A Peculiar Problem in Linear Programming Problem

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Abstract

In this paper, we aim to investigate a peculiar problem in linear programming problem. It is formed by considering even coefficients in the objective function. The constraints are involved with atleast one surplus variable in all considered cases. Some fruitful results are obtained.In this study,the optimum basic feasible solution is derived as $x_1=(n+4)/(n+1), x_2=0, x_3=0$ *with the value of objective function* $2n(n+4)/(n+1)$.

1.Introduction:

Linear programming is an important technique which is used for management problems, distribution, investment and advertising. It is also used in business and also in service sectors like education, government, hospital,libraries etc. This method is suitable in the problems distinguished by the existence of decision variables. Linear functions of the decision variables are only involved in the objective function and the constraints . An objective function represents the major goal that compute the functional of the system such as maximizing profits (or) minimizing losses. There may be always some limitations on the availability of resources like man power, machine, material (or) time for the system.

The existence of linearity in the problem is a key feature of linear programming. The utilization of linear programming arises a large number of different applications. Some models may not be rigid, but can be made linear by using certain mathematical transformations. Still some applications are not at all linear, but can be effectively approximated by linear model. The effort with which linear programming models can be generally solved makes an impressive means of business practices with other uncontrollable non-linear models.

Linear programming[1-3] started in 1947 under the forces of world war -II. One of the united states air force projects called SCOOP(Scientific Computation Of Optimum Programs) was done under the supervisions of G.B.Dantzig. That simplex algorithm and similar theory was developed by Dantzig[4,5] and his team in 1947.

Associates of Dantzig team with Jon Von Neuman led to primary insight in the mathematical theory of linear programming. Neuman highlighted the necessity of duality and could instantly found the link between linear programming and the theory of games on which he had done primary work in 1928.

In previous years, It turned into a crucial tool of analysis in the hands of economics. T.C.Koopmary and L.V.Kantaravitch in the U.S.S.R became explorers in this aspect.They were awarded with Nobel Prize in economics in 1975.

1.2 Applications of Operational Research:

Now a days, many categories of business and government use the benefits of operational research. The following are the applications of operational research which are used now a days.

In the area of Accounting: Many sectors like Assigning audit teams ,Credit policy analysis, Cash flow planning.

In the filed of Construction: Deployment of work force,Allocation of resources to the projects, Project scheduling and Monitoring and control are the ares in which OR can be used.

In Facilities planning: We can apply OR in Factory location and size design,Transportation loading and unloading and Warehouse location decision.

In the Finance sector: Dividend policy making,Investment analysis and Allocating capital among various alternatives are sloved by OR methods.

In Research and Development,OR techniques[6] are frequently applied in controling projects,project allocations and Planning of product introduction.

2. Simplex Algorithm:

Simplex method is employed to investigate the nature of the solution for the constituted LPP.

Step(1): Let us start with the objective function whether it is maximized or not. If not, we have to convert the minimization problem into maximization by using the relation min z=-max(-z)

Step(2):All b_i's in the constraints should be at positive level. Otherwise,all b_i's should be converted as positive by multiplying the constraint by negative sign.

Step(3):Introduce slack variables or surplus variables along with artificial variables in the given constrains to make them equality constraints.

Step(4):In case of introducing surplus variables along with artificial variables, we assign –M to the artificial variables in the objective function.

Step(5): For getting initial basic feasible solution, the non- basic variables are to be vanished.

Step(6): In this method, first we find the entering variable based on the minimum value of z_j-c_j. The corresponding column of entering variable is known as 'pivot column'.

Step(7): By taking the minimum ratio of the value of basic solution and respective value of pivot column, we get the leaving variable. The corresponding row of the leaving variable is known as 'Pivot row'.

Step(8):The column value of pivot row and pivot column is the pivot element. By the simplex method,the pivot element must be converted to unity and other elements in the pivot column are to be converted as zeroes by using elementary row transformations.

Step(9): The process is continued until the optimum condition(z_j-c_j>0) is satisfied including the artificial variable at zero level. Otherwise, go to step:10

Step(10):When all elements in the pivot column are at negative level or zero in any iteration the solution is unbounded solution. Otherwise, go to step:11

Step(11):When the artificial variables are at positive level / zero level even then if the optimality condition is satisfied, the current solution is called pseudo optimum solution(no solution).

