Optimal Design of Fractional Order PID controller for Non-Minimum Phase System

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Abstract

Regardless of the vibrant progression of process control in present decades, the proportional-integral-derivative (PID) controller continues to be the most often used feedback controller even today. An elegant way of ornamenting the performance of PID controllers is to use fractional-order controllers because it has two more parameters integral and derivative actions viz λ and μ respectively as compared to the conventional PID controller. There are a numerousauthenticvibrant systems the performances of which are enhanced by considering a non-integersystemwhich is related to the fractional calculus.In this paper an optimized Fractional order PID Controller is designed for a non-minimum phase system.

Keywords: Fractional calculus, FOPID controller, FOPID tuning, FOMCON,Nelder's Mead optimization algorithm

1. INTRODUCTION

Control systems have an increasingly significant role in the growth and progression of modern civilization and technology. Automatic control system maintains a defined relationship between reference input signal and desired output signal. The input –output difference of the system is called error and it is optimized by using a controller. Designing the best controller for any system is one of the most demanding research topics. Classical integer order proportional integral and the derivative gain (PID) controllers are generally used in all industrial applications due to the widely known characteristics namely, uncomplicated design, good performance, robustness, effective and comprehensive design methodology. The system performance is adjusted by the functionality of PID controller to tune the system by reducing the complexity and minimizing the error to get the required designed system[1] [2]. In 1942 Ziegler –Nichols proposed tuning method to optimize the controller parameters [3]. The main requirement of all the controllers is to design them in such a way so that the process variables are always close to desired value [4]. Many researchers have done lots of work on the optimizing technique to obtain proper tuning methods to obtain the required design specifications of the closed loop system. However it is difficult to achieve desired controller tuning because of the presence of inherent non-linearities in the plant dynamics and various uncertainties such as modeling error, measurement noise and external disturbances involved in the system. Thus, poor tuning can lead to poor control performance and even poor robustness. Hence for increasing the capabilities, PID controllers were designed by using fuzzy logic, artificial intelligence which gave better performance than traditional PID controllers. Comparison of different methodologies for tuning the PID controller has shown that fuzzy logic based PID controller is superior thanthe conventional PID controller [5]. As far as the research was going on researchers concentrated more upon different implemented optimization techniques like particle swarm optimization (PSO) in Maiti et al.2008 and Cao et al.2006,Genetic algorithm(GA) in Cao & Cao et al.2005,Neural Network based FOPID in Sadati et al.2008,hybrid fuzzy fopid in S.Das et al.2013,bacterial foraging in Luo& Li et al.2009,modified invasive weed optimization algorithm (MIWO) in Debarti et al.2009.Nelder's Mead optimization algorithm has been discussed in Nelder et al.1965.During the past few years the potential of fractional calculus and the increasing number of studies related to the applications of fractional order controllers in many areas of science and engineering have led to the importance of studying aspects such as the analysis, design, tuning and implementation of these controllers. The first Fractional order PID controller (FOPID) was introduced by I.Podlubny in 1994. To enhance the robustness and performance of PID control system, Podlubny has proposed a FOPID ($PI^{\lambda}D^{\mu}$) which is a generalization of the PID controller [6].The fractional order is an abstract notion and can be realized only through approximate estimation. Control system theory involves both fractional order controller and fractional order dynamic system which has to be controlled, but fractional order controller is commonly used in practice because plant models are generally integer order models. The main aim of using fractional order models is to accomplish a robust performance of controller despite the presence of the uncertainties of the plant model, high frequency noise & load disturbances. In FOPID controller besides the proportional, Integral and derivative parameters, it has two additional parameters, the order of fractional integration λ and the order of fractional derivative μ. Therefore it has five parameters that make the FOPID more flexible and less sensitive to parameter changes. There are different software tools also available for analysis, simulations and implementation of fractional order systems like Ninteger, Fomcon,Fopid andCrone toolbox [7] [8]. A plethora of research work is under progress in optimization technique for obtaining more significant optimized parameters which satisfy the design specifications for fractional order controlsystems.

This paper is organized as follows. In Section 2 PID controller is discussed. Section 3 and 4 covers definition of fractional order calculus and fractional order controller respectively. Section 5 deals with a design example and result discussion and then conclusion in Section 6.

