A Generalized problem in Linear programming problem

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Abstract

The paper aims to analise a generalised problem in LPP. It is constucted by taking odd coefficients in the objective function and successive terms to the coefficients of constrains. The possible cases for finding basic feasible solutions are deeply discussed .The results in various aspects are established.

1.Introduction:

The main important feature of LPP model is the existence of linearity in the problem for tracing a suitable solution.Linear programming models[1-3] appear in a broad way of different fileds with accurate mathematical transformations.Some applications may not be linear. But they can be solved with suitable approaches [6] by some techniques .The basic idea was first developed by "Lenoid kontoruich" in 1939.LP was introduced by "George B.Dantzig"[4,5] in 1947.

1.1 Advantages of liner programming:

The main advantages of linear programming are

(i).The observable advantage of linear programming is it's clarity and easy way to apply the procedure.

(ii).Linear programming makes optimal use of available resources with better quality of decision making.

1.2 Limitations of linear programming:

LP is mainly applicable to many problems where the constraints and objective functions are linear .

i.e; where they can be expressed as equations which represent straight lines .

(i).In real life situations,if the constraints or objective functions are not linear , this technique cannot be effectively applied.

(ii).LP deals with only single objective. Where as in real life situations, they may have multiple and conflicting objectives.In such cases,LP is not much useful.

1.3 Application Areas of linear programming:

LP has a wide range of applications in many sectors.

(i).It helps decision makers to use their productive resource effectively.

(ii).Some times, The decision-making approach may have more objectives and less subjectives.

2. Simplex Algorithm:

Simplex method is employed to investigate the nature of the solution for the constituted LPP.

Step(1): Let us start with the objective function whether it is maximized or not. If not, we have to convert the minimization problem into maximization by using the relation min z=-max(-z)

Step(2):All b_i's in the constraints should be at positive level. Otherwise,all b_i's should be converted as positive by multiplying the constraint by negative sign.

Step(3):Introduce slack variables or surplus variables along with artificial variables in the given constrains to make them equality constraints.

Step(4):In case of introducing surplus variables along with artificial variables, we assign –M to the artificial variables in the objective function.

Step(5): For getting initial basic feasible solution, the non- basic variables are to be vanished.

Step(6): In this method, first we find the entering variable based on the minimum value of z_j-c_j. The corresponding column of entering variable is known as 'pivot column'.

Step(7): By taking the minimum ratio of the value of basic solution and respective value of pivot column, we get the leaving variable. The corresponding row of the leaving variable is known as 'Pivot row'.

Step(8):The column value of pivot row and pivot column is the pivot element. By the simplex method,the pivot element must be converted to unity and other elements in the pivot column are to be converted as zeroes by using elementary row transformations.

Step(9): The process is continued until the optimum condition(z_j-c_j>0) is satisfied including the artificial variable at zero level. Otherwise, go to step:10

Step(10):When all elements in the pivot column are at negative level or zero in any iteration the solution is unbounded solution. Otherwise, go to step:11

Step(11):When the artificial variables are at positive level / zero level even then if the optimality condition is satisfied, the current solution is called pseudo optimum solution(no solution).

3. Basic problem:

The following LPP is formed as below:

Objective function: Max Z=$(2n+1)x_1 + (2n+3)\,x_2 + (2n+5)\,x_3$

$$\textbf{Subject to the constraints:} \; n+1)\,x_1 + (n+2)\,x_2 + (n+3)\,x_3 \le \text{or} \ge \text{or} = (n+4)$$
$$(n+2)\,x_1 + (n+3)\,x_2 + (n+4)\,x_3 \le \text{or} \ge \text{or} = (n+5)$$
$$(n+3)\,x_1 + (n+4)\,x_2 + (n+5)\,x_3 \le \text{or} \ge \text{or} = (n+6)$$
$$\& \; x_1, x_2, x_3 \ge 0$$

It is divided into four cases.