3. Basic problem:

The following LPP is constructed as below:

Objective function: $MinZ = 2nx_1+(2n+2)\,x_2+(2n+4)\,x_3$

Subject to the constraints: $nx_1 + (n+1)x_2 + (n+2)\,x_3 \le \text{ or } \ge \text{or } = (n+3)$

$(n+1)\,x_1 + (n+2)x_2+ (n+3)x_3 \le \text{ or } \ge \text{ or } = (n+4)$

$(n+2)\,x_1 + (n+3)\,x_2 + (n+4)\,x_3 \le \text{or } \ge \text{ or } = (n+5)$

and $x_1, x_2, x_3 \ge 0$

It is divided into four cases.

3.1 Case(1) :The first constraint has the type $\le$ and the remaining two have the type $\ge$.

Objective function: $Min\ Z=2nx_1+(2n+2)x_2+(2n+4)x_3$

Subject to the constraints: $nx_1+(n+1)x_2+(n+2)x_3 \le (n+3)$

$(n+1)x_1+(n+2)x_2+(n+3)x_3 \ge (n+4)$

$(n+2)x_1+(n+3)x_2+(n+4)x_3 \ge (n+5)$

and $x_1,x_2,x_3 \ge 0$

By applying simplex method ,we get the following table when n=1,2,3,4,5,6.

Table-1

Iteration	Objective function = MinZ	Entering Variable	Leaving Variable	Pivot Position (k,r)	Are any variables introduced?	Is optimality condition satisfied?	Optimum basic feasible solution	Does it contain Alternative solution?
When n=1								
Initial First Second Third	-11M (-M-36)/5 -30/4 -10/2	x_3 S_3 x_1 *	A_2 A_1 x_3 *	(3,3) (6,2) (1,3) *	Slack(1), Surplus(2) , Artificial(2) variables	No No No yes	$X=\begin{pmatrix}5/2\\0\\0\end{pmatrix}$	No
When n=2								
Initial First Second Third	-13M (-M-56)/6 -48/5 -24/3	x_3 S_3 x_1 *	A_2 A_1 x_3 *	(3,3) (6,2) (1,3) *	Slack(1), Surplus(2) , Artificial(2) variables	No No No yes	$X=\begin{pmatrix}6/3\\0\\0\end{pmatrix}$	No
When n=3								
Initial First Second	-15M (-M-64)/7 -56/6	x_3 S_3 x_1	A_2 A_1 x_3	(3,3) (6,2) (1,3)	Slack(1), Surplus(2) , Artificial(2)	No NoNo yes	$X=\begin{pmatrix}7/4\\0\\0\end{pmatrix}$	No

Third	-42/4	*	*	*	variables			
When n=4								
Initial First Second Third	-17M (-M-108)/8 -96/7 -64/5	x_3 S_3 x_1 *	A_2 A_1 x_3 *	(3,3) (6,2) (1,3) *	Slack(1), Surplus(2) , Artificial(2) variables	No No No yes	$X=\begin{pmatrix}8/5\\0\\0\end{pmatrix}$	No
When n=5								
Initial First Second Third	-19M (-M-140)/9 -126/8 -90/6	x_3 S_3 x_1 *	A_2 A_1 x_3 *	(3,3) (6,2) (1,3) *	Slack(1), Surplus(2) , Artificial(2) variables	No No No yes	$X=\begin{pmatrix}9/6\\0\\0\end{pmatrix}$	No
When n=6								
Initial First Second Third	-21M (-M-176)/10 -160/9 -120/7	x_3 S_3 x_1 *	A_2 A_1 x_3 *	(3,3) (6,2) (1,3) *	Slack(1), Surplus(2) , Artificial(2) variables	No No No yes	$X=\begin{pmatrix}10/7\\0\\0\end{pmatrix}$	No

3.1.1Conclusions:

In this Case, The following conclusions are made as below.

(i).One Slack variable, Two Surplus variables and Two Artificial variables are introduced in the Constraints.

(ii).In the first iteration: Entering Variable- x_3,Leaving Variable- A_2;

In the Second iteration: Entering Variable- s_3,Leaving Variable- A_1;

In the Third iteration: Entering Variable-x_1,,Leaving Variable-x_3

(iii).The Pivot position moves from the position (3,3) in the first iteration to the position (6,2) in second iteration and then it moves to the position (1,3) in third iteration.

(iv).The optimum feasible solution $\mathbf{x_1=(n+4)/(n+1),\ x_2=0\ \&\ x_3=0}$ is obtained at third iteration.

(v).The value of the objective function is **Min Z=2n(n+4)/n+1**

(vi). No alternative solution is obtained.

