A Categorization for Soft Sets Theory

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ABSTRACT

In this paper, it begins with a brief introduction on soft sets and then it describes many generalizations of it. The notions of generalized soft sets are defined and their properties are studied. After that, a notion of mapping, called soft mapping, in soft set setting is introduced. Later, algebraic structures on soft sets like soft group, soft ring, etc. are discussed we give a crisp and critical survey of the development of soft set theory and enumerate some of its various applications in different direction .

***Keywords**: Introduction to soft sets, Soft sets related work, Categorization of Soft Sets.*

I. INTRODUCTION

A soft set is a classification of elements of the universe with respect to some given set of parameters. It has been shown that soft set is more general in nature and has more capabilities in handling uncertain information. Also a fuzzy set or a rough set can be considered as a special case of soft sets. Research involving soft sets and its application in various fields of science and technology are currently going on in a rapid pace. Modern set theory formulated by George Cantor is fundamental for the whole of Mathematics. One issue associated with the notion of a set is the concept of vagueness. Mathematics requires that all mathematical notions including set must be exact. This vagueness or the representation of imperfect knowledge has been a problem for a long time for philosophers, logicians and mathematicians. However, recently it became a crucial issue for computer scientists, particularly in the area of artificial intelligence. To handle situations like this, many tools have been suggested. They include Fuzzy sets, Multi sets, Rough sets, Soft sets and many more. Owing to the fact that many mathematical objects such as fuzzy sets, topological spaces, rough sets [1,2] can be considered as particular types of soft sets, it is a very general tool to handle objects which are defined in terms of loose or general set of characteristics. A soft set can be considered as an approximate description of an object precisely consisting of two parts, namely predicate and approximate value set. Exact solutions to the mathematical models are needed in classical mathematics. If the model is so complicated that we cannot set an exact solution, we can derive an approximate solution and there are many methods for this. On the other hand, in soft set theory as the initial description of object it is of an approximate nature, we need not have to introduce the concept of an exact solution. Soft theory was initiated by the Russian researcher Molodtsov in 1999. Molodtsov proposed the soft set as a completely generic mathematical tool for modeling uncertainties. There is no limited condition to the description of objects; so researchers can choose the form of parameters they need, which greatly simplifies the decision-making process and make the process more efficient in the absence of partial information. There are many mathematical tools available for modeling complex systems such as probability theory, fuzzy set theory, interval mathematics etc. But there are inherent difficulties associated with each of these techniques. Probability theory is applicable only for a stochastically stable system. Interval mathematics is not sufficiently adaptable for problems with different uncertainties. Setting the membership function value is always been a problem in fuzzy set theory.

II. RELATED WORK

The origin of soft set theory could be traced to the work of Pawlak [3, 4] in 1993 titled hard sets and soft sets [5]. His notion of soft sets is a unified view of classical, rough and fuzzy sets. This motivated by Molodtsov in 1999 [6] titled soft set theory: first result, there in, the basic notions of the theory of soft sets and some of its possible applications were presented. In 1996 Lin have present a set theory for soft computing and presenting unified view of fuzzy sets via neighborhoods. This paper proposed fuzzy sets should be abstractly defined by such structures and are termed soft sets (sofsets). Based on such structures, W-sofset, F-sofset, P-sofset, B-sofset, Csofset, N-sofset, FP-sofset, and FF-sofsets have been identified. Maji et al., [7] presented a combination of fuzzy and soft set theories, fuzzy soft set theory is a more general soft set model which makes descriptions of the objective world more general, realistic, practical and accurate in some cases of decision making. In 2003 again presented soft set theory with some implementation in their work. Roy & Maji [8] presented a novel method of object recognition from an imprecise multi observer data in decision making problem. Pei & Miao [9] have discussed the relationship between soft sets and information systems. It is showed that soft sets are a class of special information systems. After soft sets are extended to several classes of general cases, the more general results also show that partition-type soft sets and information systems have the same formal structures, and that fuzzy soft sets and fuzzy information systems are equivalent. Xiao et al., [10] in his paper, an appropriate definition and method is designed for recognizing soft information patterns by establishing the information table based on soft sets theory and at the same time the solutions are proposed corresponding to the different recognition vectors. Feng et al., [11] extended the study of soft set to soft semirings. The notions of soft semirings, soft subsemirings, soft ideals, idealistic soft semirings and soft semi ring homomorphism were introduced, and several related properties were investigated. Herawan et al., [12] proposed an approach for visualizing soft maximal association rules which contains four main steps, including discovering, visualizing maximal supported sets, capturing and finally visualizing the maximal rules under soft set theory. In Acar et al., [13] introduce the basic notions of soft rings, which are actually a parameterized family of subrings of a ring, over a ring Babitha & Sunil [14], presented the concept of soft set relations are introduced as a sub soft set of the Cartesian product of the soft sets and many related concepts such equivalent soft set relation, partition, composition, function etc. Cagman & Enginoglu [15] define soft matrices and their operations which are more functional to make theoretical studies in the soft set theory and finally construct a soft max-min decision making method which can be successfully applied to the problems that contain uncertainties and also improving several new results, products of soft sets and uni-int decision function [16]. Feng et al., [17] aim of this paper is providing a framework to combine fuzzy sets, rough sets, and soft sets all together, which gives rise to several interesting new concepts such as rough soft sets, soft rough sets, and Soft–rough fuzzy sets. In Ali et al., [18] studied some important properties associated with these new operations. A collection of all soft sets with respect to new operations give rise to four idempotent monoids. Alkhazaleh et al., [19], in his paper, as a generalization of Molodtsov's soft set introduce the definitions of a soft multiset, its basic operations such as complement, union and intersection. Ghosh et al., [20] defined fuzzy soft ring and study some of its algebraic properties. Herawan & Deris [21], a soft set approach for association rule mining have define the notion of regular and maximal association rules between two sets of parameters. In Karaaslan et al., [22] defined concept of a soft lattice, soft sublattice, complete soft lattice, modular soft lattice, distributive soft lattice, soft chain and study their related properties. Ali [23], to discuss the idea of reduction of parameters in case of soft sets and studied approximation space of Pawlak associated with a soft set. Dariusz Wardowski [24] introduced a new notion of soft element of a soft set and establish its natural relation with soft operations and soft objects in soft topological spaces. Next, using the notion of soft element, they define, in a different way than in the literature, a soft mapping transforming a soft set into a soft set and provide basic properties of such mappings. The new approach to soft mappings enables us to obtain the natural first fixed- Innovative

