Application of modified least square method to study the heat transfer through porous fin with temperature dependent internal heat generation

Shatendra ku. Sahu[1*], Dr. Arun Arora[2], Dr D.S.Bal[3]
[1]*PhD scholar Department of Mechanical Engineering Bhilai Institute of Technology Durg Chhattisgarh India 490006*
[2]*Principal and ProfessorDepartment of Mechanical Engineering Bhilai Institute of Technology Durg Chhattisgarh India 490006*
[3]*Ex Professor Department of Mechanical Engineering Government Engineering College Raipur Chhattisgarh India*
[1*]shatendra24@gmail.com, [2]mechbitdurg@yahoo.co.in, [3]drdsbal@rediffmail.com

Abstract

In this study, modified least square method (MLS) is used to optimize heat transfer through porous fin with temperature dependent internal heat generation. The material considered for porous fin is Si3N4. The heat generated through fin varies linearly with temperature. The analysis of temperature distribution through porous media is based on passage velocity from the Darcy's model. For the validation of above model the Results is compared with the numerical results. A higher heat generation rate leads to higher fin temperatures since more amount of heat is dissipated to the surrounding. The temperature distribution through fin is depended to the Darcy and Rayleigh numbers.

Keywords: *modified least square method,temperature dependent internal heat generation, Darcy's model, Rayleigh numbers, porous fin*

1. Introduction

Fins are used in heat-exchangers, furnaces, super heaters, and turbine. Fins are used in aeroplanes, power plants, bikes etc. The heat transfer rate through fin depends on number parameter such as surface area, surrounding fluid, thermal conductivity of material, length of fin etc. Number of research has been made for heat transfer through porous fin. Porous media has a numerous industrial application like catalytic and inert packed bed reactors, enhancing drying efficiency, filtering, insulation, lubrication.[1]Sobamowo optimizedheat transfer study of porous fin with temperature-dependent thermal conductivity and internal heat generationisanalyzed numerically using Legendre wavelet collocation method and result is validated by Runge-Kutta method [2]. Hatami et al applied three methods to study porous fin with temperature dependent internal heat generation. Result reviles that the differential transformation method, least square method and collocation method are more effective compared with numerical method [3].Homotopy perturbation method, variation iteration method, and perturbation method are employed to approach temperature distribution of porous fins by Rostamiyan et al. The result has been validated with the analytical method [4]. Hoshyaret al has applied Homotopy Analysis Method (HAM), for the analysis of temperature distribution in a porous fin with temperature dependent internal heat generation. The Homotopy Analysis Method (HAM) has high capability to solve nonlinear problems [5]. The analytical approaches like Least Squares Method (LS), Differential Transformation Method (DTM) and Collocation Method (CM) are used for solving heat transfer through fin and to improve fin efficiency. Aziz and Bouaziz applied least square method for same type of problems and compared with numerical result [6]. Akbari et al studied the efficiency of straight fin with different methods and developed Akbari's- Ganji's method which is very effective to solve nonlinear equations [7].Darvishi et al. found the effects of radiation and convection heat transfer in a rectangular radial porous fin. This allowed for the heat flow to infiltratethe porous fin enabling a solid-fluid interaction to occur. Theyestablished that in a model containing radiation more heat is present than in a similar model without radiation [8]. Gorla and Bakier investigated natural convection and radiation in porous fins.They concluded that the radiation transfers more heat in comparison to a similar model without radiation [9].Kiwan and Zeitounfinds the performance of rectangular porous fins mounted around the inner cylinder of a cylindrical annulus by performing a finite volume type numerical study. They investigated that, in comparison to solid fins, porous fins provided higher transfer rates for similar configurations and that the heat transfer rate from the cylinder equipped with porous fins decreased as the fin inclination increased [10]. Naidu et al applie conjugate conduction-convection analysis for numerical study of natural convection from a cylindrical fin placed in a cylindrical porous enclosure for solving the heat conduction equation [11]. Jooma and Harley studied Time dependent nonlinear partial differential equation modelling heat transfer in a porous radial fin. The Differential Transformation Method is employed in order to account for the steady state condition [12]. Bouaziz et al. presented the efficiency of longitudinal fins with temperature-dependent thermo physical properties. Also, the effects of temperature-dependent thermal conductivity of a moving fin and added radiative component to the surface heat loss have been studied [13]

