BIPOLAR VALUED MULTI FUZZY SUBFIELDS OF A FIELD

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ABSTRACT:

In this paper, definition of bipolar valued multi fuzzy subfield and some Theorems of bipolar valued multi fuzzy subfield of a field are discussed.

Keywords: *Bipolar valued fuzzy subset, bipolar valued multi fuzzy subset, bipolar valued multi fuzzy subfield.*

1. INTRODUCTION:

In 1965, Zadeh [13] introduced the notion of a fuzzy subset of a set, fuzzy sets are a kind of useful mathematical structure to represent a collection of objects whose boundary is vague. Since then it has become a vigorous area of research in different domains, there have been a number of generalizations of this fundamental concept such as intuitionistic fuzzy sets, interval-valued fuzzy sets, vague sets, soft sets etc. Lee [6] introduced the notion of bipolar-valued fuzzy sets. Bipolar-valued fuzzy sets are an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [−1, 1]. In a bipolar-valued fuzzy set, the membership degree 0 means that elements are irrelevant to the corresponding property, the membership degree [0, 1] indicates that elements somewhat satisfy the property and the membership degree [−1, 0] indicates that elements somewhat satisfy the implicit counter property. Bipolar-valued fuzzy sets and intuitionistic fuzzy sets look similar each other. However, they are different each other [6, 7]. Sabu Sebastian and T.V.Ramakrishnan [8, 9] defined the multi-fuzzy sets. Anitha.M.S., Muruganantha Prasad & K.Arjunan[1] defined as Bipolar-valued fuzzy subgroups of a group. B.Yasodara and K.E.Sathappan [11, 12] defined the bipolar valued multi fuzzy subsemirings of a semiring, We introduce the concept of bipolar valued multi fuzzy subfield of a field and established some results.

2.PRELIMINARIES:

2.1 Definition:

A bipolar valued fuzzy set (BVFS) A in X is defined as an object of the form $A = \{ < x, A^{+}(x), A^{-}(x) >/ x \in X\}$, where $A^{+} : X \to [0, 1]$ and $A^{-} : X \to [-1, 0]$. The positive membership degree $A^{+}(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar-valued fuzzy set A and the negative membership degree $A^-(x)$ denotes the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar valued fuzzy set A.

2.2 Example:

A = { < a, 0.9, –0.6 >, < b, 0.8, –0.7 >, < c, 0.7, –0.5 >} is a bipolar valued fuzzy subset of X = {a, b, c}.

2.3 Definition:

A bipolar valued multi fuzzy set (BVMFS) A in X of order n is defined as an object of the form A = { < x, $A_i^+(x)$, $A_i^-(x)$ > / $x\in X$}, where $A_i^+ : X\to [0, 1]$ and $A_i^- : X\to [-1, 0]$, i = 1,2, 3, … n. The positive membership degrees $A_i^+(x)$ denote the satisfaction degree of an element x to the property corresponding to a bipolar-valued multi fuzzy set A and the negative membership degrees $A_i^-(x)$ denote the satisfaction degree of an element x to some implicit counter-property corresponding to a bipolar-valued multi fuzzy set A.

2.4 Example:

A = { < a, 0.5, 0,6, 0.3, –0.3, –0.6, –0.5 >, < b, 0.1, 0.4, 0.7, –0.7, –0.3, –0.6 >, < c, 0.5, 0.3, 0.8, –0.4, –0.5, –0.3 >} is a bipolar-valued multi fuzzy subset of order 3 in X = { a, b, c }.

2.5 Definition:

Let A and B be two bipolar valued multi fuzzy subsets of a set X. We define the following relations and operations:

(i) $A \subset B$ if and only if $A_i^+(x) \le B_i^+(x)$ and $A_i^-(x) \ge B_i^-(x)$ for all i and for all $x\in X$.

(ii) A = B if and only if $A_i^+(x) = B_i^+(x)$ and $A_i^-(x) = B_i^-(x)$ for all i and for all $x\in X$.

(iii) $A\cap B = \{ \langle x, \min (A_i^+(x), B_i^+(x)), \max (A_i^-(x), B_i^-(x)) \rangle / x\in X$ and for all i }.

