A Special Case Study in LPP

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Abstract

The paper is intended to study a special problem in LPP. It is constituted by taking even coefficients in the objective function and successive terms of coefficients in the constrains. Few cases are thoroughly investigated. Some considerable results are obtained.In this study, the generalised optimum basic feasible solutions are traced in three cases.

1. Introduction:

The applications of some operations research techniques are useful to find out the possibility along several methods of working to get an optimal value of the course of action. In this way,it is necessary to prepare a mathematical model[2,6]. This model exhibits the solution of system used for decision analysis. Forming a mathematical model means transforming the stated description and numerical data into mathematical expressions. This shows relation among various decision variables, objective function and constraints. Generally, the basis of optimality may be performance, profit, cost, time utility,etc.

George B Dantzing[4] started LP model to trace suitable solution for the military logistic problems during his work with the US Air Force at the time of 2nd world war in 1947. Now, it is widely applied in all zones of management, agriculture, military operations, education, transportation planning and scheduling, research and development, health care systems etc. Even though, these implementations are different, all LP models have some common properties[1,3] that are needed for the decision makers to understand the constraints properly.

Initially, let us see the meaning of the words linear and programming. The word "linear" speaks of the relation among variables in a model and the word "programming" speaks of the mathematical modeling and solving of a problem that includes the plan of order to achieve the objective.

1.1 Advantages of liner programming:

The linear programming methods have the following advantages:

(i). It upgrades the decision quality.

(ii). It helps the decision makers to make the use of their available capital.

(iii). It also helps to obtain an optimum solution.

1.2 Limitations of linear programming:

Through LP has many advantages and applications in various zones, there are certain limitations in this method. They are:

(i).Linear programming supposes linear relationship amid decision variables. But in real-life problems it is not so.

(ii). There is no assurance of getting integer values for decision variables.

(iii).This method does not consider the effect of time and uncertainty.

(iv). Linear programming works with only single objective while in real-life cases, there may be multiple objectives.

1.3 Application Areas of linear programming:

Linear programming technique is extensively used for decision making in business, industry and many other areas such as Applications in Agriculture,Military,Production Management,Financial Management & Personal Management.

1.4 Simplex Method:

When the real life problem is formed into a linear programming, if that problem has more than one variable then there is need to find out an optimum solution. For such cases, we have a mechanism called Simplex Method. In 1947,GB Dantzing[5] brought this method into existence.

2. Simplex Algorithm:

Simplex method is employed to investigate the nature of the solution for the constituted LPP.

Step(1): Let us start with the objective function whether it is maximized or not. If not, we have to convert the minimization problem into maximization by using the relation min z=-max(-z)

Step(2):All b_i's in the constraints should be at positive level. Otherwise,all b_i's should be converted as positive by multiplying the constraint by negative sign.

Step(3):Introduce slack variables or surplus variables along with artificial variables in the given constrains to make them equality constraints.

Step(4):In case of introducing surplus variables along with artificial variables, we assign –M to the artificial variables in the objective function.

Step(5): For getting initial basic feasible solution, the non- basic variables are to be vanished.

Step(6): In this method, first we find the entering variable based on the minimum value of z_j-c_j. The corresponding column of entering variable is known as 'pivot column'.

Step(7): By taking the minimum ratio of the value of basic solution and respective value of pivot column, we get the leaving variable. The corresponding row of the leaving variable is known as 'Pivot row'.

Step(8):The column value of pivot row and pivot column is the pivot element. By the simplex method,the pivot element must be converted to unity and other elements in the pivot column are to be converted as zeroes by using elementary row transformations.

Step(9): The process is continued until the optimum condition(z_j-c_j>0) is satisfied including the artificial variable at zero level. Otherwise, go to step:10

Step(10):When all elements in the pivot column are at negative level or zero in any iteration the solution is unbounded solution. Otherwise, go to step:11

Step(11):When the artificial variables are at positive level / zero level even then if the optimality condition is satisfied, the current solution is called pseudo optimum solution(no solution).

3. Basic problem:

The following LPP is formed as below:

Objective function: Max Z = $2nx_1+(2n+2)x_2+(2n+4)x_3$

Subject to the constraints: $nx_1+(n+1)x_2+(n+2)x_3 \leq \text{ or } \geq \text{ or } = (n+3)$

$(n+1)x_1+(n+2)x_2+(n+3)x_3 \leq \text{ or } \geq \text{ or } = (n+4)$

$(n+2)x_1+(n+3)x_2+(n+4)x_3 \leq \text{ or } \geq \text{ or } = (n+5)$

and $x_1, x_2, x_3 \geq 0$

It is divided into four cases.

