A Variety Problem in Linear Programming

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Abstract

The purpose of this paper is to find out an optimum basic feasible solution for a variety problem in LPP. It is framed by taking strictly odd coefficients in the objective function with successive coefficients to all the constrains. In this study, the optimum basic feasible solution is successfully derived as $x_1 = 0, x_2 = 0, x_3 = \frac{n+5}{n+2}$ *with the objective fuction value* $MinZ = (2n+1)(n+5)/(n+2)$*only in one case out of four.*

1.Introduction:

An organized group of people with a particular purpose as a business or government departments faces the problem on an amount of resources assigned to particular recipients such as labour, machine, material etc. Most of these resolutions reach the solution after the required considerations according to the constrains. The Operation Research model exhibits the solution of system with the help of decision analysis.

Linear programming [1,3] is the most adaptable and extensively used quantitative technique. This model gives well-organized methods for resoluting the optimal decision. The optimal solution is that meets the objective of management based on varies limitations and restrictions.

1.1 Advantages of liner programming:

We have the following benifits with the Linear programming.

(i). Maximizes the profit of organization by selection of minimum product.

(ii).It minimizes the distribution cost for firms.

(iii). By selection of variety of stocks,it will increase the return investment by adopting suitable decision.

1.2 Application Areas of linear programming:

Linear programming technique is effectively employed for decision making in business, industry and many other areas such as Applications in Agricultural,Military, Production Management[2,6],Financial Management & Personal Management.

1.3 Simplex Method:

This method was invested by George Dantzing [3,4] in 1947.The method is also known as simplex algorithm. The name of the algorithm is derived from the concept of a simplex and was suggested by T.S.Motzkin

2. Simplex Algorithm:

Simplex method is employed to investigate the nature of the solution for the constituted LPP.

Step(1): Let us start with the objective function whether it is maximized or not. If not, we have to convert the minimization problem into maximization by using the relation min z=-max(-z)

Step(2):All b_i's in the constraints should be at positive level. Otherwise,all b_i's should be converted as positive by multiplying the constraint by negative sign.

Step(3):Introduce slack variables or surplus variables along with artificial variables in the given constrains to make them equality constraints.

Step(4):In case of introducing surplus variables along with artificial variables, we assign –M to the artificial variables in the objective function.

Step(5): For getting initial basic feasible solution, the non- basic variables are to be vanished.

Step(6): In this method, first we find the entering variable based on the minimum value of z_j-c_j. The corresponding column of entering variable is known as 'pivot column'.

Step(7): By taking the minimum ratio of the value of basic solution and respective value of pivot column, we get the leaving variable. The corresponding row of the leaving variable is known as 'Pivot row'.

Step(8):The column value of pivot row and pivot column is the pivot element. By the simplex method,the pivot element must be converted to unity and other elements in the pivot column are to be converted as zeroes by using elementary row transformations.

Step(9): The process is continued until the optimum condition(z_j-c_j>0) is satisfied including the artificial variable at zero level. Otherwise, go to step:10

Step(10):When all elements in the pivot column are at negative level or zero in any iteration the solution is unbounded solution. Otherwise, go to step:11

Step(11):When the artificial variables are at positive level / zero level even then if the optimality condition is satisfied, the current solution is called pseudo optimum solution(no solution).

3. Basic problem:

The following LPP is constructed as below:

Objective function: Min/MaxZ =$(2n+1)x_1+(2n+3)\,x_2+(2n+5)\,x_3$

Subject to the constraints :$(n+1)x_1 + (n+2)x_2 + (n+3)\,x_3 \leq$ or $\geq$ or = $(n+3)$

$$(n+2)\,x_1 + (n+3)x_2 + (n+4)x_3 \leq \text{ or } \geq \text{ or } = (n+4)$$

$$(n+3)\,x_1 + (n+4)\,x_2 + (n+5)x_3 \leq \text{ or } \geq \text{ or } = (n+5)$$

and $x_1, x_2, x_3 \geq 0$

It is divided into four cases.