(iv) $A\cup B = \{ \langle x, \max (A_i^+(x), B_i^+(x)), \min (A_i^-(x), B_i^-(x)) \rangle / x\in X$ and for all i }.

2.6 Definition:

Let F be a field. A bipolar valued multi fuzzy subset A of F is said to be a bipolar valued multi fuzzy subfield of F if the following conditions are satisfied,

(i) $A_i^+(x-y) \ge \min\{ A_i^+(x), A_i^+(y) \}$ for all x, y in F

(ii) $A_i^-(x-y) \le \max\{ A_i^-(x), A_i^-(y) \}$ for all x, y in F

(iii) $A_i^+(xy^{-1}) \ge \min\{ A_i^+(x), A_i^+(y) \}$ for all x, $y\neq 0$ in F

(iv) $A_i^-(xy^{-1}) \le \max\{ A_i^-(x), A_i^-(y) \}$ for all x, $y\neq 0$ in F.

2.7 Example:

Let F = Z_3 = { 0, 1, 2 } be a field with respect to the ordinary addition and multiplication. Then A = { < 0, 0.5, 0.8, 0.6, – 0.6, – 0.5, – 0.7 >, < 1, 0.4, 0.7, 0.5, – 0.5, – 0.4, – 0.6 >, < 2, 0.4, 0.7, 0.5, – 0.5, – 0.4, – 0.6 > } is a bipolar valued multi fuzzy subfield of order 3 in F.

Note: In this paper, $A = \langle A_i^+, A_i^- \rangle$ is a bipolar valued multi fuzzy subfield of order (dimension) n.

3. PROPERTIES:

3.1 Theorem:

Let A = $\langle A_i^+, A_i^- \rangle$ be a bipolar valued multi fuzzy subfield of a field F. Then $A_i^+(-x) = A_i^+(x)$, $A_i^-(-x) = A_i^-(x)$, $A_i^+(x) \le A_i^+(0)$, $A_i^-(x) \ge A_i^-(0)$, for all i and for all x in F and $A_i^+(x^{-1}) = A_i^+(x)$ and $A_i^-(x^{-1}) = A_i^-(x)$, $A_i^+(x) \le A_i^+(1)$ and $A_i^-(x) \ge A_i^-(1)$, for all i and for all $x \ne 0$ in F, where 0 and 1 are identity elements in F.

Proof:

Let x be in F. Now, $A_i^+(x) = A_i^+(-(-x)) \ge A_i^+(-x) \ge A_i^+(x)$ for all i and for all x in F. And $A_i^-(x) = A_i^-(-(-x)) \le A_i^-(-x) \le A_i^-(x)$ for all i and for all x in F. And, $A_i^+(0) = A_i^+(x-x) \ge \min\{A_i^+(x), A_i^+(-x)\} = A_i^+(x)$ for all i and for all x in F. And $A_i^-(0) = A_i^-(x-x) \le \max\{A_i^-(x), A_i^-(-x)\} = A_i^-(x)$ for all i and for all x in F. Let $x \ne 0$ be in F. Now $A_i^+(x) = A_i^+((x^{-1})^{-1}) \ge A_i^+(x^{-1}) \ge A_i^+(x)$ for all i and for all $x \ne 0$ in F. And $A_i^-(x) = A_i^-((x^{-1})^{-1}) \le A_i^-(x^{-1}) \le A_i^-(x)$ for all i and for all $x \ne 0$ in F. And $A_i^+(e) = A_i^+(xx^{-1}) \ge \min\{A_i^+(x), A_i^+(x^{-1})\} = A_i^+(x)$ for all i and for all $x \ne 0$ in F. And $A_i^-(e) = A_i^-(xx^{-1}) \le \max\{A_i^-(x), A_i^-(x^{-1})\} = A_i^-(x)$ for all i and for all $x \ne 0$ in F.

3.2 Theorem:

Let A = $\langle A_i^+, A_i^- \rangle$ be a bipolar valued multi fuzzy subfield of a field F. Then

(i) $A_i^+(x-y) = A_i^+(0)$ implies that $A_i^+(x) = A_i^+(y)$ for all i and for x, y in F.

(ii) $A_i^-(x-y) = A_i^-(0)$ implies that $A_i^-(x) = A_i^-(y)$ for all i and for x, y in F.