3.1 Case (1) :All the constrains are of ≤ type.

Objective function: Max Z = $2nx_1+(2n+2)x_2+(2n+4)x_3$

Subject to the constraints: $nx_1+(n+1)x_2+(n+2)x_3 \leq (n+3)$

$(n+1)x_1+(n+2)x_2+(n+3)x_3 \leq (n+4)$

$(n+2)x_1+(n+3)x_2+(n+4)x_3 \leq (n+5)$

and $x_1, x_2, x_3 \geq 0$

By applying simplex method, we get the following table when n=1,2,3,4,5&6.

Table-1

Iteration	Objective function =MaxZ	Entering Variable	Leaving Variable	Pivot Position (k,r)	Are any variables introduced?	Is optimality condition satisfied?	Optimum basic feasible solution	Does it contain Alternative solution?
When n=1								
Initial	0	x_2	s_3	(3,3)	Slack Variables(3)	No	$X=\begin{pmatrix}0\\0\\6/5\end{pmatrix}$	No
First	36/5	*	*	*		Yes		
When n=2								
Initial	0	x_3	s_3	(3,3)	Slack Variables(3)	No	$X=\begin{pmatrix}0\\0\\7/6\end{pmatrix}$	No
First	28/3	*	*	*		Yes		
When n=3								
Initial	0	x_3	s_3	(3,3)	Slack	No		No

First	80/7	*	*	*	Variables(3)	Yes	$X=\begin{pmatrix}0\\0\\8/7\end{pmatrix}$	
When n=4								
Initial First	0 27/2	x_3 *	s_3 *	(3,3) *	Slack Variables(3)	No Yes	$X=\begin{pmatrix}0\\0\\9/8\end{pmatrix}$	No
When n=5								
Initial First	0 140/9	x_3 *	s_3 *	(3,3) *	Slack Variables(3)	No Yes	$X=\begin{pmatrix}0\\0\\10/9\end{pmatrix}$	No
When n=6								
Initial First	0 28/5	x_3 *	s_3 *	(3,3) *	Slack Variables(3)	No Yes	$X=\begin{pmatrix}0\\0\\11/10\end{pmatrix}$	No

3.1.1 Conclusions:

In this case, the following conclusions are established:

(i).Three slack variables are introduced in the Constraints.

(ii).Only x_3 enters into the Basis and s_3 leaves from the Basis.

(iii).The Pivot position is always occured as (3,3)

(iv).The optimum basic feasible solution is obtained as $x_1=0$, $x_2=0$, $x_3= (n+5)/(n+4)$

(v).The value of the objective function is Max $Z=(2n+4)(n+5)/(n+4)$

(vi).No alternative solution is obtained.

3.2 Case (II):

First constraint is of type ≤ and the remaining two are of type ≥ .

