N. $\overline{\lambda}$ is an $(\in, \in\vee q_{\overline{k}}^{\overline{\delta}})$ i-v fuzzy quasi-ideal of N if and only if $\overline{\lambda}_{inet}$ is a quasi-ideal of N, for all $\overline{t} \in (\overline{0}, \frac{\overline{\delta}-\overline{k}}{2}]$, $k \in [0,1)$.

Proof. Let $\overline{\lambda}$ be an $(\in, \in\vee q_{\overline{k}}^{\overline{\delta}})$ i-v fuzzy quasi-ideal of N. Let $a, b \in N$. Suppose $a, b \in \overline{\lambda}_{\overline{t}}$, $\overline{t} \in (\overline{0}, \frac{\overline{\delta}-\overline{k}}{2})$, $øverlinek \in D[0,1)$. Then $\overline{\lambda}(a) \geq \overline{t}$ and $\overline{\lambda}(b) \geq \overline{t}$. This implies that $min\{\overline{\lambda}(a), \overline{\lambda}(b), \frac{\overline{\delta}-\overline{k}}{2}\} \geq t$. Since $\overline{\lambda}$ is an $(\in, \in\vee q_{\overline{k}}^{\overline{\delta}})$ i-v fuzzy quasi-ideal, $\overline{\lambda}(a-b) \geq \overline{t}$ and hence $a-b \in \overline{\lambda}_{\overline{t}}$. Next, let $x \in \overline{\lambda}_{\overline{t}}N \cap N\overline{\lambda}_{\overline{t}} \cap N * \overline{\lambda}_{\overline{t}}$. Then there exist $x, y, z \in \overline{\lambda}_{\overline{t}}$ and $n_1, n_2, n_3, n_4 \in N$ such that $a = xn_1 = n_2y = n_3(n_4+c) - n_3n_4$. Thus $\overline{\lambda}(x) \geq \overline{t}$, $\overline{\lambda}(y) \geq \overline{t}$ and $\overline{\lambda}(z) \geq \overline{t}$. Then

$(\overline{\lambda}N \cap N\overline{\lambda} \cap N * \overline{\lambda})(a) = min^i\{(\overline{\lambda}N)(a), (N\overline{\lambda})(a), (N * \overline{\lambda})(a)\}$

$= min\{sup_{a=xn_1}\overline{\lambda}(x), sup_{a=n_2b}\overline{\lambda}(b), sup_{a=n_3(n_4+z)-n_3n_4}\overline{\lambda}(z)\} \geq \overline{t}$.

Now

$min^i\{(\overline{\lambda}N \cap N\overline{\lambda} \cap N * \overline{\lambda})(a), \frac{\overline{\delta}-\overline{k}}{2} = min^i[min\{sup_{a=xn_1}\overline{\lambda}(x),$

$sup_{a=n_2y}\overline{\lambda}(y), sup_{a=n_3(n_4+c)-n_3n_4}\overline{\lambda}(z)\}, \frac{\overline{\delta}-\overline{k}}{2}] \geq min\{\overline{t}, \frac{\overline{\delta}-\overline{k}}{2}\} = \overline{t}$.

Since $\overline{\lambda}$ is an $(\in, \in\vee q_{\overline{k}}^{\overline{\delta}})$ i-v fuzzy quasi-ideal of N, $\overline{\lambda}(a) \geq \overline{t}$. Thus $a \in \overline{\lambda}_{\overline{t}}$ and hence $\overline{\lambda}_{\overline{t}}$ is an quasi-ideal of N.

Conversely, let us assume that $\overline{\lambda}_{\overline{t}}$, $\overline{t} \in (\overline{0}, \frac{\overline{\delta}-\overline{k}}{2}]$, $\overline{k} \in D[0,1)$, is a quasi-ideal of N.$\overline{\lambda}$ is an i-v fuzzy quasi-ideal of N. By Remark efR4.3, $\overline{\lambda}$ is an $(\in, \in\vee q_{\overline{k}}^{\overline{\delta}})$ i-v fuzzy quasi-ideal of N. □

Remark 4.11. Let $\overline{\lambda}$ be an $(\in, \in\vee q_{\overline{k}}^{\overline{\delta}})$ i-v fuzzy quasi-ideal of N. Then the level subset $\overline{\lambda}_{\overline{t}}$ is not necessarily a quasi-ideal in N.$\overline{\lambda}$ is an $(\in, \in\vee q_{0.1}^{ne0,2})$ i-v fuzzy quasi-ideal of N. If we take

$\bar{t} = [0.42, o.43]$ then $\bar{\lambda}_{\bar{t}} = \{2\}$ and $\bar{\lambda}_{\bar{t}}N \cap N\bar{\lambda}_{\bar{t}} \cap N * \bar{\lambda}_{\bar{t}} = \{\bar{0}\} not \subseteq \{2\} = \bar{\lambda}_{\bar{t}}$. Hence $\lambda_{\bar{t}}$ is not a quasi-ideal in N, because $\bar{t} \notin \bar{0}$ to $\overline{0.4}$.