(iii) $A_i^+(xy^{-1}) = A_i^+(1)$ implies that $A_i^+(x) = A_i^+(y)$ for all i and for $x \ne 0$, $y \ne 0$ in F.

(iv) $A_i^-(xy^{-1}) = A_i^-(1)$ implies that $A_i^-(x) = A_i^-(y)$ for all i and for $x \ne 0$, $y \ne 0$ in F.

Proof:

(i) $A_i^+(x) = A_i^+(x-y+y) \ge \min\{A_i^+(x-y), A_i^+(y)\} = \min\{A_i^+(0), A_i^+(y)\} = A_i^+(y) = A_i^+(y-x+x) \ge \min\{A_i^+(y-x), A_i^+(x)\} = \min\{A_i^+(0), A_i^+(x)\} = A_i^+(x)$ for all i and for x, y in F. (ii) $A_i^-(x) = A_i^-(x-y+y) \le \max\{A_i^-(x-y), A_i^-(y)\} = \max\{A_i^-(0), A_i^-(y)\} = A_i^-(y) = A_i^-(y-x+x) \le \max\{A_i^-(y-x), A_i^-(x)\} = \max\{A_i^-(0), A_i^-(x)\} = A_i^-(x)$ for all i and for x, y in F. (iii) $A_i^+(x) = A_i^+(xy^{-1}y) \ge \min\{A_i^+(xy^{-1}), A_i^+(y)\} = \min\{A_i^+(1), A_i^+(y)\} = A_i^+(y) = A_i^+(yx^{-1}x) \ge \min\{A_i^+(yx^{-1}), A_i^+(x)\} = \min\{A_i^+(1), A_i^+(x)\} = A_i^+(x)$ for all i and for $x \ne 0$, $y \ne 0$ in F. (iv) $A_i^-(x) = A_i^-(xy^{-1}y) \le \max\{A_i^-(xy^{-1}), A_i^-(y)\} = \max\{A_i^-(1), A_i^-(y)\} = A_i^-(y) = A_i^-(yx^{-1}x) \le \max\{A_i^-(yx^{-1}), A_i^-(x)\} = \max\{A_i^-(1), A_i^-(x)\} = A_i^-(x)$ for all i and for $x \ne 0$, $y \ne 0$ in F.

3.3 Theorem:

Let A = $\langle A_i^+, A_i^- \rangle$ be a bipolar valued multi fuzzy subfield of a field F.

(i) If $A_i^+(x-y) = 1$, then $A_i^+(x) = A_i^+(y)$ for all i and for x, y in F.

(ii) If $A_i^-(x-y) = -1$, then $A_i^-(x) = A_i^-(y)$ for all i and for x, y in F.

(iii) If $A_i^+(xy^{-1}) = 1$, then $A_i^+(x) = A_i^+(y)$ for all i and for $x \ne 0$, $y \ne 0$ in F.

(iv) If $A_i^-(xy^{-1}) = -1$, then $A_i^-(x) = A_i^-(y)$ for all i and for $x \ne 0$, $y \ne 0$ in F.

Proof:

(i) $A_i^+(x) = A_i^+(x-y+y) \geq \min\{ A_i^+ (x-y), A_i^+ (y) \}= \min\{1, A_i^+(y) \}= A_i^+(y) = A_i^+(-y) = A_i^+(-x+x-y) \geq \min \{ A_i^+(-x), A_i^+(x-y) \} = \min\{ A_i^+ (-x), 1 \} = A_i^+ (-x) = A_i^+ (x)$ for all i and for x, y in F. (ii) $A_i^-(x) = A_i^-(x-y+y) \leq \max \{A_i^-(x-y), A_i^-(y) \}= \max \{-1, A_i^-(y) \} = A_i^-(y) = A_i^-(-y) = A_i^-(-x+x-y) \leq \max\{ A_i^-(-x), A_i^-(x-y) \}= \max\{ A_i^-(-x), -1\} = A_i^-(-x) = A_i^-(x)$ for all i and for x, y in F. (iii) $A_i^+(x) = A_i^+(xy^{-1}y) \geq \min\{ A_i^+ (xy^{-1}), A_i^+ (y) \}= \min\{1, A_i^+(y) \}= A_i^+(y) = A_i^+(y^{-1}) = A_i^+(x^{-1}xy^{-1}) \geq \min \{ A_i^+(x^{-1}), A_i^+(xy^{-1}) \} = \min\{ A_i^+ (x^{-1}), 1 \} = A_i^+ (x^{-1}) = A_i^+ (x)$ for all i and for $x \neq 0$, $y \neq 0$ in F. (iv) $A_i^-(x) = A_i^-(xy^{-1}y) \leq \max \{A_i^-(xy^{-1}), A_i^-(y)\}= \max\{-1, A_i^-(y) \} = A_i^-(y) = A_i^-(y^{-1}) = A_i^-(x^{-1}xy^{-1}) \leq \max\{ A_i^-(x^{-1}), A_i^-(xy^{-1}) \} = \max\{ A_i^-(x^{-1}), -1\} = A_i^-(x^{-1}) = A_i^-(x)$ for all i and for $x \neq 0$, $y \neq 0$ in F.