As we have seen a sufficient amount of investigation for porous fin has been done and various methods have been applied. In this study A method called Modified least square method has been applied to porous fin with internal heat generation. The analysis of temperature distribution through porous media is based on passage velocity from the Darcy's model. For the validation of above model the Results is compared with the numerical results. Rao and Bal developed modified least square method for curve fitting which has advantage over least square method when applied to many engineering problems. Number of numerical problem has been solved and result is compared with least square method [14][15]. Till now very less number of investigations has been done using modified least square problem like Bal and Bal used modified least square method for optimization of space curve generation mechanism whose equations are nonlinear and compared with the result obtained from least square method. Hence modified least square method is powerful tool to solve nonlinear differential equation [16].

2. Description of Problem

The problem is studied with the example of porous fin with rectangular profile and has temperature-dependent internal heat generation. The dimensions of this fin are length L, width w and thickness t. The cross section area of the fin is A is constant. Few assumptions have been made for simplicity of problem like porous medium is homogeneous and isotropic and both fluid and solid have same phase.The another assumption is finite-length fin with insulated tip, so the heat loss from the tip of the fin compared with the top and bottom surfaces of the fin is assumed to be negligible. Since the transverse Biot number should be small for the fin to be effective [6] [17], the temperature variation in the transverse direction are neglected.

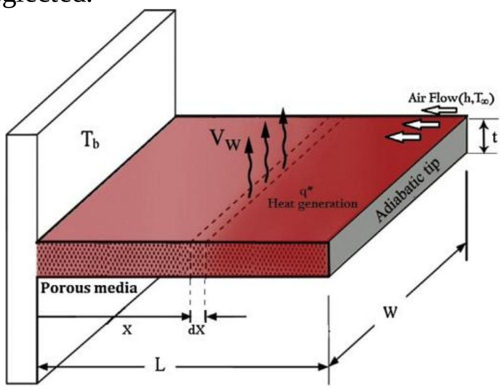

Figure 1.Convective porous fin with temperature-dependent heat generation [3].

The temperature inside the fin is only a function of *length*, thus heat conduction is assumed to occur solely in the longitudinal direction. Energy balance can be written as:

$$q_x - q_{x+\Delta x} + q^* A.\Delta x = \dot{m} c_p (T_x - T_\infty) + hp\Delta x (T_x - T_\infty) \quad [1]$$

The mass flow rate of fluid passing through porous material is

$$\dot{m} = \rho V_w \Delta x w \quad [2]$$

Where $V_w = \frac{gk\beta}{v}(T_x - T_\infty)$ from Darcy's model [3]

Substitution of equation (2) and (3) in to equation (1) yields

$$\frac{q_x - q_{x+\Delta x}}{\Delta x} + q^* A = \frac{\rho c_p gk\beta w}{v}(T_x - T_\infty)^2 + hp(T_x - T_\infty) \quad [4]$$

As Δx tends to zero equation 4 becomes

$$\frac{dq}{dx} + q^* A = \frac{\rho c_p gk\beta w}{v}(T_x - T_\infty)^2 + hp(T_x - T_\infty) \quad [5]$$

From Fourier's law of conduction $q_{conduction}$ is given by,

$$q_{conduction} = k_{eff}\, A \frac{dT}{dx} \quad [6]$$

Where A is area of cross section of fin A= (w.t) and k_{eff} is the effective thermal conductivity of porous fin that can be obtained from following equation

$$k_{eff} = \emptyset . k_f + (1 - \emptyset) k_s \quad [7]$$

Where ϕ is porosity of porous fin. Substitution of equation (6) into equation (5) leads to:

$$\frac{d^2T}{dx^2} + \frac{q^*}{k_{eff}} - \frac{\rho c_p gk\beta}{t.k_{eff}.v}(T_x - T_\infty)^2 + \frac{hp}{k_{eff}.A}(T_x - T_\infty) = 0 \quad [8]$$

It is assumed that heat generation in the fin varies with temperature as

$$q^* = q^*_\infty (1 + \varepsilon (T_x - T_\infty)) \quad [9]$$

Where q^*_∞ is internal heat generation at temperature T_∞.

