# Analysis of Parallel Concatenated Convolutional Code with Generator Polynomials of Mixed Constraint Length

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## ABSTRACT

Communication systems require, information to be transmitted from transmitter to receiver in such a way that the information received is of the same quality as that of transmitted. Thus same has been tried to achieve by using the parallel concatenated convolutional codes for short distance communication. In this paper, asymmetric turbo codes are analyzed as an attractive solution to improve link budgets and reduce systems costs by relaxing the requirements on expensive optical devices in high capacity optical transport systems. The performance of the parallel concatenated convolutional codes can be improved by varying parameters such as frame size, number of decoding iterations, constraint lengthetc. This paper proposes a different class of convolutional codes using generator polynomials of mixed constraint length. Bit error rate (BER) decreases rapidly to the value of nearly one error per 10<sup>5</sup> bits at a reasonably good value of energy per bit to the noise power spectral density (Eb/No). The simulation results show better BER performance of parallel concatenated codes for 1500 frame size, five number of decoding iterations, code-rate of 1/3 and Logarithmic Maximum-A-Posteriori (Log-MAP) decoding algorithm over AWGN channel with BPSK modulation by using MATLAB.

## Keywords

Parallel concatenated convolutional codes, bit error rate, energy per bit to the noise power spectral density, Maximum-A-Posteriori.

## **INTRODUCTION**

In future, there is a big challenge to fulfil the demands of data for rapidly growing number of users. The main focus is on reliable transmission of data such that canbe received in the error free form. To achieve this, Bit Error Rate (BER) should be very less. In 1948, Claude Shannon [1] proposed that the information is a set of all possible messages, where the goal is to send messages over a noisy channel and to receive them with lower probability of error. Hamming codes were family of linear error-correcting codes, invented by Richard Hamming in 1950[2]. Due to the limited redundancy of the data; they can only detect and correct errors when the error rate is low. After this,forward error correction convolutional codes were introduced by Elias in 1955 [3]. Then in 1962, Robert Gallager [4] introduced a method of transmitting message over a noisy transmission channel named as low density parity check (LDPC) codesbut due to some drawbacks these codes are rediscovered in the mid-1990s by MacKay and Neal [5].

In 1993, turbo codes, also called as parallel concatenated convolutional codes, were introduced as one of the most powerful error control codes. These were the first practical codes to closely approach the channel capacity [6]. They are basically constructed by two or more parallel Recursive Systematic Convolutional (RSC) encoders, which are linked by a pseudo-random interleaver[7]. The parallel concatenated codes using non-identical component codes are referred as asymmetric turbo codes. It has different constraint lengths

and/or different generator polynomials [8].The decoding of a turbo codes can be done in an iterative way usingvarious decoding algorithms such as Maximum A posteriori Probability (MAP), Log-MAP [9], Soft Output Viterbi Algorithm (SOVA) [10] etc.

This paper proposes analysis of the asymmetric turbo codes that uses the mixed type of the component codes with different constraint length as well as generator polynomial. In the present work, the same has been tried to achieve by using the parallel concatenated convolutional codes for short distance communication. Here, BPSK (Binary Phase Shift Keying) modulation is considered over AWGN channel.

## 2. PARALLEL CONCATENATED CONVOLUTIONAL CODES

The parallel concatenated code encoder is shown in Figure 1. Here, both the RSC encoders having short constraint lengths are arranged parallel symmetry, in order to avoid excessive decoding complexity [11]. These two component encoders are separated by an interleaver. The first encoder takes the input information bits and generates the parity bits. The interleaver interleaves the information bits to generate interleaved information. The second encoder uses inter-leaved information and generates parity bits.



Figure 1. Parallel concatenated convolutional code encoder

These codes are basically consists of two parameters; the code rate r and the constraint length K. The code rate of 1/2 is used for puncturing, while 1/3 is for unpucturing of parallel concatenated codes. The constraint length parameter K denotes the length of the convolutional encoder i.e. the maximum number of input bits onwhich output depends. Basic structure of simulated turbo codes having generator polynomial (1,37/21,15/17) with mixed constraint length is shown in Figure2. Here, the one message bit is converted into three transmitted bits. The first bit is message bit represented as 1. Second bit is parity bit generated by passing message bits through the encoder I with generator polynomial (37/21) having constraint length equals to five while the third bit is parity bit generated by passing message bit through the encoder II with generator polynomial (15/17) having a constraint length equals to four.



Figure 2. Generator Polynomial (1,37/21,15/17)

The performance of the communication system depends upon the used decoding algorithm. A typical diagram showing the parallel concatenated convolutional decoder is shown in Figure3. Since parallel decoding is an iterative process, it requires a soft output algorithm like the logarithmic maximum a-posteriori algorithm (Log-MAP) or the Soft Output Viterbi Algorithm (SOVA) for decoding [9][10].



Figure3.Parallel concatenated convolutional code decoder

# **3. SIMULATION SETUP**

 Table 1.Simulation Specifications

Parameters	Value	
Number of Frames	500,1500,2500	
Channel Used	AWGN	
Generator polynomials	(1,7/5,15/17), $(1,15/17,37/21),(1,23/35,15/17),$ $(1,37/21,7/5),(1,37/21,15/17)$	
Algorithm	Log-MAP, SOVA	
Modulation Technique	BPSK	
Eb/No (dB)	0 to 3 dB	

Various parameters considered during simulation are given in table 1. The proposed asymmetric turbo code uses parallel concatenated component encoders with mixed type of

generator polynomials and different constraint lengths.During the simulation process, AWGN channel using BPSK modulation is considered for analyzing frame sizes, constraint length, un-punctured pattern, varying number of iterations and decoding algorithm.