3.4 Theorem:

Let $A = \langle A_i^+, A_i^- \rangle$ be a bipolar valued multi fuzzy subfield of a field F.

(i) If $A_i^+ (x-y) = 0$, then either $A_i^+(x) = 0$ or $A_i^+(y) = 0$ for all i and for x, y in F.

(ii) If $A_i^- (x-y) = 0$, then either $A_i^- (x) = 0$ or $A_i^- (y) = 0$ for all i and for x, y in F.

(iii) If $A_i^+ (xy^{-1}) = 0$, then either $A_i^+(x) = 0$ or $A_i^+(y) = 0$ for all i and for $x \neq 0$, $y \neq 0$ in F.

(iv) If $A_i^- (xy^{-1}) = 0$, then either $A_i^- (x) = 0$ or $A_i^- (y) = 0$ for all i and for $x \neq 0$, $y \neq 0$ in F.

Proof:

(i) Let x and y in F. By the definition $A_i^+(x-y) \geq \min\{A_i^+(x), A_i^+(y) \}$ which implies that $0 \geq \min \{A_i^+(x), A_i^+(y)\}$. Thus either $A_i^+(x) = 0$ or $A_i^+(y) = 0$ for all i. (ii) By the definition $A_i^-(x-y) \leq \max\{A_i^-(x), A_i^-(y) \}$ which implies that $0 \leq \max \{A_i^-(x), A_i^-(y) \}$. Thus either $A_i^-(x) = 0$ or $A_i^-(y) = 0$ for all i. (iii) Let $x \neq 0$ and $y \neq 0$ in F. By the definition $A_i^+(xy^{-1}) \geq \min \{ A_i^+(x), A_i^+(y) \}$ which implies that $0 \geq \min \{ A_i^+(x), A_i^+(y) \}$. Thus either $A_i^+(x) = 0$ or $A_i^+(y) = 0$ for all i. (iv) By the definition $A_i^-(xy^{-1}) \leq \max \{ A_i^-(x), A_i^-(y) \}$ which implies that $0 \leq \max \{ A_i^-(x), A_i^-(y) \}$. Thus either $A_i^-(x) = 0$ or $A_i^-(y) = 0$ for all i.

3.5 Theorem:

Let $A = \langle A_i^+, A_i^- \rangle$ be a bipolar valued multi fuzzy subfield of a field F. Then

(i) $A_i^+ (x+y) = A_i^+ (y+x)$ if and only if $A_i^+ (x) = A_i^+ (-y+x+y)$ for all i and for x, y in F.

(ii) $A_i^- (x+y) = A_i^- (y+x)$ if and only if $A_i^- (x) = A_i^- (-y+x+y)$ for all i and for x, y in F.

(iii) $A_i^+ (xy) = A_i^+ (yx)$ if and only if $A_i^+ (x) = A_i^+ (y^{-1}xy)$ for all i and for $x \neq 0$, $y \neq 0$ in F.

(iv) $A_i^- (xy) = A_i^- (yx)$ if and only if $A_i^- (x) = A_i^- (y^{-1}xy)$ for all i and for $x \neq 0$, $y \neq 0$ in F.