Systems Design and Engineering point results in the soft set theory. In China Xiaolong Xin and Wenting Li [25] initiate the study of soft congruence relations by using the soft set theory.The notions of soft quotient rings, generalized soft ideals and generalized soft quotient rings, are introduced, and several related properties are investigated.

III. CATEGORIZATION FOR SOFT TEST

Let 𝔉 be a collection of fuzzy sets on U. Each fuzzy set is defined by a neighborhood (may be a singleton) in the membership function space; let 𝒞ℴℓ be a neighborhood system [Lin, 1996]. Let MF(U) be the total space of membership functions under consideration, i.e. MF(U) is the total union of all membership functions representing 𝔉, namely the union of members in 𝒞ℴℓ: MF(U) = ∪ 𝒞ℴℓ. 𝒞ℴℓ is a neighborhood system on MF(U). Depending on the nature of 𝒞ℴℓ, we have various types of fuzzy sets. To avoid confusing, here will call the fuzzy sets defined by 𝒞ℴℓ soft sets (or sofset). Here is a list of soft sets.

3.1. W-Sofset Weighted Soft Set (Mathematical Soft Set or Quantitative Soft Set), if 𝒞ℴℓ consists of singletons. Every membership function is treated as a characteristic function of a soft set. Many classical fuzzy theorists have implicitly taken this view. For clarity, when refer their works, will use, instead of fuzzy set, weighted soft set, also we will say each element in a soft set has a weight (instead of grade). soft set theory is essentially a "new" mathematical analysis of real variables where one studies functions from the point of view of weighted memberships.

3.2. F-Sofset Finite-multi Soft Set, if 𝒞ℴℓ consists of finite sets.

3.3. P-Sofset Partitioned Soft Set, if 𝒞ℴℓ forms a crispy partition. Mathematically this is a most beautiful theory, realizing that a fuzzy set (that tolerates perturbation) has to be represented by a set of membership functions. Each set represents one and only one fuzzy set. Then the space of membership functions is partitioned into equivalence classes. So P-sofset theory is very elegant and beautiful. However, one may wonder how could there be a natural partition in a "continuous" membership function space.

3.4. B-Sofset Basic Neighborhood Soft Set, if 𝒞ℴℓ forms a basic neighborhood system (i.e., the neighborhood system is defined by a binary relation). Basic neighborhoods are geometric view of binary relation. Intuitively, related membership functions are "geometrically" near to each other. A basic neighborhood system is an abstract binary relation, so it may include some "unexpected" cases.

3.5. C-Sofset Covering Soft Set, if $\mathcal{Col}$ forms a covering.

3.6. N-Sofset Neighborhood Soft Set (Real World Soft Set or Qualitative Soft Set), if $\mathcal{Col}$ forms a neighborhood system.

3.7. FP-Sofset Fuzzy-Partitioned Soft Sets, if $\mathcal{Col}$ forms a fuzzy (weighted) partition.

3.8. **FF-Softset** Double Fuzzy Soft Sets, if $\mathcal{Col}$ forms a fuzzy (weighted) covering.

IV. CONCLUSION

Soft Sets represent a powerful tool for decision making about information systems, data mining and drawing conclusions from data, especially in those cases where some uncertainty exists in the data. Its efficiency in dealing with uncertainty problems is as a result of its parameterized concept.

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