Nano δg-Interior And Nano δg-Closure In Nano Topological Spaces

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Abstract

Lellis Thivagar introduced Nano topological spaces and studied some of their properties. Nano Gδ closed sets and Nano δG closed sets introduced by R. Vijayalakshmi,et.,al in Nano topological spaces. Aim of the present paper is we introduce Nano-δG Interior and Nano δG -Closure in Nano topological spaces. Also we investigate some of their relation and characterizations.

Keywords: *Nano δg closed sets, Nano δg open sets ,Nano-δG Interior , Nano δG -Closure, Nano topological spaces*

1. Introduction

The concept of Nano topology was introduced by Lellis Thivagar [5]which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it. Nano δ closed sets , Nano δg closed sets and Nano gδ closed sets introduced by R. Vijayalakshmi[6],et.,al in Nano topological spaces and studied some of their properties. Also we investigate the relationships between the other existing Nano closed sets.Aim of the present paper is we introduce Nano-δG Interior And Nano δG-Closure in Nano topological spaces. Also we investigate some of their relation and characterizations.

2. Preliminaries

Definition 2.1 [4]:

Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.

(i). The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$

That is $L_R(X) = \bigcup_{x\in U}\{R(x): R(x) \subseteq X\}$.

Where R(x) denotes the equivalence class determined by $x \in U$.

(ii). The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by

$$U_R(X) = \bigcup_{x\in U}\{R(x): R(x) \cap X \neq \phi\}$$

(iii). The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$. That is $B_R(X) = U_R(X) - L_R(X)$.

Property 2.2 [5]:
If(U, R) is an approximation space and X, Y ⊆ U, then

i) $L_R(X) \subseteq X \subseteq U_R(X)$
ii) $L_R(\phi) = U_R(\phi) = \phi$
iii) $L_R(U) = U_R(U) = U$
iv) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
v) $U_R(X \cap Y) \subseteq U_R(X) \cup U_R(Y)$
vi) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
vii) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
viii) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever X ⊆ Y.
ix) $U_R(X^C) = [L_R(X)]^C$ and $L_R(X^C) = [U_R(X)]^C$
x) $U_R(U_R(X)) = L_R(U_R(X)) = U_R(X)$
xi) $L_R(L_R(X)) = U_R(L_R(X)) = L_R(X)$

Definition 2.3 [5]:
Let U be a non-empty, finite universe of objects and R be an equivalence relation on U. Let X ⊆ U. Let $\tau_R(X) = N\tau = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$. Then Nτ is a topology on U , called as the Nano topology with respect to X.
Elements of the Nano topology are known as the Nano-open sets in U and (U, Nτ) is called the Nano topological space. $[N\tau]^C$ is called as the dual Nano topology of Nτ.
Elements of $[N\tau]^C$ are called as Nano closed sets. `

Definition 2.4[6]
Let (U, Nτ) be a Nano topological space with respect to X where X ⊆ U and if A ⊆U, then A is said to be
(i).Nano δ-closed
if A=Ncl_δ(A), where Ncl_δ (A)={x∈U:Nint(Ncl(Q))∩A ≠ϕ, Q∈Nτ and x∈Q}.
(ii).Nano δG-closed set if Nδcl(A)⊆Q whenever A⊆Q,Q is Nano open in (U, Nτ).
(iii).Nano Gδ-closed set if Ncl(A)⊆Q whenever A⊆Q , Q is Nδ-open in (U,Nτ).

3. Nano δG Interior In Nano Topological Space.

Defintion 3.1:
Let (U, Nτ) be a Nano topological space and let x ∈U. A subset N of U is said to be N-δG-neighbourhood of x if there exists a N-δG-open set G such that x ∈G⊂ N.

Definition 3.2
Let A be a subset of (U, Nτ). A point x ∈A is said to be N-δG-interior point of A is a N-δG-neighbourhood of x. The set of all N-δG-interior points of A is called the N-δG-interior of A and is denoted by N-δG-int(A).

Theorem 3.3
If A be a subset of (U, Nτ) .Then N-δG-int(A) = ∪{ G : G is a N-δG-open, G⊂ A}.