Theorem 4.12. Let $\bar{\lambda}$ be an $(\in, \in\vee\, q_{\bar{k}}^{\bar{\delta}})$ i-v fuzzy subset of N. $\bar{\lambda}$ is an $(\in, \in\vee\, q_{\bar{k}}^{\bar{\delta}})$ i-v fuzzy bi-ideal of N, if and only $\bar{\lambda}_{\bar{t}}$ is a bi-ideal in N, for all $\bar{t} \in (0, \frac{\bar{\delta}-\bar{k}}{2}]$, $\bar{k} \in D[0,1)$. and $\bar{\delta} \in D[0,1)$.

Proof. Let $\bar{\lambda}$ be an $(\in, \in\vee\, q_{\bar{k}}^{\bar{\delta}})$ i-v fuzzy bi-ideal of N. Let $\bar{t} \in (\bar{0}, \frac{\bar{\delta}-\bar{k}}{2}]$, $k \in D[0,1)$. Suppose $a, b \in N$ such that $a, b \in \bar{b}da_{\bar{t}}$. Then $\bar{\lambda}(a) \geq \bar{t}$; $\bar{\lambda}(b) \geq \bar{t}$ and $min\{\bar{\lambda}(a), \bar{\lambda}(b), \frac{\bar{\delta}-\bar{k}}{2}\} \geq \bar{t}$. Since $\bar{\lambda}$ is an $(\in, \in\vee\, q_{\bar{k}}^{\bar{\delta}})$ i-v fuzzy bi-ideal,$\bar{\lambda}(a-b) \geq \bar{t}$ and hence $a - b \in \bar{\lambda}_{\bar{t}}$. Let $c \in N$. Suppose $a \in \bar{\lambda}_{\bar{t}}N\bar{\lambda}_{\bar{t}} \cap \bar{\lambda}_{\emptyset verlinet}N * \bar{\lambda}_{\bar{t}}$. Then there exist $a, b, x_1, x_2, y \in \bar{\lambda}_{\bar{t}}$ and $n_1, n_2, n_3 \in N$ such that $c = an_1b = x_1n_2(x_2n_3 + b) - x_1n_2x_2n_3$.

Thus $\bar{\lambda}(a) \geq \bar{t}$, $\bar{\lambda}(b) \geq \bar{t}$, $\bar{\lambda}(x_1) \geq \bar{t}$, $\bar{\lambda}(x_2) \geq \bar{t}$ and $\bar{\lambda}(y) \geq \bar{t}$. Now

$(\bar{\lambda}N\bar{\lambda} \cap \bar{\lambda}N * \bar{\lambda})(c) = min^i\{(\bar{\lambda}N\bar{\lambda})(c), (\bar{\lambda}N * \bar{\lambda})(c)\}$

$$= min^i\{sup_{c=zn_1b}\{min\{\bar{\lambda}(a), \bar{\lambda}(b)\}\}, sup_{c=x_1n_2(x_2n_3+y)-x_1n_2x_2n_3}\{min\{\bar{\lambda}(x_1), \bar{\lambda}(y)\}\}\} \geq \bar{t}.$$

We have

$$min^i\{(\bar{\lambda}N\bar{\lambda} \cap \bar{\lambda}N * \bar{\lambda})(a), \frac{\bar{\delta}-\bar{k}}{2} = min^i[min\{sup_{c=an_1b}\{min\{\bar{\lambda}(a), \bar{\lambda}(b)\}\}, sup_{c=x_1n_2(x_2n_3+b)-x_1n_2x_2n_3}\{min\{\bar{\lambda}(x_1), \bar{\lambda}(y)\}\}\}, \frac{\bar{\delta}-\bar{k}}{2}]\}$$

$$\geq min\{\bar{t}, \frac{\bar{\delta}-\bar{k}}{2}\}$$

$$= \bar{t}.$$

Since $\bar{\lambda}$ is an $(\in, \in\vee\, q_{\bar{k}}^{\bar{\delta}})$ i-v fuzzy bi-ideal of N, $\bar{\lambda}(c) \geq \bar{t}$. Thus $c \in \bar{\lambda}_{\bar{t}}$ and hence $\lambda_{\bar{t}}$ is a bi-ideal of N.