3.1 Case(1) : First constraint is of type ≤ and the remaining two are of type ≥ .

Objective function: MinZ= $(2n+1)x_1+(2n+3)x_2+(2n+5)x_3$

Subject to the constraints :$(n+1)x_1 + (n+2)x_2 + (n+3)x_3 \leq (n+4)$

$$(n+2)x_1 + (n+3)x_2 + (n+4)x_3 \geq (n+5)$$

$$(n+3)x_1 + (n+4)x_2 + (n+5)x_3 \geq (n+6)$$

and $x_1, x_2, x_3 \geq 0$

By applying simplex method, we get the following table when n=1,2,3,4,5 & 6.

Table-1

Iteration	Objective function =MinZ	Entering Variable	Leaving Variable	Pivot Position (k,r)	Are any variables introduced?	Is optimality condition satisfied?	Optimum basic feasible solution	Does it contain Alternative solution?
When n=1								
Initial First Second Third	-13M (-M-49)/6 -42/5 -6	x_3 s_3 x_1 *	A_2 A_1 x_3 *	(3,3) (6,2) (5,1) *	Slack (1) Surplus(2) Artificial(2) variables	No No No Yes	$X=\begin{pmatrix}2\\0\\0\end{pmatrix}$	No
When n=2								
Initial First Second Third	-15M (-M-42)/7 -21/2 -35/4	x_3 s_3 x_1 *	A_2 A_1 x_3 *	(3,3) (6,2) (5,1) *	Slack (1) Surplus(2) Artificial(2) variables	No No No Yes	$X=\begin{pmatrix}7/4\\0\\0\end{pmatrix}$	No
When n=3								
Initial First Second Third	-17M (-M-99)/8 -88/7 -56/5	x_3 s_3 x_1 *	A_2 A_1 x_3 *	(3,3) (6,2) (5,1) *	Slack (1) Surplus(2) Artificial(2) variables	No No No Yes	$X=\begin{pmatrix}8/5\\0\\0\end{pmatrix}$	No
When n=4								
Initial First Second Third	-19M (-M-113)/9 -117/8 -27/2	x_3 s_3 x_1 *	A_2 A_1 x_3 *	(3,3) (6,2) (5,1) *	Slack (1) Surplus(2) Artificial(2) variables	No No No Yes	$X=\begin{pmatrix}3/2\\0\\0\end{pmatrix}$	No
When n=5								
Initial First Second Third	-21M (-M-165)/8 -50/3 -110/7	x_3 s_3 x_1 *	A_2 A_1 x_3 *	(3,3) (6,2) (5,1) *	Slack (1) Surplus(2) Artificial(2) variables	No No No Yes	$X=\begin{pmatrix}10/7\\0\\0\end{pmatrix}$	No
When n=6								
Initial First Second	-23M (-M-204)/11 -187/10	x_3 s_3 x_1	A_2 A_1 x_3	(3,3) (6,2) (5,1)	Slack (1) Surplus(2) Artificial(2)	No No No		No

Third	-143/8	*	*	*	variables	Yes	X= $\begin{pmatrix} 11/8 \\ 0 \\ 0 \end{pmatrix}$	

3.1.1 Conclusions:

In this Case, The following conclusions are obtained.

(i).One Slack variable, two Surplus Variables & two Artificial Variables are introduced in the Constraints.

(ii).In the first iteration: Entering Variable- x_3, Leaving Variable- A_2;

In the Second iteration: Entering Variable- s_3, Leaving Variable- A_1;

In the Third iteration: Entering Variable- x_1, Leaving Variable-x_3

(iii).The Pivot position moves from first iteration (3,3) to second iteration (6,2) and then it moves to third iteration (5,1).

(iv).The optimum basic feasible solution is obtained as $x_1 = 0, x_2 = 0$ & $x_3 = n + 5/(n+2)$ at third iteration.