3.1 Case (1): All the constrains are of ≤ type

$$\textbf{Objective Function: } \text{Max } Z = 2n+1)x_1 + (2n+3)\,x_2 + (2n+5)\,x_3$$
$$\textbf{Subject to the constraints:} \; (n+1)\,x_1 + (n+2)\,x_2 + (n+3)\,x_3 \le (n+4)$$
$$(n+2)\,x_1 + (n+3)\,x_2 + (n+4)\,x_3 \le (n+5)$$
$$(n+3)\,x_1 + (n+4)\,x_2 + (n+5)\,x_3 \le (n+6)$$
$$\& \; x_1, x_2, x_3 \ge 0$$

By applying simplex method, we get the following table when n=1,2,3,4,5&6

Table-1

Iteration	Objective function =MaxZ	Entering Variable	Leaving Variable	Pivot Position (k,r)	Are any variables introduced?	Is optimality condition satisfied?	Optimum basic feasible solution	Does it contain Alternative solution?
When n=1								
Initial	0	x_2	s_3	(3,3)	Slack Variables(3)	No	$X=\begin{pmatrix}0\\0\\7/6\end{pmatrix}$	No
First	49/6	*	*	*		Yes		
When n=2								
Initial	0	x_3	s_3	(3,3)	Slack Variables(3)	No	$X=\begin{pmatrix}0\\0\\8/7\end{pmatrix}$	No
First	72/7	*	*	*		Yes		
When n=3								
Initial	0	x_3	s_3	(3,3)	Slack Variables(3)	No	$X=\begin{pmatrix}0\\0\\9/8\end{pmatrix}$	No
First	99/8	*	*	*		Yes		
When n=4								
Initial	0	x_3	s_3	(3,3)	Slack Variables(3)	No	$X=\begin{pmatrix}0\\0\\10/9\end{pmatrix}$	No
First	130/9	*	*	*		Yes		
When n=5								
Initial	0	x_3	s_3	(3,3)	Slack Variables(3)	No	$X=\begin{pmatrix}0\\0\\11/10\end{pmatrix}$	No
First	165/10	*	*	*		Yes		
When n=6								
Initial	0	x_3	s_3	(3,3)	Slack Variables(3)	No	$X=\begin{pmatrix}0\\0\\12/11\end{pmatrix}$	No
First	204/11	*	*	*		Yes		

3.1.1Conclusions:

In this Case, The following conclusions are observed.

(i)Three Slack variables are introduced in the Constraints.

(ii).only x_3 enters in to the Basis and s_3 leaves from the Basis.

(iii).The Pivot position is always fixed as (3,3)

(iv).The optimum basic feasible solution is obtained as $\mathbf{x_1=0}$, $\mathbf{x_2=0}$, $\mathbf{x_3=}$ **(n+6/n+5)** at first iteration.

(v).The value of the objective function is **Max Z=(2n+5)(n+6)/(n+5).**

(vi).No alternative solution is obtained.

3.2 Case(II):

First constraint is of type $\le$ and the remaining two are of type $\ge$.

Objective function: $\mathbf{Max\,Z = (2n+1)x_1 + (2n+3)x_2 + (2n+5)x_3}$

Subject to the constraints:

$$(n+1)x_1 + (n+2)x_2 + (n+3)x_3 \le (n+4)$$
$$(n+2)x_1 + (n+3)x_2 + (n+4)x_3 \ge (n+5)$$
$$(n+3)x_1 + (n+4)x_2 + (n+5)x_3 \ge (n+6)$$
$$\& \; x_1, x_2, x_3 \ge 0$$

By applying simplex method, we get the following table when n=1,2,3,4,5& 6

Table-2

Iteration	Objective function =MaxZ	Entering Variable	Leaving Variable	Pivot Position (k,r)	Are any variables introduced?	Is optimality condition satisfied?	Optimum basic feasible solution	Does it contain Alternative solution?
				When n=1				
Initial First Second Third	-13M 49-M/6 42/5 35/4	x_3 s_3 s_2 *	A_2 A_1 s_1 *	(3,3) (6,2) (5,1) *	Slack (1) Surplus(2) Artificial(2) variables	No No No Yes	$X=\begin{pmatrix}0\\0\\5/4\end{pmatrix}$	No
				When n=2				
Initial First Second Third	-15M 72-M/7 63/6 54/5	x_3 s_3 s_2 *	A_2 A_1 s_1 *	(3,3) (6,2) (5,1) *	Slack (1) Surplus(2) Artificial(2) variables	No No No Yes	$X=\begin{pmatrix}0\\0\\6/7\end{pmatrix}$	No
				When n=3				
Initial First Second Third	-17M 99-M/8 88/7 77/6	x_3 s_3 s_2 *	A_2 A_1 s_1 *	(3,3) (6,2) (5,1) *	Slack (1) Surplus(2) Artificial(2) variables	No No No Yes	$X=\begin{pmatrix}0\\0\\7/6\end{pmatrix}$	No
				When n=4				
Initial First Second	-19M 130-M/9 117/8	x_3 s_3 s_2	A_2 A_1 s_1	(3,3) (6,2) (5,1)	Slack (1) Surplus(2) Artificial(2)	No No No		No