Conversely, let us assume that $\bar{\lambda}_{\bar{t}}$, $\bar{0} < \bar{t} \leq \frac{\bar{\delta}-\bar{k}}{2}]$, $k \in D[0,1)$, is a bi-ideal of N. $\bar{\lambda}$ is an i-v fuzzy bi-ideal of N. By Remark 4.3, $ine\lambda$ is an $(\in, \in\vee\, q_{\overline{0.1}}^{\overline{0.2}})$ i-v fuzzy bi-ideal of N. □

Theorem 4.13. Let $\bar{\mu}$ and $\bar{\lambda}$ be any two $(\in, \in\vee\, q_{\bar{k}}^{\bar{\delta}})$ i-v fuzzy quasi-ideals of N. Then $\bar{\mu} \cap \bar{\lambda}$ is also an $(\in, \in\vee\, q_{\bar{k}}^{\bar{\delta}})$ i-v fuzzy quasi-ideal of N.

Proof. Let $\bar{\mu}$ and $\bar{\lambda}$ be any two $(\in, \in\vee\, q_{\bar{k}}^{\bar{\delta}})$ i-v fuzzy quasi-ideals of N. Then, by a proof similar to that of Theorem 3.14?, we can show that $(\bar{\mu} \cap \bar{\lambda})$ is an $(\in, \in\vee\, q_{\bar{k}}^{\bar{\delta}})$ i-v fuzzy subgroup of N. Let $a \in N$. Choose $x, y, x_1, y_1, z \in N$ such that $a = xy = x_1(y_1 + c) - x_1y_1$. Since $\bar{\mu}$ and $\bar{\lambda}$ are the $(\in, \in\vee\, q_{\emptyset verlinek}^{\bar{\delta}})$ i-v fuzzy quasi-ideals of N, we have

(1) $min^i\{min[sup_{a=xy}\{\bar{\lambda}(x)\}, sup_{a=xy}\{\bar{\lambda}(y), sup_{a=x_1(y_1+z)-x_1y_1}\bar{\lambda}(z)\}], \frac{\bar{\delta}-\bar{k}}{2}\} \leq \bar{\lambda}(a)$

and

(2) Now

$$min[((\overline{\mu} \cap \overline{\lambda})N \cap N(\overline{\mu} \cap \overline{\lambda}) \cap N * (\overline{\mu} \cap \overline{\lambda}))(a), \frac{\overline{\delta}-\overline{k}}{2}]$$

$$= min^i[min\{sup_{a=xy}\{(\overline{\mu} \cap \overline{\lambda})(x)\}, sup_{a=xy}\{(\overline{\mu} \cap \overline{\lambda})(y)\},$$

$$sup_{a=a_1(b_1+c)-x_1y_1}\{(\overline{\mu} \cap \overline{\lambda})(z)\}\}, \frac{\overline{\delta}-\overline{k}}{e2}]$$

$$= min^i[min\{sup_{a=xy}\{min\{(\overline{\mu}(x), \overline{\lambda})(x)\}\}\}, sup_{a=xy}\{min\{(\overline{\mu}(y), \overline{\lambda})(y)\},$$

$$sup_{a=a_1(b_1+c)-x_1y_1}\{min\{(\overline{\mu}(z), \overline{\lambda})(z)\}\}\}, \frac{\overline{\delta}-}{linek\overline{2}}]$$

$$\leq min^i[min\{min[\{sup_{a=xy}\{(\overline{\mu}(x)\}, sup_{a=xy}\overline{\mu}(y)\},$$

$$sup_{a=a_1(b_1+c)-x_1y_1}\{(\overline{\mu}(z), \frac{\overline{\delta}-\overline{k}}{2}\}, min\{min[\{sup_{a=xy}\{(\overline{\lambda}(x)\}, sup_{a=xy}\overline{b}da)(y)\},$$

$$sup_{a=a_1(b_1+c)-x_1y_1}\{(\overline{\lambda}(z), \frac{\overline{\delta}-\overline{k}}{2}\}]$$

$$\leq min\{\overline{\mu}(a), \overline{\lambda}(a)\},$$

from (1) and (2). $= (\overline{\mu} \cap \overline{\lambda})(a)$

Thus $\overline{\mu} \cap \overline{\lambda}$ is an $(\in, \in\vee\, q_{\frac{\delta}{k}})$ i-v fuzzy quasi-ideal of N. □

Theorem 4.14. Let $\overline{\mu}$ and $\overline{\lambda}$ be any two $(\in, \in\vee\, q_{\frac{\delta}{k}})$ i-v fuzzy bi-ideal of N. Then $\overline{\mu} \cap \overline{\lambda}$ is also an $(\in, \in\vee\, q_{\frac{\delta}{k}})$ i-v fuzzy bi-ideal of N.

Proof. The proof is similar to that of Theorem 4.13 □

5. Conclusion

In this paper we have presented the notion of $(\in, \in\vee\, q_{\frac{\delta}{k}})$-i-v fuzzy bi-ideals, $(\in, \in\vee\, q_{\frac{\delta}{k}})$-i-v fuzzy quasi-ideals of near-rings and derived the properties of these ideals.

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