For simplifying above equations some dimensionless parameters are introduced as follows:

$$\boldsymbol{\theta} = \frac{(T_x - T_\infty)}{(T_b - T_\infty)} \quad \boldsymbol{X} = \frac{x}{L} \quad \boldsymbol{M^2} = \frac{hPL^2}{K_0 A} \quad \boldsymbol{S_h} = \frac{D_a.x.R_a}{k_r}\left(\frac{L}{t}\right)^2 \quad \boldsymbol{G} = \frac{q^*}{hP(T_x - T_\infty)} \quad \boldsymbol{\varepsilon_G} = \boldsymbol{\varepsilon}(T_b - T_\infty) \quad [10]$$

Where S_h is a porous parameter that indicates the effect of the permeability of the porous medium as well as buoyancy effect so higher value of S_h indicates higher permeability of the porous medium or higher buoyancy forces. M is a convection parameter that indicates the effect of surface convecting of the fin. Finally equation (8) can be rewritten as:

$$\frac{d^2\theta}{dx^2} - \boldsymbol{M^2\theta} + \boldsymbol{M^2 G(1 + \varepsilon_G \theta)} - \boldsymbol{S_h \theta^2} = \boldsymbol{0} \quad [11]$$

In this paper we assumed finite-length fin with insulated tip. For this case, the fin tip is insulated so that there will not be any heat transfer at the insulated tip and boundary condition will be,

$$\theta(0) = 1, \qquad \theta'(1) = 0 \quad [12]$$

3.Modified Least Square Method

Least square method is used to solve linear and nonlinear differential equation approximately and very conveniently. Since it gives approximate solution to problem it should be close to the exact. Modified least square method is (Rao and Bal[14][15]) developed over least square method to solve nonlinear differential equation and gives accurate solution. Bal et al [15] solved some example like rigid body dynamics, problems of chemical solution, with modified least square method and compared with the result of least square method. Application of modified least square method to achieve the minimum to total sum of square is given by

$f = \sum_{i=1}^{n} y_i^2 (\overline{y_i} - y_i)^2$ [13]

Between the fixed value and approximated value where y_i as experimental data and $\overline{y_i}$ as approximated value. To achieve minimum value the f, derivative of f with respect to all unknown parameter must be zero.

Application of modified least square method

The boundary conditions for the porous fin are according to assumptions are

$\theta(0) = 1, \theta'(1) = 0$. To satisfy the boundary condition the solution of given differential equation, the trail function can be approximated as $\left[X - \left(\frac{1}{n+1}\right).X^{n+1}\right]$. It is to be noted that other polynomial equation which satisfy above boundary condition can also be considered. So it is taken as,

$$\theta(X) = 1 - A\left(x - \frac{1}{2}x^2\right) + B\left(x - \frac{1}{3}x^3\right) \quad [14]$$

Where A, B are constants. To solve trial function using modified least square method four points in the domain can be taken as

$$x_1 = \frac{1}{5}, x_2 = \frac{2}{5}, x_3 = \frac{3}{5}, x_4 = \frac{4}{5} \quad [15]$$

To determine constants A and B we consider following coefficients for porous material Si_3N_4

G=0.4; M=1; Da=0.0001; L/t=10; Ra=10,000;

$\varepsilon_G = 0.6$; L=1; k_s=954 [16]

S_h can be calculated. From equation (xi) (xii) (xiii) (xiv) we get the temperature distribution as

$$\theta(x) := 1 - 0.3110069394x + 0.1768431015x^2 - 0.01422641976x^3 [17]$$

4. Result and Discussion

In this paper heat transfer through porous fin with internal heat generation is considered. The fin is of rectangular profile. Modified least square method is used to find heat transfer through porous fin. To validate the above model the result has been compared with the numerical analysis done by Hatami et al [3] as shown in table 1.