(v).The value of the objective function is $\mathbf{Min\ Z} = (n+5)(2n+1)/(n+2)$

(vi).No alternative solution is traced.

3.2 Case(II):

First and Third constraints are of type $\geq$ and the remaining second one is of type $\leq$.

Objective function: MinZ = $(2n+1)x_1+(2n+3)x_2+(2n+5)x_3$

Subject to constraints: $(n+1)x_1 + (n+2)x_2 + (n+3)x_3 \geq (n+4)$

$$(n+2)x_1 + (n+3)x_2 + (n+4)x_3 \leq (n+5)$$

$$(n+3)x_1 + (n+4)x_2 + (n+5)x_3 \geq (n+6)$$

$$\text{and } x_1, x_2, x_3 \geq 0$$

By applying simplex method, we get the following table when n=1,2,3,4,5& 6

Table-2

Iteration	Objective function =MinZ	Entering Variable	Leaving Variable	Pivot Position (k,r)	Are any variables introduced?	Is optimality condition satisfied?	Optimum basic feasible solution	Does it contain Alternative solution?
When n=1								
Initial First Second	-12M (-2M-49)/6 (-M-45)/5	x_3 s_3 *	A_2 s_2 *	(3,3) (6,2) *	Slack (1) Surplus(2) Artificial(2) variables	No No Yes	No	No
When n=2								
Initial First Second	-14M (-2M-72)/7 (-M-63)/6	x_3 s_3 *	A_2 s_2 *	(3,3) (6,2) *	Slack (1) Surplus(2) Artificial(2) variables	No No Yes	No	No
When n=3								

Initial First Second	-16M (-2M-99)/8 (-M-88)/7	x_3 s_3 *	A_2 s_2 *	(3,3) (6,2) *	Slack (1) Surplus(2) Artificial(2) variables	No No Yes	No	No
When n=4								
Initial First Second	-18M 2(-M-65)/9 (-M-88)/7	x_3 s_3 *	A_2 s_2 *	(3,3) (6,2) *	Slack (1) Surplus(2) Artificial(2) variables	No No Yes	No	No
When n=5								
Initial First Second	-20M (-2M-165)/10 (-M-150)/9	x_3 s_3 *	A_2 s_2 *	(3,3) (6,2) *	Slack (1) Surplus(2) Artificial(2) variables	No No Yes	No	No
When n=6								
Initial First Second	-22M (-2M-204)/11 (-M-63)/6	x_3 s_3 *	A_2 s_2 *	(3,3) (6,2) *	Slack (1) Surplus(2) Artificial(2) variables	No No Yes	No	No

3.2.1 Conclusions:

In this case, The below conclusions are made.

(i).One Slack variable, two Surplus Variables & two Artificial Variables are introduced in the Constraints.

(ii).In the first iteration: Entering Variable- x_3,Leaving Variable- A_2;

In the Second iteration: Entering Variable- s_3,Leaving Variable- s_2;

(iii). The Pivot position moves from first iteration (3,3) to second iteration (6,2) .

(iv).The optimality condition is satisified.But it has no basic feasible solution.

3.3 Case(III):

First and second constraints are of type $\geq$ and the remaining third one is of type $\leq$.

Objective function: $\text{Min}Z = (2n+1)x_1+(2n+3)x_2+(2n+5)x_3$

Subject to constraints:

$$(n+1)x_1 + (n+2)x_2 + (n+3)x_3 \geq (n+4)$$
$$(n+2)x_1 + (n+3)x_2 + (n+4)x_3 \geq (n+5)$$
$$(n+3)x_1 + (n+4)x_2 + (n+5)x_3 \leq (n+6)$$
$$\text{and } x_1, x_2, x_3 \geq 0$$