Third	104/7	*	*	*	variables	Yes	$X=\begin{pmatrix}0\\0\\8/7\end{pmatrix}$	
				When n=5				
Initial First Second Third	-21M 165-M/10 150/9 135/8	x_3 s_3 s_2 *	A_2 A_1 s_1 *	(3,3) (6,2) (5,1) *	Slack (1) Surplus(2) Artificial(2) variables	No No No Yes	$X=\begin{pmatrix}0\\0\\9/8\end{pmatrix}$	No
				When n=6				
Initial First Second Third	-23M 204-M/11 187/10 170/9	x_3 s_3 s_2 *	A_2 A_1 s_1 *	(3,3) (6,2) (5,1) *	Slack (1) Surplus(2) Artificial(2) variables	No No No Yes	$X=\begin{pmatrix}0\\0\\10/9\end{pmatrix}$	No

3.2.1 Conclusions:

In this Case, The following conclusions are observed.

(i).One Slack variable, two Surplus Variables & two Artificial Variables are introduced in the Constraints.

(ii).In the first iteration: Entering Variable- x_3 , Leaving Variable- A_2;

In the Second iteration: Entering Variable- s_3, Leaving Variable- A_1;

In the Third iteration: Entering Variable- s_2, Leaving Variable- s_1

(iii).The Pivot position moves from first iteration (3,3) to second iteration (6,2) & from second iteration (6,2) to third iteration (5,1).

(iv).No alternative solution is obtained.

(v).The optimum basic feasible solution **x_1= 0, x_2=0, x_3=(n+4/n+3)** is obtained at third iteration.

(vi).The value of the objective function is **Max Z= (2n+5)(n+4)/(n+3).**

3.3 Case(III):

First and second constraints are of type $\leq$ and the remaining third one is of type $\geq$.

Objective function: Max Z = $(2n+1)x_1+(2n+3)x_2+(2n+5)x_3$

Subject to the constraints:

$$(n+1)x_1+(n+2)x_2+(n+3)x_3 \leq (n+4)$$
$$(n+2)x_1+(n+3)x_2+(n+4)x_3 \leq (n+5)$$
$$(n+3)x_1+(n+4)x_2+(n+5)x_3 \geq (n+6)$$

$$\& \; x_1, x_2, x_3 \geq 0$$

By applying simplex method, we get the following table when n=1, 2 ,3, 4, 5 & 6

Table-3

Iteration	Objective function =MaxZ	Entering Variable	Leaving Variable	Pivot Position (k,r)	Are any variables introduced?	Is optimality condition satisfied?	Optimum basic feasible solution	Does it contain Alternative solution?

When n=1								
Initial First Second	-7M 49/6 42/5	x_3 s_3 *	A_1 s_2 *	(3,3) (6,2) *	Slack (2) Surplus(1) Artificial(1) variables	No No Yes	$X=\begin{pmatrix}0\\0\\7/6\end{pmatrix}$	No
When n=2								
Initial First Second	-8M 72/7 63/6	x_3 s_3 *	A_1 s_2 *	(3,3) (6,2) *	Slack (2) Surplus(1) Artificial(1) variables	No No Yes	$X=\begin{pmatrix}0\\0\\8/7\end{pmatrix}$	No
When n=3								
Initial First Second	-9M 99/8 88/7	x_3 s_3 *	A_1 s_2 *	(3,3) (6,2) *	Slack (2) Surplus(1) Artificial(1) variables	No No Yes	$X=\begin{pmatrix}0\\0\\9/8\end{pmatrix}$	No
When n=4								
Initial First Second	-10M 130/9 117/8	x_3 s_3 *	A_1 s_2 *	(3,3) (6,2) *	Slack (2) Surplus(1) Artificial(1) variables	No No Yes	$X=\begin{pmatrix}0\\0\\10/9\end{pmatrix}$	No
When n=5								
Initial First Second	-11M 165/10 150/9	x_3 s_3 *	A_1 s_2 *	(3,3) (6,2) *	Slack (2) Surplus(1) Artificial(1) variables	No No Yes	$X=\begin{pmatrix}0\\0\\11/10\end{pmatrix}$	No
When n=6								
Initial First Second	-12M 204/11 187/10	x_3 s_3 *	A_1 s_2 *	(3,3) (6,2) *	Slack (2) Surplus(1) Artificial(1) variables	No No Yes	$X=\begin{pmatrix}0\\0\\12/11\end{pmatrix}$	No

3.3.1 Conclusions:

In this Case, The following conclusions are made:

(i).Two Slack variables, One Surplus Variable & One Artificial Variable are introduced in the Constraints.