### 4. RESULTS AND DISCUSSION

Turbo codes with larger constraint length will achieve good performance but by using this computational complexity also increases. Thus, it is necessary to decrease the constraint length of one of component encoder in order to obtain good BER performance.

Thus the variations of BER with respect to Eb/No for two decoding algorithms namely SOVA and Log-MAP are shown in Figure 4.It is observed that BER reduces to  $10^{-4}$  at Eb/No of 0.85 dB for Log-MAP while for SOVA it is approximately 1.85 dB. The Log-MAP is far better than the SOVA decoding algorithm. Therefore, for further implementation the Log MAP decoding algorithm is considered for simulation.



Figure 4.Variation of BER with Eb/No for different Decoding algorithms

The effect of puncturing on the performance of turbo codes are shown in Figure 5. It concludes that unpuncturing of turbo codes approaches to the minimum value of BER at a comparatively lesser value of Eb/No i.e. 1.2 dB. In the present work, short distance communications is considered in which accurate results are required. Therefore, unpunctured method is used for further simulation of parallel concatenated codes.



Figure 5.Variation of BER with Eb/No for punctured and un-punctured parallel concatenated convolutional codes.

In Figure 6, the variation of frames sizes for the polynomial (1,15/17,37/21) is shown by varying the number of frames send i.e. 500,1500,2500 frames. It has been observed that  $10^{-5}$  BER is achieved at lower Eb/No value for 2500 frame size. Thus, it is seen that on increasing the frame size, same BER is obtained at lower Eb/No. But while simulating, it is observed that it takes too much time for 2500 frames, therefore 1500 frames are considered for further

simulation of parallel concatenated convolutional codes which closely approaches the higher 2500 frame size performance.



#### Figure 6.Variation of BER with Eb/No for different frame sizes

The polynomials selected here consist of mixed constraint lengths i.e. of value k=2,3,4, Here, encoder I and II both are consisted of generator polynomials having different constraint length as shown in table 2.

Code	Polynomial	Encoder I		Encoder II	
110.		Constraint length	Polynomial	Constraint length	Polynomial
1.	(1,7/5,15/17)	K=3	7/5	K=4	15/17
2.	(1,15/17,37/21)	K=4	15/17	K=5	37/21
3.	(1,23/35,15/17)	K=5	23/35	K=4	15/17
4.	(1,37/21,15/17)	K=5	37/21	K=4	15/17
5.	(1,37/21,7/5)	K=5	37/21	K=3	7/5

Table 2.Different sets of generator polynomials employed

In Figure7, the variation of BER with Eb/No is presented for the generator polynomial(1,7/5,15/17). It is seen that with the increase in number of iterations, BER of  $10^{-5}$  at 1.42 dB Eb/No is achieved for fifth iteration.



Figure 7. Variation of BER with Eb/No for polynomial (1,7/5,15/17)

Variations by polynomial (1,15/17,37/21) in the simulation of parallel concatenated codes are shown in the Figure8. It is noticed that BER of  $10^{-5}$  at Eb/No of 1.21 dB is achieved for the fifth iteration.



Figure8.Variation of BER with Eb/No for polynomial (1,15/17,37/21)

In Figure9, the effect of BER with Eb/No is shown for polynomial (1,23/35,15/17). Here, performance of BER improves rapidly with number of increasing iterations especially after second iteration. BER minimum is achieved for 1.4 dB, 1.18 dB and 0.97 dB values of Eb/No for iteration 3,4,5 respectively.Variation by polynomial (1,37/21,15/17) on the simulation of parallel concatenated codes is shown in Figure 10. It is seen that, BER minimum of $10^{-5}$  is achieved corresponding to 0.92 dB of Eb/No for fifth iteration. Thus the value obtained at the fifth iteration is concluded as one of the best values of Eb/No for turbo codes under some specified conditions.



Figure 9. Variation of BER with Eb/No for Polynomial (1,23/35,15/17)



Figure 10. Variation of BER with Eb/No for Polynomial (1,37/21,15/17)

The Eb/No after fifth iteration comes out to be 1.41 dB for BER  $10^{-5}$  as shown in Figure 12. Here, the polynomial used for the simulation is consisted of constraint length k=5 for the encoder I and k=3 for the encoder II.





By comparing all the polynomials, it has been analyzed that the polynomial (1,37/21,15/17) shows the best results out of the all used polynomials as shown in Figure 13.



Figure 11.Variation of BER with Eb/No for all the polynomials at five number of iterations

#### **CONCLUSIONS**

In this paper, the asymmetric parallel concatenated convolutional codes are studied andevaluated for optimum performance under AWGN channel. Generator polynomial isimportant in designing the turbo code system where the performance of the system can beimproved without any complexity of the system. From the error performance results, it is observed that parallel codes are quite suitable for the wireless communications applications. In this system, the encoder architecture is basedon parallel concatenation of random interleaver of iterative cooperation between soft-inputsoft- outputs (SISO) decoder. It is concluded that by changing the frame size, generatorpolynomial, code rate and also the number of iterations, the performance of turbo encoderand decoder gets affected. The achieved results concludes that the polynomial (1,37/21,15/17) is best for optimizing the asymmetric parallel concatenated codes. These asymmetric turbo codes can be used as forward-error correction method in high-capacity fiber optic transport systems i.e. in OTN and PON network systems. Thus, Noise free short distance communication without having bandwidth loss can be achieved by using parallel concatenated convolutional codes. These codes are also helpful in mobile communication and wireless grid.

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