Proof:
Let A be a subset of (U, Nτ)
x∈N-δG-int(A)⇔x is a N-δG-interior point of A.
⇔A is a N-δG-nbhd of point x.
⇔there exists N-δG-open set G such that x∈G⊂A
⇔x ∈∪{G:G is a N-δG-open, G⊂A}
Hence N-δG-int(A)= ∪{G:G is a N-δG-open, G⊂A}.

Theorem 3.4:
Let A and B be subsets of (U, Nτ) . Then

(i).N-δG-int(U)=U and N-δG-int(ϕ)= ϕ
(ii). N-δG-int(A) ⊂ A.
(iii). If B is any N-δG-open set contained in A, then B ⊂N-δG - int(A).
(iv).If A ⊂B, then N-δG-int(A) ⊂N-δG-int(B).
(v). N-δG-int(N-δG-int(A)) = N-δG-int(A).

Proof:

(i). Since U and ϕ are N-δG open sets,
by Theorem 3.3N-δG-int(U) =∪{ G : G is a N-δG-open, G⊂U}
= U∪{all N-δG open sets}=U.
(ie) int(U)=U.
Since ϕ is the only N-δG-open set contained in ϕ,
N-δG-int(ϕ)= ϕ

(ii). Let x∈N-δG-int(A) ⟹x is a N-δG interior point of A.
⟹A is a nbhd of x.
⟹x ∈A.
Thus, x∈N-δGint(A) ⟹x∈A.
Hence N-δG-int(A)⊂A.

(iii). Let B be any N-δG-open sets such that B⊂A.Let Let x∈B.
Since B is a N-δG-open set contained in A. x is a N-δG-interior point of A.
(ie) x∈N-δG-int(A).Hence B ⊂N-δG-int(A).

(iv). Let A and B be subsets of (U, Nτ) such that A ⊂B. Let x∈N-δG-int(A). Then x is a N- δG-interior point of A and so A is a N-δG-nbhd of x. Since B⊃A, B is also N-δG-nbhd of x.⟹x∈N-δG-int(B).
Thus we have shown that x∈N-δG-int(A) ⟹x∈N-δG-int(B).

(v).Proof is obvious

Theorem 3.5:

If a subset A of space (U, Nτ) is N-δG-open, then N-δG-int(A)=A.

Proof:

Let A be N-δG-open subset of (U, Nτ). We know that N-δG-int(A)⊂A. Also, A is N-δG-open set contained in A. From Theorem (iii) A⊂N-δG-int(A). Hence N-δG-int(A)=A. The converse of the above theorem need not be true, as seen from the following example.

Example 3.6:

Let U= {a_1, a_2, a_3,a_4} with U/R= {{a_1, a_2},{ a_3,a_4}}
Let X={a_1, a_2}⊆U.
Then Nτ ={U, ϕ,{a_1, a_2}}.
Nano δG-open set ={U, ϕ, {a_1},{a_2}{a_3},{a_4}, {a_1, a_2},{a_1, a_3},{a_1, a_4},{a_2, a_3},{a_2, a_4}, {a_1, a_2,a_4},{a_1, a_2,a_3}}
N-δG-int({a_1, a_3,a_4}) = ϕ ∪{ a_1}∪{a_3}∪{ a_4}∪{a_1, a_3}∪{a_1, a_4}} ={ a_1, a_3,a_4}.
But { a_1, a_3,a_4} is not N-δG-open set in U.

Theorem 3.7:

If A and B are subsets of (U, Nτ),then N-δG-int(A)∪N-δG-int(B)⊂N-δG-int(A∪ B).

Proof.

We know that A⊂A∪ B and B⊂A∪B.
We have Theorem 3.4 (iv)
N-δG-int(A)⊂N-δG-int(A∪B), N-δG-int(B)⊂N-δG-int(A∪B).
This implies thatN-δG-int(A)∪N-δG-int(B)⊂N-δG-int(A∪B).

Theorem 3.8

If A and B are subsets of (U, Nτ) ,then N-δG-int(A∩B)=N-δG-int(A)∩N-δG-int(B).