Proof:

(i) Let x, y be in F. Assume that $A_i^+(x+y) = A_i^+(y+x)$, so, $A_i^+(-y+x+y) = A_i^+(-y+y+x) = A_i^+(0+x) = A_i^+(x)$ for all i and for x, y in F. Conversely, assume that $A_i^+(x) = A_i^+(-y+x+y)$, then $A_i^+(x+y) = A_i^+(x+y+x-x) = A_i^+(y+x)$ for all i and for x, y in F. (ii) Assume $A_i^-(x+y) = A_i^-(y+x)$, then $A_i^-(-y+x+y) = A_i^-(-y+y+x) = A_i^-(0+x) = A_i^-(x)$ for all i and for x, y in F. Conversely, assume that $A_i^-(x) = A_i^-(-y+x+y)$, so, $A_i^-(x+y) = A_i^- (x+y+x-x) = A_i^-(y+x)$ for all i and for x, y in F. (iii) Let $x \neq$

0, $y \neq 0$ be in F. Assume that $A_i^+(xy) = A_i^+(yx)$, so, $A_i^+(y^{-1}xy) = A_i^+(y^{-1}yx) = A_i^+(1x) = A_i^+(x)$ for all i and for $x \neq 0$, $y \neq 0$ in F. Conversely, assume that $A_i^+(x) = A_i^+(y^{-1}xy)$, then $A_i^+(xy) = A_i^+(xyxx^{-1}) = A_i^+(yx)$ for all i and for $x \neq 0$, $y \neq 0$ in F. (iv) Assume $A_i^-(xy) = A_i^-(yx)$, then $A_i^-(y^{-1}xy) = A_i^-(y^{-1}yx) = A_i^-(1x) = A_i^-(x)$ for all i and for $x \neq 0$, $y \neq 0$ in F. Conversely, assume that $A_i^-(x) = A_i^-(y^{-1}xy)$ so $A_i^-(xy) = A_i^-(xyxx^{-1}) = A_i^-(yx)$ for all i and for $x \neq 0$, $y \neq 0$ in F.

3.6 Theorem:

If $A = \langle A_i^+, A_i^- \rangle$ is a bipolar valued multi fuzzy subfield of a field F, then

$H = \{ x \in F \mid A_i^+(x) = 1, A_i^-(x) = -1 \text{ for all } i \}$ is either empty or is a subfield of F.

Proof:

If no element satisfies this condition, then H is empty. If x and y in H, then $A_i^+(x-y) \geq \min\{A_i^+(x), A_i^+(y)\} = \min\{1, 1\} = 1$. Therefore $A_i^+(x-y) = 1$ for all i. And $A_i^-(x-y) \leq \max\{A_i^-(x), A_i^-(y)\} = \max\{-1, -1\} = -1$. Therefore $A_i^-(x-y) = -1$ for all i. That is $x-y \in H$. If x and $y \neq 0$ in H, then $A_i^+(xy^{-1}) \geq \min\{A_i^+(x), A_i^+(y)\} = \min\{1, 1\} = 1$. Thus $A_i^+(xy^{-1}) = 1$ for all i. And $A_i^-(xy^{-1}) \leq \max\{A_i^-(x), A_i^-(y)\} = \max\{-1, -1\} = -1$. Therefore $A_i^-(xy^{-1}) = -1$ for all i. That is $xy^{-1} \in H$. Hence H is a subfield of F. Hence H is either empty or is a subfield of F.

3.7 Theorem:

If $A = \langle A_i^+, A_i^- \rangle$ is a bipolar valued multi fuzzy subfield of a field F, then $H = \{ x \in F \mid A_i^+(x) = A_i^+(0) = A_i^+(1) \text{ and } A_i^-(x) = A_i^-(0) = A_i^-(1) \text{ for all } i \}$ is a subfield of F, where 0 and 1 are identity elements.