Objective function: $\text{Max}Z = 2nx_1+(2n+2)x_2+(2n+4)x_3$

Subject to the constraints: $nx_1 + (n+1)x_2 + (n+2)x_3 \leq (n+3)$

$$(n+1)x_1 + (n+2)x_2 + (n+3)x_3 \geq (n+4)$$

$$(n+2)x_1 + (n+3)x_2 + (n+4)x_3 \geq (n+5)$$

and $x_1, x_2, x_3 \geq 0$

By applying simplex method, we get the following table when n=1,2,3,4,5& 6

Table-2

Iteration	Objective function =MaxZ	Entering Variable	Leaving Variable	Pivot Position (k,r)	Are any variables introduced?	Is optimality condition satisfied?	Optimum basic feasible solution	Does it contain Alternative solution?
When n=1								
Initial	-11M	x_3	A_2	(3,3)	Slack (1)	No		
First	(-M+36)/5	s_3	A_1	(6,2)	Surplus(2)	No	$X=\begin{pmatrix}0\\0\\4/3\end{pmatrix}$	$X=\begin{pmatrix}4\\0\\0\end{pmatrix}$
Second	15/2	s_2	s_1	(5,2)	Artificial(2)	No		
Third	8	*	*	*	variables	Yes		
When n=2								
Initial	-13M	x_3	A_2	(3,3)	Slack (1)	No		
First	(-M+56)/6	s_3	A_1	(6,2)	Surplus(2)	No	$X=\begin{pmatrix}0\\0\\5/4\end{pmatrix}$	$X=\begin{pmatrix}5/2\\0\\0\end{pmatrix}$
Second	48/5	s_2	s_1	(5,1)	Artificial(2)	No		
Third	10	*	*	*	variables	Yes		
When n=3								
Initial	-15M	x_3	A_2	(3,3)	Slack (1)	No		
First	(-M+80)/7	s_3	A_1	(6,2)	Surplus(2)	No	$X=\begin{pmatrix}0\\0\\6/5\end{pmatrix}$	$X=\begin{pmatrix}2\\0\\0\end{pmatrix}$
Second	35/3	s_2	s_1	(5,1)	Artificial(2)	No		
Third	12	*	*	*	variables	Yes		
When n=4								
Initial	-17M	x_3	A_2	(3,3)	Slack (1)	No		
First	(-M+108)/8	s_3	A_1	(6,2)	Surplus(2)	No	$X=\begin{pmatrix}0\\0\\7/6\end{pmatrix}$	$X=\begin{pmatrix}7/4\\0\\0\end{pmatrix}$
Second	96/7	s_2	s_1	(5,1)	Artificial(2)	No		
Third	14	*	*	*	variables	Yes		
When n=5								
Initial	-19M	x_3	A_2	(3,3)	Slack (1)	No		
First	(-M+140)/9	s_3	A_1	(6,2)	Surplus(2)	No	$X=\begin{pmatrix}0\\0\\8/7\end{pmatrix}$	$X=\begin{pmatrix}8/5\\0\\0\end{pmatrix}$
Second	63/4	s_2	s_1	(5,1)	Artificial(2)	No		
Third	16	*	*	*	variables	Yes		
When n=6								
Initial	-21M	x_3	A_2	(3,3)	Slack (1)	No		
First	(-	s_3	A_1	(6,2)	Surplus(2)	No		

Second Third	M+176)/10 160/9 18	s_2 *	s_1 *	(5,1) *	Artificial(2) variables	No Yes	$X=\begin{pmatrix}0\\0\\9/8\end{pmatrix}$	$X=\begin{pmatrix}3/2\\0\\0\end{pmatrix}$

3.2.1 Conclusions:

In this Case, the following conclusions are drawn:

(i).One Slack variable,two Surplus Variables & two Artificial Variables are introduced in the Constraints.

(ii).In the first iteration: Entering Variable- x_3, Leaving Variable- A_2;

In the Second iteration: Entering Variable- S_3, Leaving Variable- A_1;

In the Third iteration: Entering Variable- S_2, Leaving Variable- S_1

(iii).The Pivot position moves from the position (3,3) in the first iteration to the position(6,2) in the second iteration & from the position (6,2) in the Second iteration to the position(5,1) in the Third iteration.

(iv).The optimum basic feasible solution is obtained as $x_1=0, x_2=0$, $x_3= (n+3) / (n+2)$ at third iteration itself.

(v).The value of the objective function is Max Z=2n+6

(vi).An alternative solution is obtained as $x_1=(n+3)/ n$, $x_2=0$, $x_3= 0$

3.3 Case(III):

First and Second constraints are of type $\leq$ and the remaining third one is of type $\geq$.

Objective fuction: Max Z = $2nx_1+(2n+2)x_2+(2n+4)x_3$

Subject to the constraints:

$$nx_1 + (n+1)x_2 + (n+2) x_3 \leq (n+3)$$

$$(n+1) x_1 + (n+2)x_2 + (n+3)x_3 \leq (n+4)$$

$$(n+2) x_1 + (n+3) x_2 + (n+4)x_3 \geq (n+5)$$

$$\text{and } x_1, x_2, x_3 \geq 0$$

By applying simplex method, we get the following table when n=1,2,3,4,5& 6

Table-3

Iteration	Objective function =MaxZ	Entering Variable	Leaving Variable	Pivot Position (k,r)	Are any variables introduced?	Is optimality condition satisfied?	Optimum basic feasible solution	Does it contain Alternative solution?
When n=1								
Initial First Second	-6M 36/5 15/2	x_3 s_3 *	A_1 s_2 *	(3,3) (6,2) *	Slack (2) Surplus(1) Artificial(1) variables	No No Yes	$X=\begin{pmatrix}0\\0\\5/4\end{pmatrix}$	No

When n=2								
Initial First Second	-7M 28/3 48/5	x_3 s_3 *	A_1 s_2 *	(3,3) (6,2) *	Slack (2) Surplus(1) Artificial(1) variables	No No Yes	$X=\begin{pmatrix}0\\0\\6/5\end{pmatrix}$	No
When n=3								
Initial First Second	-8M 80/7 35/3	x_3 s_3 *	A_1 s_2 *	(3,3) (6,2) *	Slack (2) Surplus(1) Artificial(1) variables	No No Yes	$X=\begin{pmatrix}0\\0\\7/6\end{pmatrix}$	No
When n=4								
Initial First Second	-9M 27/2 96/7	x_3 s_3 *	A_1 s_2 *	(3,3) (6,2) *	Slack (2) Surplus(1) Artificial(1) variables	No No Yes	$X=\begin{pmatrix}0\\0\\8/7\end{pmatrix}$	No
When n=5								
Initial First Second	-10M 140/9 63/4	x_3 s_3 *	A_1 s_2 *	(3,3) (6,2) *	Slack (2) Surplus(1) Artificial(1) variables	No No Yes	$X=\begin{pmatrix}0\\0\\9/8\end{pmatrix}$	No
When n=6								
Initial First Second	-11M 88/5 160/9	x_3 s_3 *	A_1 s_2 *	(3,3) (6,2) *	Slack (2) Surplus(1) Artificial(1) variables	No No Yes	$X=\begin{pmatrix}0\\0\\10/9\end{pmatrix}$	No