Proof:

We know that A∩B⊂A and A∩B⊂B. We have N-δG-int(A∩B)⊂N-δG-int(A) and N-δG-int(A∩B)⊂N-δG-int(B). This implies that
N-δG-int(A∩B)⊂N-δG-int(A)∩N-δG-int(B) -----(1)

Again let x∈N-δG-int(A)⊂N-δG-int(B). Then x∈N-δG-int(A) and x∈N-δG-int(B). Hence x is a N-δG-int point of each of sets A and B. It follows that A and B is N-δG-nbhds of x, so that their intersection A∩ B is also a N-δG-nbhds of x. Hence x ∈N-δG-int(A∩B). Thus x∈N-δG-int(A) ∩N-δG-int(B) implies that x ∈N-δG-int(A∩B).
Therefore N-δG-int(A)∩N-δG-int(B)⊂N-δG-int(A∩B) ------(2)
From (1) and (2), We get N-δG-int(A∩B)=N-δG-int(A)∩N-δG-int(B).

Theorem 3.9

If A is a subset of U, then Nint(A)⊂N-δG-int(A).

Proof:

Let A be a subset of U.
Let x∈ Nint(A) ⟹x∪{G : G is nano open, G⊂ A}.
⟹there exists an nano open set G such that x∈ G ⊂A.
⟹there exist a N-δG-open set G such that x∈G ⊂A, as every open set is a N-δG-open set in U .
⟹x ∈∪{G:G is N-δG- open,G⊂A}.
⟹ x∈N-δG-int(A).
Thus x∈Nint(A) ⟹x∈N-δG-int(A).
Hence Nint(A)⊂N-δG-int(A).

Remark.**3.10**

Containment relation in the above theorem may be proper as seen from the following example.

Example 3.11

Let U= $\{a_1, a_2, a_3\}$ with U/R= $\{\{a_1,a_3\},\{a_2\}\}$
Let X=$\{a_1,a_3\}$⊆U.
Then Nτ =$\{U, \phi,\{a_1,a_3\}\}$.
N-δG-open =$\{U, \phi,\{a_1\},\{a_3\},\{a_1,a_3\}\}$
Let A = $\{a_3\}$. Now N-δG-int(A)=$\{a_3\}$ and Nint(A)= ϕ

4. Nano δG Closure In Nano Topological Space.

Definition 4.1:

Let A be a subset of a space U. We define the N-δG-closure of A to be the intersection of all N-δG-closed sets containing A.
In symbols, N-δG-cl(A) = ∩{F:A⊂ F ∈N-δGC(U)}

Theorem 4.2:

If A and B are subsets of a space U. Then
(i) N-δG-cl(U) = Uand N-δG-cl(ϕ) = ϕ
(ii) A⊂N-δG-cl(A).
(iii)If B is any N-δG-closed set containing A, then N-δG-cl(A) ⊂ B.
(iv) If A⊂ B then N-δG-cl(A) ⊂N-δG-cl(B).

Proof:

(i) By the definition of N-δG-closure, U is the only N-δG-closed set containing U.Therefore N-δG-cl(U)=Intersection of all the N-δG-closed sets containing U =∩{U}=U.That is N-δG-cl(U)=U.
By the definition of N-δG-closure, N-δG-cl(ϕ)=Intersection of all the N-δG-closed sets containing= ϕ ∩{ ϕ}= ϕ . That is N-δG-cl(ϕ)=ϕ .
(ii) By the definition of N-δG-closure of A, it is obvious that A ⊂N-δG-cl(A).
(iii) Let B be any N-δG-closed set containing A. Since N-δG-cl(A) is the intersection of all
N-δG-closed sets containing A, N-δG-cl(A) is contained in every N-δG-closed set containing A. Hence in particular N-δG-cl(A)⊂B
(iv) Let A and B be subsets of U such that A⊂B. By the definition N-δG-cl(B) = ∩{ F: B ⊂F∈ N-δGC(U)}. If B⊂ F ∈N-δGC(U), then N-δG-cl(B) ⊂ F. Since A ⊂B, A⊂ B ⊂F ∈

N-δGC(U), we have N-δG-cl(A) ⊂F. Therefore N-δG-cl(A)⊂{ ∩F:B⊂ F∈N-δGC(U)} = N-δG-cl(B). (i.e) N-δG-cl(A) ⊂N-δG-cl(A).