(ii).In the first iteration: Entering Variable- x_3,Leaving Variable- A_1;

In the Second iteration: Entering Variable- s_3,Leaving Variable- s_2;

(iii).The Pivot position moves from first iteration (3,3) to second iteration (6,2)

(iv).The optimum basic feasible solution is obtained as **$x_1 = 0$, $x_2=0$, $x_3=$(n+6/n+5)** at second iteration.

(v).The value of the objective function is **MaxZ= (2n+4)(n+6/n+4).**

(vi). No alternative solution is obtained.

3.4 Case (IV):

The first constraint is of type ≤ , the second one is of type ≥ and third one is of type =.

Objective function: Max Z= $(2n+1)x_1 + (2n+3)x_2 + (2n+5)x_3$

Subject to the constraints:

$$(n+1)x_1 + (n+2)x_2 + (n+3)x_3 \le (n+4)$$
$$(n+2)x_1 + (n+3)x_2 + (n+4)x_3 \ge (n+5)$$
$$(n+3)x_1 + (n+4)x_2 + (n+5)x_3 = (n+6)$$
$$\& \; x_1, x_2, x_3 \ge 0$$

By applying simplex method, we get the following table when n=1,2,3,4,5 &6

Table-4

Iteration	Objective function =MaxZ	Entering Variable	Leaving Variable	Pivot Position (k,r)	Are any variables introduced?	Is optimality condition satisfied?	Optimum basic feasible solution	Does it contain Alternative solution?
When n=1								
Initial	-13M	x_3	A_2	(3,3)	Slack (1) Surplus(1) Artificial(2) variables	No	No	No
First	49-M/6	*	*-	*		Yes		
When n=2								
Initial	-15M	x_3	A_2	(3,3)	Slack (1) Surplus(1) Artificial(2) variables	No	No	No
First	72-M/7	*	*	*		Yes		
When n=3								
Initial	-17M	x_3	A_2	(3,3)	Slack (1) Surplus(1) Artificial(2) variables	No	No	No
First	99-M/8	*	*	*		Yes		
When n=4								
Initial	-19M	x_3	A_2	(3,3)	Slack (1) Surplus(1) Artificial(2) variables	No	No	No
First	130-M/9	*	*	*		Yes		
When n=5								
Initial	-21M	x_3	A_2	(3,3)	Slack (1) Surplus(1)	No	No	No

First	165-M/10	*	*	*	Artificial(2) variables	Yes		
When n=6								
Initial	-23M	x_3	A_2	(3,3)	Slack (1) Surplus(1) Artificial(2) variables	No	No	No
First	204-M/11	*	*	*		Yes		

3.4.1 Conclusions:

In this Case, The following conclusions are established.

(i).One Slack variable, one Surplus Variable & two Artificial Variables are introduced in the Constraints.

(ii).In the first iteration: Entering Variable- x_3, Leaving Variable- A_2

(iii).The Pivot position in the first iteration is (3,3)

(iv).No optimum basic feasible solution is obtained (Pseudo optimal solution).

4.Over all Conclusions:

Case	**Optimum Basic Feasible Solution (X)**	**The value of the objective function (Z)**	**An Alternative Solution (if exists)**
I	$X=\begin{pmatrix}0\\0\\(n+6/n+5)\end{pmatrix}$	**Max Z= (2n+5)(n+6/n+5)**	**No**
II	$X=\begin{pmatrix}0\\0\\(n+4/n+3)\end{pmatrix}$	**Max Z= (2n+5)(n+4/n+3)**	**No**
III	$X=\begin{pmatrix}0\\0\\(n+5/n+4)\end{pmatrix}$	**Max Z= (2n+4)(n+6/n+4)**	**No**
IV	**No**	**No**	**No**

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