Table 1.Dimensionless temperature values for Si_3N_4

x	MLS	LS
0.0	**1**	**1**
0.1	**0.9706535107**	**0.9771365539**
0.2	**0.9447585248**	**0.9570592833**
0.3	**0.9222296840**	**0.9396788027**
0.4	**0.9029816295**	**0.9249057265**
0.5	**0.8869290032**	**0.9126506687**
0.6	**0.8739864462**	**0.9028242440**
0.7	**0.8640686001**	**0.8953370665**
0.8	**0.8570901066**	**0.8900997505**
0.9	**0.8529656067**	**0.8870229104**
1.0	**0.8516097423**	**0.8860171606**

The above result is based on G=0.4; M=1; Da=0.0001; L/t=10; Ra=10,000; $\varepsilon_G = 0.6$; L=1. The heat transfer through porous fin is affected by Darcy number and Rayleigh numbers. When Rayleigh no. changes, heat transfer through porous fin also changes this effects is plotted as shown below. The effect of increasing the Ra number which represents the buoyancy force isshownin Figure 2. It means that by increasing in the Ra number leads to more effect of Buoyancy force and consequently heat transfer rate (Figure-3) due to convection. So, higher value of Ra makes more heat transfer between the solid fin and the air flow and subsequently leads to more value of temperature.

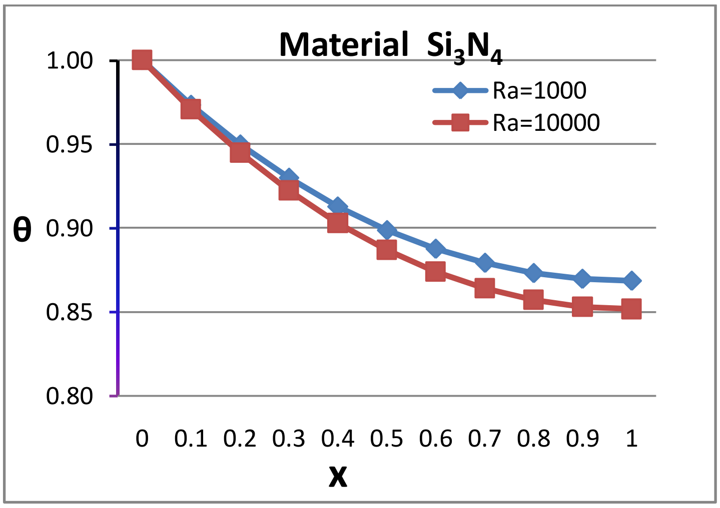

Figure-2 Temperature Distribution versus Ra variation at G = 0.4, M = 1, ε_G = 0.6 and Da = 10^{-3}.

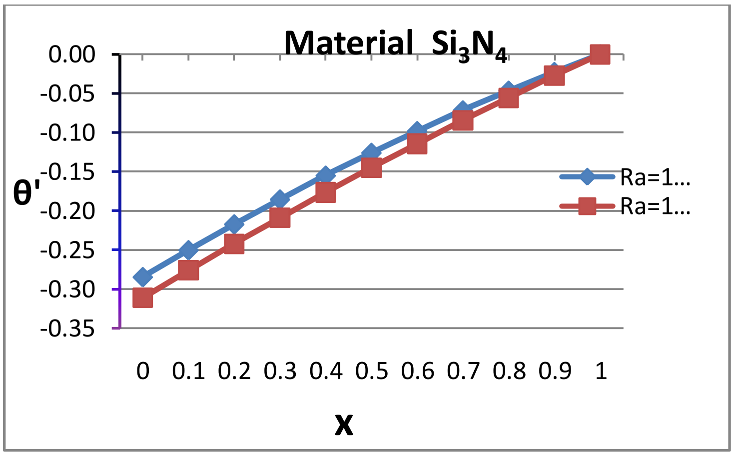

Figure-3 Rate of heat transfer versus Ra variation at G = 0.4, M = 1, ε_G = 0.6 and Da = 10^{-3}.