Theorem 4.3:
If A⊂ (U, Nτ) is N-δG-closed, then N-δG-cl(A)=A.
Proof:
Let A be N-δG-closed subset of U. We know that A ⊂N-δG-cl(A). Also A⊂ A and A is N-δG-closed. By theorem 4.2 (iii) N-δG-cl(A) ⊂A.
Hence N-δG-cl(A) = A.
Remarks 4.4:
The converse of the above theorem need not be true as seen from the following example.
Example:4.5
Let U= {a_1, a_2, a_3,a_4} with U/R= {{a_1, a_2},{ a_3,a_4} }
Let X={a_1, a_2}⊆ U.
Then Nτ ={U, ϕ ,{ a_1, a_2}}.
Nano δG-closed set ={ U, ϕ,{a_3},{a_4}, {a_1, a_3},{a_1, a_4},{a_2, a_3},{a_2, a_4},{a_3, a_4},{a_1, a_2,a_3},
{a_1, a_3,a_4}, {a_1, a_2,a_4},{a_2, a_3,a_4}}
N-δG-cl({a_1})={ a_1, a_3} ∩ { a_1, a_4}∩{a_1,a_2,a_3}∩{ a_1, a_3,a_4}∩{a_1, a_2,a_4}={ a_1}.
But { a_1} is not s N-δG-closed set in U.
Theorem 4.6
If A and B are subsets of a space (U, Nτ) then N-δG-cl(A∩ B)⊂N-δG-cl(A)∩N-δG-cl(B).
Proof:
Let A and B be subsets of U. Clearly A∩B ⊂A and A∩B⊂ B. By theorem N-δG-cl(A∩B)⊂N-δG-cl(A) and N-δG-cl(A∩B)⊂N-δG-cl(B). Hence N-δG-cl(A∩ B)⊂N-δG-cl(A) ∩N-δG-cl(B).
Theorem 4.7
If A and B are subsets of a space (U, Nτ) then N-δG-cl(A∪B)= N-δG-cl(A)∪N-δG-cl(B).
Proof:
Let A and B be subsets of U. Clearly A⊂A∪B and B⊂A∪B.
We have N-δG-cl(A) ∪N-δG-cl(B) ⊂N-δG-cl(A∪ B) ----(3)
Now to prove N-δG-cl(A∪B) ⊂N-δG-cl(A)∪N-δG-cl(B).
Let x∈N-δG-cl(A∪ B) and
suppose x∉N-δG-cl(A) ∪N-δG-cl(B). Then there exists N-δG-closed sets A_1 and B_1 with A⊂A_1, B ⊂B_1 and x∉A_1∪B_1. We have A∪B⊂A_1∪B_1 and A_1∪B_1 is N-δG-closed set by theorem such that x∉ A_1∪B_1.Thus x ∉N-δG-cl(A∪ B) which is a contradiction to x ∈N-δG-cl(A∪ B). Hence N-δG-cl(A∪B) N-δG-cl(A) ∪N-δG-cl(B) ----(4)
From (3) and (4), we have N-δG-cl(A∪B)=N-δG-cl(A) ∪N-δG-cl(B).
Theorem 4.8
For an x∈U,x ∈N-δG-cl(A) if and only if V∩ A≠ $\phi$for every N-δG- open sets V containing x.
Proof:
Let x ∈U and x∈N-δG-cl(A). To prove V∩A≠ ϕ for every N-δG-open set V containing x. Prove the result by contradiction. Suppose there exists a N-δG-open set V containing x such that V∩ A =ϕ . Then A ⊂U-V and U-V is N-δG-closed. We have N-δG-cl(A)⊂U-V. This shows that x∉N-δG-cl(A), which is a contradiction. Hence V∩A≠ $\phi$for every N-δG-open set V containing x. Conversly; let V∩ A ≠ $\phi$for every N-δG-open set V containing x. To prove x∈N-δG-cl(A). We prove the result by contradiction. Suppose x ∉N-δG-cl(A).
Then x ∈U – F and S –F is N-δG-open. Also (U–F)∩A = ϕ, which is a contradiction.
Hence x ∈N-δG-cl(A).
Theorem 4.9
If A is a subset of a space (U, Nτ) , then N-δG-cl(A) ⊂Ncl(A).
Proof:

Let A be a subset of a space (U, Nτ) .By the definition of nano closure, Ncl(A) = ∩{F: U⊂F ∈C(U)}.
If A⊂ F ∈NC(U)}, Then A⊂ F∈N-δG(U), because every closed set is N-δG-closed. That is N-δG-cl(A) ⊂ F. Therefore N-δG-cl(A) ⊂∩{F⊂ X :F ∈NC(U)} = Ncl(A). Hence N-δG-cl(A) ⊂Ncl(A).