3.2 Case(II):

Second constraint is of the type ≤ and the remaining two are of type ≥.

Objective function: MinZ = $2nx_1+(2n+2)x_2+(2n+4)x_3$

Subject to the constraints:

$$nx_1 + (n+1)x_2 + (n+2)\,x_3 \geq (n+3)$$
$$(n+1)\,x_1 + (n+2)x_2 + (n+3)x_3 \leq (n+4)$$
$$(n+2)\,x_1 + (n+3)\,x_2 + (n+4)\,x_3 \geq (n+5)$$
$$\text{and}\ \ x_1,\ x_2,\ x_3 \geq 0$$

By applying simplex method, we get the following table when n=1,2 ,3,4,5 &6

Table-2

Iteration	Objective function = MinZ	Entering Variable	Leaving Variable	Pivot Position (k,r)	Are any variables introduced?	Is optimality condition satisfied?	Optimum basic feasible solution	Does it contain Alternative solution?
When n=1								
Initial First Second	-10M (-2M-36)/5 (-M-30)/4	x_3 S_3 *	A_2 S_2 *	(3,3) (6,2) *	Slack(1), Surplus(2) , Artificial(2) variables	No Yes	No	No
When n=2								
Initial First Second	-12M (-2M-56)/6 (-M-48)/5	x_3 S_3 *	A_2 S_2 *	(3,3) (6,2) *	Slack(1), Surplus(2) , Artificial(2) variables	No No Yes	No	No
When n=3								
Initial First Second	-14M (-2M-80)/7 (-M-70)/6	x_3 S_3 *	A_2 S_2 *	(3,3) (6,2) *	Slack(1), Surplus(2) , Artificial(2) variables	No No Yes	No	No
When n=4								
Initial First Second	-16M (-2M-108)/8 (-M-96)/7	x_3 S_3 *	A_2 S_2 *	(3,3) (6,2) *	Slack(1), Surplus(2) , Artificial(2) variables	No No Yes	No	No
When n=5								
Initial First Second	-18M (-2M-140)/9 (-M-126)/8	x_3 S_3 *	A_2 S_2 *	(3,3) (6,2) *	Slack(1), Surplus(2) , Artificial(2) variables	No No Yes	No	No
When n=6								
Initial First Second	-20M (-2M-126)/10 (-M-160)/9	x_3 S_3 *	A_2 S_2 *	(3,3) (6,2) *	Slack(1), Surplus(2) , Artificial(2) variables	No No Yes	No	No

3.2.1 Conclusions:

In this Case, The following conclusions are drawn.

(i).One Slack variable, two Surplus Variables & two Artificial Variables are introduced in the Constraints.
(ii).In the first iteration: Entering Variable- x_3,Leaving Variable- $A_{2;}$
In the second iteration: Entering Variable-s_3, Leaving variable-$s_{2;}$
(iii).The Pivot position moves from first iteration (3,3) to second iteration(6,2)
(iv). Theoptimum basic feasible solution is satisfied.But it has no feasible solution.

3.3 Case(III):

First and second constraints are of type ≥ and the remaining third one is of type ≤.

Objective function: $MinZ=2nx_1+(2n+2)x_2+(2n+4)x_3$

Subject to the constraints:

$$nx_1 + (n+1)x_2 + (n+2)x_3 \geq (n+3)$$
$$(n+1)x_1 + (n+2)x_2 + (n+3)x_3 \geq (n+4)$$
$$(n+2)x_1 + (n+3)x_2 + (n+4)x_3 \leq (n+5)$$
$$\text{and } x_1, x_2, x_3 \geq 0$$

By applying simplex method, we get the following table when n=1,2,3,4,5 & 6

Table-3

Iteration	Objective function =MinZ	Entering Variable	Leaving Variable	Pivot Position (k,r)	Are any variables introduced?	Is optimality condition satisfied?	Optimum basic feasible solution	Does it contain Alternative solution?
When n=1								
Initial First	-9M (-3M-36)/5	x_3 *	S_3 *	(3,3) *	Slack(1), Surplus(2) , Artificial(2) variables	No Yes	No	No
When n=2								
Initial First	-11M (-3M-56)/6	x_3 *	S_3 *	(3,3) *	Slack(1), Surplus(2) , Artificial(2) variables	No Yes	No	No
When n=3								
Initial First	-13M (-3M-80)/7	x_3 *	S_3 *	(3,3) *	Slack(1), Surplus(2) , Artificial(2) variables	No Yes	No	No
When n=4								
Initial First	-15M (-3M-	x_3 *	S_3 *	(3,3) *	Slack(1), Surplus(2) ,	No Yes	No	No