2. PID CONTROLLER

PID is a well-known controller frequentlyusedasatremendous form of a phase lead-lag compensator with onepole at the origin and the other at infinity [1] [9].A traditional PID controller has three mode functionality controller action proportional (k_p), integrating (k_I) & derivate (k_D).The transfer function and general structure of PID controller is given by as follow:

$$C(s) = \frac{U(s)}{E(s)} = K_p + \frac{K_i}{s} + K_d . s$$

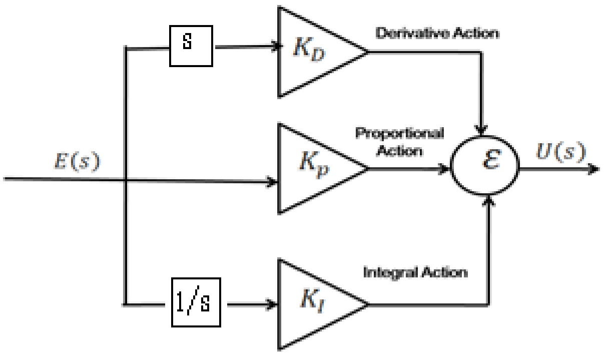

Fig.1 General Structure of PID controller

3. Definitions of Fractional Order Calculus

The following definitions of fractional calculus are used widely in the area of control system.

3.1Grunwald-Letnikov Definition

According toGrunwald-Letnikov (GL) definition the expression used is as given below:

$$ {}_aD_t^{\alpha} f(t) = \lim_{h\to 0} \frac{1}{h^{\alpha}} \sum_{j=0}^{\left[\frac{t-a}{h}\right]} (-1)^j \binom{\alpha}{j} f(t-jh) (1)$$

where$w_j^{\alpha} = (-1)^j \binom{\alpha}{j}$represents the coefficients of the polynomial $(1-z)^{\alpha}$.Thecoefficientscan also be obtained recursivelyfrom

$w_0^{\alpha} = 1$, $w_j^{\alpha} = \left(1 - \frac{\alpha+1}{j}\right) w_{j-1}^{\alpha}$ j=1,2,… (2)

3.2 Riemann-Liouville Definition

TheRiemann-Liouville definition is defined as [10]

$$ {}_aD_t^{-\alpha} f(t) = \frac{1}{\Gamma(\alpha)} \int_a^t (t-\tau)^{\alpha-1} f(\tau) d\tau \qquad (3)$$

Where, $\Gamma(\alpha)$, is the gamma function, 0< α < 1and a is the initial time instance, often assumed to be zero, i.e., a = 0.The differentiation is then denoted as$D_t^{-\alpha}$f(t)

3.3 Caputo's Definition

Caputo's definition isgiven by

$$ {}_0D_t^{\alpha} y(t) = \frac{1}{\Gamma(1-\gamma)} \int_0^t \frac{y^{(m+1)}(\tau)}{(t-\tau)^{\gamma}} d\tau \qquad (4)$$

where $\alpha = m + \gamma$, m is a integer, and $0 < \gamma \leq 1$. Similarly Caputo's Fractional Order integration is defined as

$$ {}_0D_t^{\gamma} y(t) = \frac{1}{\Gamma(-\gamma)} \int_0^t \frac{y(\tau)}{(t-\tau)^{\gamma+1}} d\tau, \gamma<0 \qquad (5)$$

3.4 Cauchy'sDefinition

This definition is a general extension of the integer-order Cauchy formula

$$ \mathrm{D}^{\gamma} \mathrm{f}(\mathrm{t}) = \frac{\Gamma(\gamma+1)}{2\pi \mathrm{j}} \int_C \frac{\mathrm{f}(\tau)}{(\tau-\mathrm{t})^{\gamma+1}} \mathrm{d}\tau \qquad (6)$$

Where, C is the smooth curve encircling the single-valued function f (t).

4. FRACTIONAL ORDER CONTROLLER

Fractional order controllers have attracted more and more attention in various fields and have increased the interest in analysis and implementation of the non-integer order controllers. A fractional order PID controller is represented as $PI^{\lambda}D^{\mu}$ [10]. It allows us to adjust integral (λ) and derivative (μ) order in addition to the proportional, integral and derivative constants where the values of λ and μ lie between 0 and 2.This also provides more flexibility and opportunity to better adjust the dynamical properties of the control system. The fractional order controller revels good robustness. The transfer functional of fractional order controller is given by

$$ C(s) = \frac{U(s)}{E(s)} = K_p + \frac{K_i}{s^{\lambda}} + K_d . s^{\mu} \qquad (7)$$