Remark 4.10
Containment relation in the above theorem may be proper as seen from the following example.
Example 4.11
Let U= $\{a_1, a_2, a_3\}$ with U/R= $\{\{a_1,a_3\},\{a_2\}\}$
Let X=$\{a_1,a_3\}$⊆U.
Then Nτ ={U, ϕ,$\{a_1,a_3\}$}.
N-δG-Closed ={U, ϕ,$\{a_2\}$,$\{a_1,a_2\}$,$\{a_2,a_3\}$}
Let A=$\{a_1,a_2\}$. Now N-δG-cl(A)=$\{a_1,a_2a_3\}$ and N-cl(a_1,a_2)= U
It follows N-δG-cl(A)⊂N-cl(A) and N-δG-cl(A) ≠N-cl(A) .
Corrolory 4.12
Let A be any subset of (U, Nτ) . Then
(i) N-δG-int(A))^C = N-δG-cl(A^C)
(ii) N-δG-int(A) = (N-δG-cl(A^C))
(iii) N-δG-cl(A) = (N-δG-cl(A^C))
Proof:
Let x ∈N-δG-int(A))^C. Then x∉N-δG-int(A). That is every N-δG-open set V containing x is such that V⊄ A. That is every N-δG-open set V containing x is such that V⊄ A^C . By theorem x∈N-δG-int(A))^C and therefore N-δG-int(A))^C⊂N-δG-cl(A^C). Conversely, let x∈N-δG-cl(A^C). Then by theorem, every N-δG-open set V containing x is such that V∩A^C≠ ϕ. That is every N-δG-open set V containing x is such that V⊄A. This implies by definition of N-δG-interior of A, x∉N-δG-int(A). That is x∈N-δG-int(A))^C and N-δG-cl(A^C)⊂(N-δG-int(A))^C. Thus N-δG-int(A))^C=N-δG-cl(A^C)
(ii) Follows by taking complements in (i).
(ii) Follows by replacing A by A^C in (i).

CONCLUSIONS

Many different forms of concepts have been introduced over the years. Various interesting problems arise when one considers openness. Its importance is significant in various areas of mathematics and related sciences, this paper we studied the concept of Nano δG-Interior and Nano δG-Closure in Nano topological spaces. This shall be extended in the future Research with some applications

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REFERENCES

[1] S.Chandrasekar, T Rajesh Kannan, M Suresh,δωα-Closed Sets in Topological Spaces, Global Journal of Mathematical Sciences: Theory and Practical.9 (2), 103-116,**(2017)**

[2] S.Chandrasekar,T Rajesh Kannan,R.Selvaraj, δωα closed functions and δωα Closed Functions in Topological Spaces, International Journal of Mathematical Archive ,8(11),**2017**, 1-6.

[3] S.Chandrasekar, M.Suresh and T.Rajesh Kannan, Nano Sg-Interior And Nano Sg-Closure In Nano Topological Spaces, International Journal of Mathematical Archive, 8(4) ,94-100,(**2017**).

[4] S. Jeyashri, S. Chandrasekar and G. Ramkumar,Totally nano sg continuous functions And slightly nano sg continuous functions in nano topological spaces, International

Journal of Mathematical Archive-9(5), **2018**, 13-18

[5] M. L. Thivagar and C. Richard, On Nano forms of weakly open sets, International Journal of Mathematics and Statistics Invention, 1 (1) **2013**, 31-37.

[6] R. Vijayalakshmi, A.P.Mookambika,Nano δG-Closed Sets in Nano Topological Spaces, International Journal for Research in Engineering Application & Management (communicated).