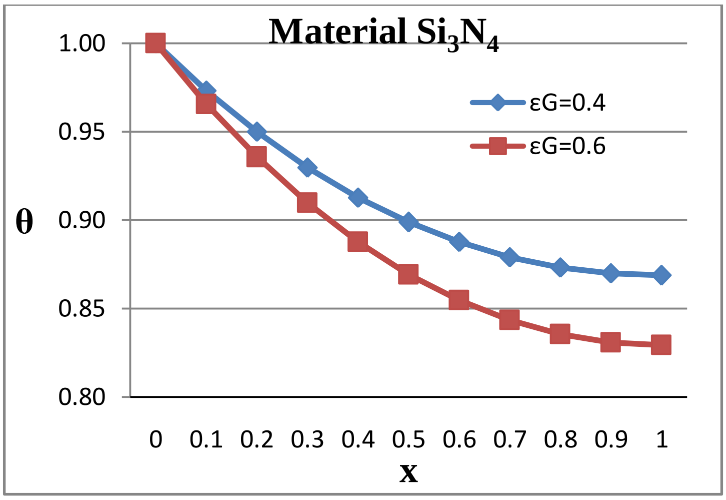

Figure-4 Temperature profile versus $\varepsilon_G$variation at G = 0.4, M = 1, Ra=10000 and Da = 10^{-3}.

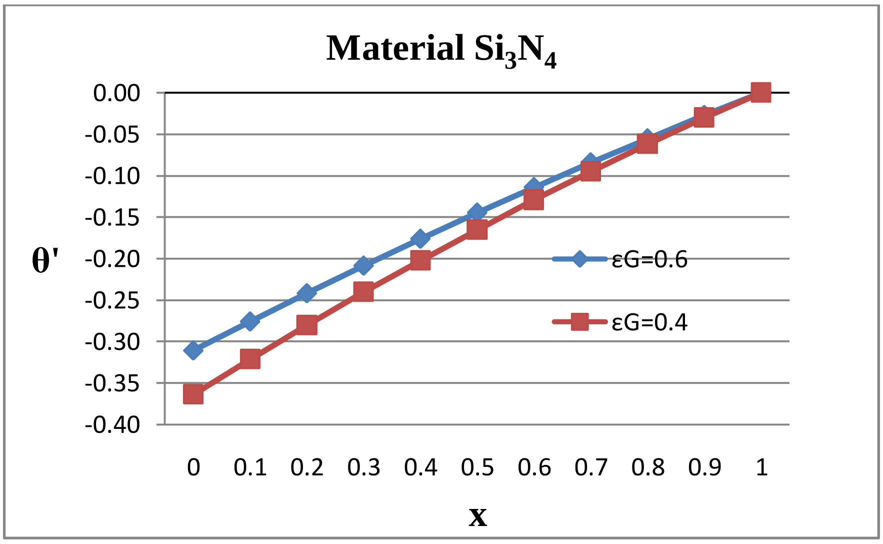

Figure-5 plot for θ' versus ε_G variation at G = 0.4, M = 1, Ra=10000 and Da = 10^{-3}.

The effect of variation in internal heat transfer parameter is plotted in Figure 4. The other parameters are taken as constant and values are G = 0.4, M = 1, Ra=10000 and Da = $10^{-3.}$ The variation of heat generation is directly related to internal heat transfer parameter. Thus by increasing this parameter, the heat generation through the fin also increases. Bychanging the value of the heat generation parameter, the temperature profile reaches to the higher value. The effect of variation in ε_G on rate of heat transfer is as shown in figure-5. As ε_G increases rate of Heat transfer also increase along the length of the fin.

5. Conclusions

In this research, Modified least square method is applied to study the heat transfer in a rectangular porous fin with the temperature dependent internal heat by considering the assumption of adiabatic tip. It has been found that modified least square method is one of the effective method to solve nonlinear problem.The results indicate that the temperature distribution is depending on the Darcy and Rayleigh numbers. The higher heat generation inside fin leads to higher fin temperatures, because the fin has to dissipate a larger amount of heat to the atmosphere under steady state conditions.

Nomenclature

a constant	M convective parameter
A section area of fin	n number of iteration
c_p specific heat	q conducted heat
G generation number, dimensionless	q* internal heat generation
h convection coefficient	Ra Rayleigh number
k thermal conductivity	S_h porous parameter
A, B are constants	T temperature
K permeability	Tb temperature at fin base
L length of fin	T_∞ sink temperature for convection
LS Least Square Method	t thickness of the fin
MLS Modified least square Method	ũ trial function
β coefficient of volumetric thermal expansion	V_w velocity of fluid passing through the fin
Δ temperature difference	w width of the fin
ε_G internal heat generation parameter	x axial coordinate
θ dimensionless temperature	X dimensionless axial coordinate [x/L]
ν kinematic viscosity	f fluid properties
ρ density	eff porous properties
ϕ porosity variable	s solid properties

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