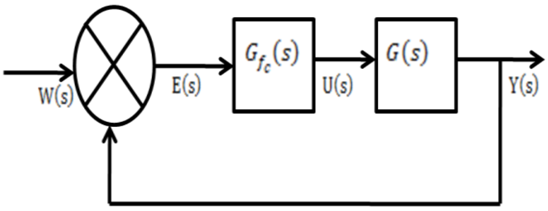

Fig.2 Block diagram of a Fractional order system

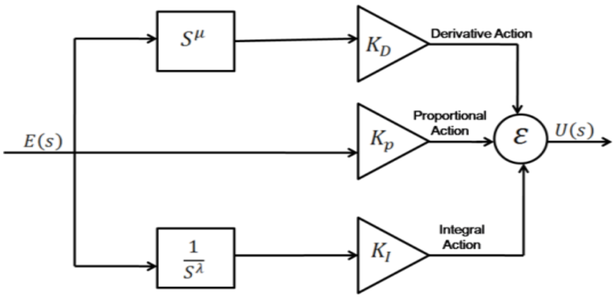

Fig.3 General Structure of $PI^{\lambda}D^{\mu}$ controller

A fractional PID controller is an extension of the conventional PID controller. All the conventional PID controllers are particular cases of the fractional controller.

- If λ=1and μ=1 then it is conventional PID controller,
- if λ=0 and μ=1 then it is conventional PD controller,
- if λ=1 and μ=0 then it is PI controller and
- if λ=0 and μ=0 then it is conventional P controller.

To get faster and effective response FOPID controller is designed. The parameters of FOPID are optimized using NM-optimization algorithm [11].The parameters of FOPID controller canbe optimized by minimizing the performance indices namely [12]

$$\text{Integral of square error ISE} = \int_0^\infty e^2(t)dt \tag{8}$$

$$\text{Integral of absolute error IAE} = \int_0^\infty e(t)dt \tag{9}$$

$$\text{Integral of time square error ITSE} = \int_0^\infty te^2(t)dt \tag{10}$$

$$\text{Integral of time absolute error ITAE} = \int_0^\infty t|e(t)|dt \tag{11}$$

To reduce the error between the reference and the actual value of the system output is the main objective of the control systems. As including the error straight away would lead to a zero result, the squared or the absolute value of the error is involved. ISE can be used when large errors exist in the system and have to be penalized quickly, however this criterion withstands small errors which results in oscillatory response. IAE produces less oscillatory response but results in slow response, further it is rarely used as it is not analytical. ITAE and ITSE include a time factor multiplied and therefore keep a check on errors that exist for a long period, these criteria are preferred as compared to the other two as these methods lead to faster responses.

The purpose of designing a fractional order controller is mainly to achieve design specifications, by removing unwanted noise and load disturbances in the system. These are following merits of fractional order controller[13].

- Attainment of improved phase margins and gain crossover frequency.
- More robust to variations in system gain.
- Rejection of noise of high frequency.
- Assured elimination of output disturbance.
- Minimization of steady state errors.

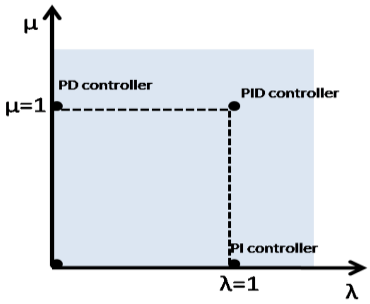

Fig.4 Converge of FOPID controller

5.RESULT AND DISCUSSION

The processes of any industry can be classified according a suitable controller is to be adopted on the basis of stability, uncertainty, nonlinearity, delay, order of transfer function, disturbances and noise. An example of a non-minimum phase system, has been taken in this paper, the transfer function of the system is as follow:

$$G_{NMP}(s) = \frac{3-s}{(s+1)(s+5)} \tag{12}$$

A transfer function which has one or more poles/zeros in the right half of the s-plane is called non-minimum phase transfer function.