	108)/8				Artificial(2) variables			
When n=5								
Initial First	-17M (-3M-140)/9	x_3 *	S_3 *	(3,3) *	Slack(1), Surplus(2) , Artificial(2) variables	No Yes	No	No
When n=6								
Initial First	-19M (-3M-176)/10	x_3 *	S_3 *	(3,3) *	Slack(1), Surplus(2) , Artificial(2) variables	No Yes	No	No

3.3.1 Conclusions:

In this Case, the following conclusions are found

(i). One Slack variable, two Surplus Variables&two Artificial Variables are introduced in the Constraints.

(ii).In the first iteration: Entering Variable- x_3,Leaving Variable-$s_{3;}$

(iii).The Pivot position in first iteration is (3,3)

(iv).The optimum basic feasible solution is satisfied and it has no feasible solution.

3.4 Case (IV):

All the constraintsare of type ≥ .

Objective function: $\text{Max}Z = 2nx_1+(2n+2)x_2+(2n+4)x_3$

Subject to the Constraints :

$$nx_1+(n+1)x_2+(n+2)x_3 \geq (n+3)$$
$$(n+1)x_1+(n+2)x_2+(n+3)x_3 \geq (n+4)$$
$$(n+2)x_1+(n+3)x_2+(n+4)x_3 \geq (n+5)$$
$$\text{and } x_1 , x_2, x_3 \geq 0$$

By applying simplex method, we get the following table when n=1,2 ,3,4,5 & 6

Table-4

Iteration	Objective function =MaxZ	Entering Variable	Leaving Variable	Pivot Position (k,r)	Are any variables introduced?	Is optimality condition satisfied?	Optimum basic feasible solution	Does it contain Alternative solution?
When n=1								
Initial First Second Third	-15M (-3M+36)/5 (-M+30)/4 8	x_3 S3 S2 *	A_3 A_2 A_1 *	(3,3) (6,2) (5,1) *	surplus(3) , artificial(3) variables	No	No	No
When n=2								
Initial First	-18M (-3M+56)/6	x_3 S3	A_3 A_2	(3,3) (6,2)	surplus(3) , artificial(3)	No	No	No

Second Third	(-M+48)/5 10	S2 *	A_1 *	(5,1) *	variables			
When n=3								
Initial First Second Third	-21M (-3M+80/7 (-M+70)/6 12	x_3 S3 S2 *	A_3 A_2 A_1 *	(3,3) (6,2) (5,1) *	surplus(3) , artificial(3) variables	No	No	No
When n=4								
Initial First Second Third	-24M (-3M+108)/6 (-M+96)/7 14	x_3 S3 S2 *	A_3 A_2 A_1 *	(3,3) (6,2) (5,1) *	surplus(3) , artificial(3) variables	No	No	No
When n=5								
Initial First Second Third	-27M (-3M+140)/9 (-M+126)/8 16	x_3 S3 S2 *	A_3 A_2 A_1 *	(3,3) (6,2) (5,1) *	surplus(3) , artificial(3) variables	No	No	No
When n=6								
Initial First Second Third	-30M (-3M+176)/10 (-M+160)/9 18	x_3 S3 S2 *	A_3 A_2 A_1 *	(3,3) (6,2) (5,1) *	surplus(3) , artificial(3) variables	No	No	No

3.4.1 Conclusions:

In this Case, The following conclusions are obtained

(i).Three Surplus Variables &Three Artificial Variables are introduced in the Constraints.

(ii).In the first iteration: Entering Variable- x_3,Leaving Variable-A_3;

In the second iteration Entering variable-s_3,Leaving Variable-A_2:

In the third iteration Entering variable-s_2,Leaving Variable-A_1:

(iii).The Pivot position moves from first iteration (3,3) to second iteration (6,2) & from second iteration (6,2) to third iteration (5,1).

(iv).No optimum basic feasible solution is obtained.

4.Over all Conclusions:

Case	**Optimum Basic Feasible Solution(X)**	**The value of the objective function(Z)**	**An Alternative Solution (if exists)**
I	**x_1=(n+4)/(n+1), x_2 =0 & x_3=0**	**Min Z=2n(n+4)/n+1,**	**No**
II	**Not Satisfied**	*	*

III	**Not Satisfied**	*	*
IV	**Unbounded Solution**	*	*

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