3.3.1 Conclusions:

In this Case, the following conclusions are established:

(i).Two Slack variables, One Surplus Variable & One Artificial Variable are introduced in the Constraints.

(ii).In the first iteration: Entering Variable- x_3, Leaving Variable- $A_{1;}$

In the Second iteration: Entering Variable- S_3,Leaving Variable- S_2;

(iii).The Pivot position moves from the position (3,3) in the first iteration to the position (6,2) in the second iteration.

(iv).The optimum basic feasible solution $x_1=0$, $x_2=0$, $x_3= (n+4) / (n+3)$ is obtained at second iteration.

(v).The value of the objective function is MaxZ=(2n+4) (n+4)/(n+3)

(vi).No alternative solution is obtained.

3.4 Case (IV):

The first constraint is of type ≤ and the second one is of type ≥ and third one is of type =.

Objective fuction: MaxZ = $2nx_1+(2n+2)x_2+(2n+4)x_3$

Subject to the constraints:

$$nx_1 + (n+1)x_2 + (n+2) x_3 \leq (n+3)$$

$$(n+1)x_1 + (n+2)x_2 + (n+3)x_3 \geq (n+4)$$

$$(n+2)x_1 + (n+3)x_2 + (n+4)x_3 = (n+5)$$

and $x_1, x_2, x_3 \geq 0$

By applying simplex method, we get the following table when n=1, 2 ,3, 4, 5 & 6

Table-4

Iteration	Objective function =MaxZ	Entering Variable	Leaving Variable	Pivot Position (k,r)	Are any variables introduced?	Is optimality condition satisfied?	Optimum basic feasible solution	Does it contain Alternative solution?
When n=1								
Initial First	-11M (-M+36)/5	x_3 *	s_3 *-	(3,3) *	Slack (1) Surplus(1) Artificial(2) variables	No Yes	No	No
When n=2								
Initial First	-13M (-M+56)/6	x_3 *	s_3 *	(3,3) *	Slack (1) Surplus(1) Artificial(2) variables	No Yes	No	No
When n=3								
Initial First	-15M (-M+80)/7	x_3 *	s_3 *	(3,3) *	Slack (1) Surplus(1) Artificial(2) variables	No Yes	No	No
When n=4								

Initial First	-17M (-M+108)/8	x_3 *	s_3 *	(3,3) *	Slack (1) Surplus(1) Artificial(2) variables	No Yes	No	No
When n=5								
Initial First	-19M (-M+140)/9	x_3 *	s_3 *	(3,3) *	Slack (1) Surplus(1) Artificial(2) variables	No Yes	No	No
When n=6								
Initial First	-21M (-M+176)/10	x_3 *	s_3 *	(3,3) *	Slack (1) Surplus(1) Artificial(2) variables	No Yes	No	No

3.4.1 Conclusions:

In this Case, the following conclusions are Obtained:

(i).One Slack variable, One Surplus Variables & Two Artificial Variables are introduced in the Constraints.

(ii).In the first iteration: Entering Variable- x_3, Leaving Variable-S_3.

(iii).The Pivot position in the first iteration is (3,3).

(iv).No optimum basic feasible solution is obtained.

4. Over all Conclusions: The following over all conclusions are made through this resaerch work.

Case	Optimum Basic Feasible Solution (X)	The value of the objective function(Z)	An Alternative Solution(if exists)
I	$x_1=0$, $x_2=0$, $x_3=(n+5)/(n+4)$	MaxZ= (2n+4) (n+5) / (n+4)	**No**
II	$x_1=0$, $x_2=0$, $x_3=(n+3)/(n+2)$	MaxZ=2n+6	$x_1=(n+3)/n$, $x_2=0$, $x_3=0$
III	$x_1=0$, $x_2=0$, $x_3=(n+4)/(n+3)$	MaxZ=(2n+4) (n+4)/(n+3)	**No**
IV	**No**	Max Z= (-M+(2n+4)(n+5))/(n+4)	**No**

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