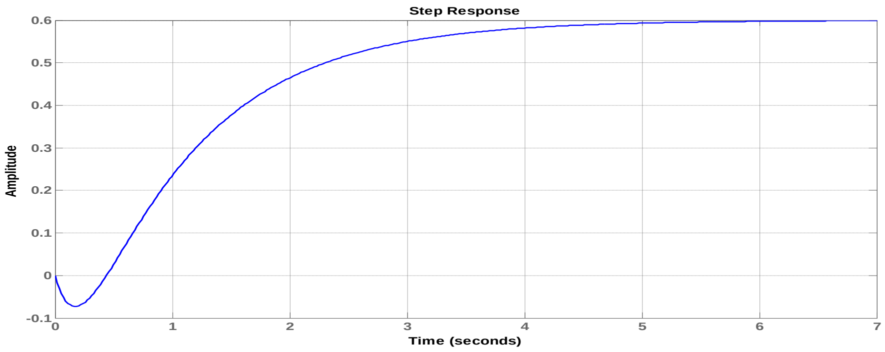

Fig 5.1 Step response of nmp system without controller

First, the step response of the Non-Minimum Phase system is designed without any controlleras shown in Figure 5.1 and thena PID controller is designed using Sisotool in MATLAB by ZN techniqueas shown in Figure 5.2. The performance parameters of this controller are given in Table I**.**

TableI. Performance parametersof PID Controller

Kp	Ki	Kd	Settling time(ts)	Max. Overshoot
1.779	3.558	0.342	5.21	31.6%

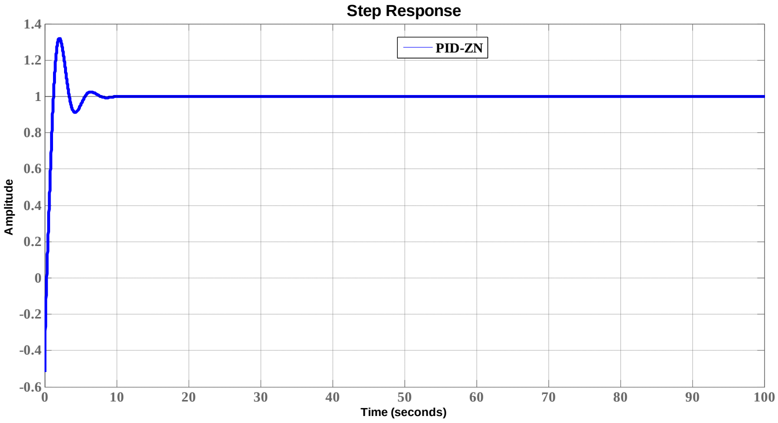

Fig 5.2 Step response of NMP system with PID controller

PID controller is not able to get better closed loop performance ofNMP system and the step response with a PID controller depicts a peak overshoot of 30 % which is not desirable.Therefore a fractional order PID controller is designed using FOMCON (fractional Order Modeling and Control) toolbox in MATLAB [14].The parameters of FOPID controller are optimized using NM-optimization algorithm with an objective of minimizing the IAE of the system. The obtained parameters of the FOPID controller after optimization are given in Table II.

Table II.ParametersofFOPID Controller

Kp	Ki	Kd	λ	μ
0.0049	2.889	0.96753	0.6732	0.5657

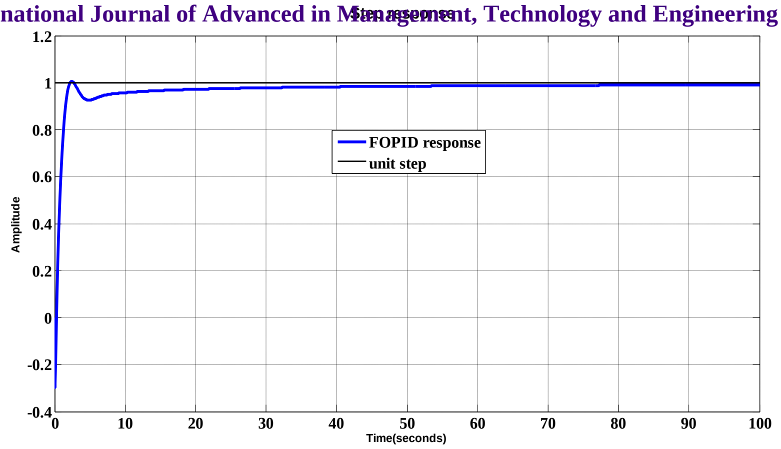

Fig 5.3 Step response of NMP system with FOPID controller

6. Conclusions

In this paper Fractional Order PID Controller is used to get the better performance of a Non minimum Phase system as compared to the classical PID controller. FOPID controller have five parameters for tuning that means fractional order controllers provide two extra more parameters for tuning than the classical PID controllers, which improves the overall performance of the system. Simulation result shows that FOPID controller gives better result as compared to classical PID controller.

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