Proof:

By Theorem 2.1, $A_i^+(-x) = A_i^+(x) = A_i^+(0)$ and $A_i^-(-x) = A_i^-(x) = A_i^-(0)$ for all i. Thus $-x \in H$. If x, $y \in H$, then $A_i^+(x-y) \geq \min\{A_i^+(x), A_i^+(y)\} = \min\{A_i^+(0), A_i^+(0)\} = A_i^+(0)$ and $A_i^+(0) \geq A_i^+(x-y)$. Thus $A_i^+(0) = A_i^+(x-y)$ for all i. Also $A_i^-(x-y) \leq \max\{A_i^-(x), A_i^-(y)\} = \max\{A_i^-(0), A_i^-(0)\} = A_i^-(0)$ and $A_i^-(0) \leq A_i^-(x-y)$. Thus $A_i^-(0) = A_i^-(x-y)$ for all i. Therefore $x-y \in H$. By Theorem 2.1, $A_i^+(x^{-1}) = A_i^+(x) = A_i^+(1)$ and $A_i^-(x^{-1}) = A_i^-(x) = A_i^-(1)$ for all i. Thus $x^{-1} \in H$. If x, $y \neq 0 \in H$, then $A_i^+(xy^{-1}) \geq \min\{A_i^+(x), A_i^+(y)\} = \min\{A_i^+(1), A_i^+(1)\} = A_i^+(1)$ and $A_i^+(1) \geq A_i^+(xy^{-1})$. Thus $A_i^+(1) = A_i^+(xy^{-1})$ for all i. Also $A_i^-(xy^{-1}) \leq \max\{A_i^-(x), A_i^-(y)\} = \max\{A_i^-(1), A_i^-(1)\} = A_i^-(1)$ and $A_i^-(1) \leq A_i^-(xy^{-1})$. Thus $A_i^-(1) = A_i^-(xy^{-1})$ for all i. Therefore $xy^{-1} \in H$. Hence H is a subfield of H.

3.8 Theorem:

If $A = \langle A_i^+, A_i^- \rangle$ and $B = \langle B_i^+, B_i^- \rangle$ are two bipolar valued multi fuzzy subfields of a field F, then their intersection $A \cap B$ is a bipolar valued multi fuzzy subfield of G.

Proof:

Let $A = \{\langle x, A_i^+(x), A_i^-(x) \rangle / x \in F\}$, $B = \{\langle x, B_i^+(x), B_i^-(x) \rangle / x \in F\}$ for all i. Let $C = A \cap B$ and $C = \{\langle x, C_i^+(x), C_i^-(x) \rangle / x \in F\}$. Then $C_i^+(x-y) = \min\{A_i^+(x-y), B_i^+(x-y)\} \geq \min\{\min\{A_i^+(x), A_i^+(y)\}, \min\{B_i^+(x), B_i^+(y)\}\} \geq \min\{\min\{A_i^+(x), B_i^+(x)\}, \min\{A_i^+(y), B_i^+(y)\}\} = \min\{C_i^+(x),$

$C_i^+(y)$ } for all i. Also $C_i^-(x-y) = \max \{ A_i^-(x-y), B_i^-(x-y) \} \leq \max \{ \max \{A_i^-(x), A_i^-(y)\}, \max \{B_i^-(x), B_i^-(y)\}\} \leq \max \{ \max \{A_i^-(x), B_i^-(x)\}, \max \{A_i^-(y), B_i^-(y)\}\} = \max \{ C_i^-(x), C_i^-(y) \}$ for all i. And $C_i^+(xy^{-1}) = \min \{ A_i^+(xy^{-1}), B_i^+(xy^{-1}) \} \geq \min\{ \min \{A_i^+(x), A_i^+(y) \}, \min \{B_i^+(x), B_i^+(y) \}\} \geq \min \{ \min \{A_i^+(x), B_i^+(x) \}, \min \{A_i^+(y), B_i^+(y) \}\} = \min \{ C_i^+(x), C_i^+(y) \}$ for all i. Also $C_i^-(xy^{-1}) = \max \{A_i^-(xy^{-1}), B_i^-(xy^{-1}) \} \leq \max \{ \max \{A_i^-(x), A_i^-(y)\}, \max \{B_i^-(x), B_i^-(y) \}\} \leq \max \{ \max \{A_i^-(x), B_i^-(x)\}, \max \{A_i^-(y), B_i^-(y)\}\} = \max \{ C_i^-(x), C_i^-(y) \}$ for all i. Hence $A \cap B$ is a bipolar valued multi fuzzy subfield of F.

3.9 Theorem:

The intersection of a family of bipolar valued multi fuzzy subfields of a field F is a bipolar valued multi fuzzy subfield of F.

Proof:

The proof follows from Theorems 2.8.

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