By applying simplex method, we get the following table when n=1,2,3,4,5 & 6

Table-3

Iteration	Objective function Z=Max/Min	Entering Variable	Leaving Variable	Pivot Position (k,r)	Are any variables introduced?	Is optimality condition satisfied?	Optimum basic feasible solution	Does it contain Alternative solution?
When n=1								
Initial First	-11M (-3M-49)/6	x_3 *	s_3 *-	(3,3) *	Slack (1) Surplus(2) Artificial(2) variables	No Yes	No	No
When n=2								
Initial First	-13M (-3M-72)/7	x_3 *	s_3 *-	(3,3) *	Slack (1) Surplus(2) Artificial(2) variables	No Yes	No	No
When n=3								
Initial First	-15M (-3M-99)/8	x_3 *	s_3 *-	(3,3) *	Slack (1) Surplus(2) Artificial(2) variables	No Yes	No	No
When n=4								
Initial First	-17M (-3M-130)/9	x_3 *	s_3 *-	(3,3) *	Slack (1) Surplus(2) Artificial(2) variables	No Yes	No	No
When n=5								
Initial First	-19M (-3M-165)/10	x_3 *	s_3 *-	(3,3) *	Slack (1) Surplus(2) Artificial(2) variables	No Yes	No	No
When n=6								
Initial First	-21M (-3M-204)/11	x_3 *	s_3 *-	(3,3) *	Slack (1) Surplus(2) Artificial(2) variables	No Yes	No	No

3.3.1 Conclusions:

In this Case, The conclusions are establised as below.

(i).One Slack variable, Two Surplus Variable &Two Artificial Variable are introduced in the Constraints.

(ii). In the first iteration: Entering Variable- x_3, Leaving Variable- $s_{3;}$

(iii).The Pivot position in the first iteration is (3,3).

(iv).The optimality condition is satisfied. But it has no basic feasible solution.

3.4 Case (IV): All the constrains are of type ≥.

Objective function: MaxZ = $(2n+1)x_1+(2n+3)x_2+(2n+5)x_3$

Subject to constraints : $(n+1)x_1 + (n+2)x_2 + (n+3)x_3 \geq (n+4)$

$$(n+2)x_1 + (n+3)x_2 + (n+4)x_3 \geq (n+5)$$

$$(n+3)x_1 + (n+4)x_2 + (n+5)x_3 \geq (n+6)$$

and $x_1, x_2, x_3 \geq 0$

By applying simplex method, we get the following table when n=1, 2 ,3, 4, 5 & 6

Table-4

Iteration	Objective function =MaxZ	Entering Variable	Leaving Variable	Pivot Position (k,r)	Are any variables introduced?	Is optimality condition satisfied?	Optimum basic feasible solution	Does it contain Alternative solution?
When n=1								
Initial First Second Third	-19M (-M+140)/9 63/4 16	x_3 s_3 s_2 *	A_3 A_2 A_1 *	(3,3) (6,2) (5,1) *	Surplus(3) Artificial(3) variables	No No No Yes	No	No
When n=2								
Initial First Second Third	-21M (-3M+72)/7 (-M+63)/6 54/5	x_3 s_3 s_2 *	A_3 A_2 A_1 *	(3,3) (6,2) (5,1) *	Surplus(3) Artificial(3) variables	No No No Yes	No	No
When n=3								
Initial First Second Third	-24M (-3M+990)/8 (-M+88)/7 77/6	x_3 s_3 s_2 *	A_3 A_2 A_1 *	(3,3) (6,2) (5,1) *	Surplus(3) Artificial(3) variables	No No No Yes	No	No
When n=4								
Initial First Second Third	-27M (-3M+130)/9 (-M+117)/8 104/7	x_3 s_3 s_2 *	A_3 A_2 A_1 *	(3,3) (6,2) (5,1) *	Surplus(3) Artificial(3) variables	No No No Yes	No	No
When n=5								
Initial First Second Third	-30M (-3M+165)/10 (-M+150)/9 135/8	x_3 s_3 s_2	A_3 A_2 A_1	(3,3) (6,2) (5,1)	Surplus(3) Artificial(3) variables	No No No	No	No

		*	*	*		Yes		
When n=6								
Initial First Second Third	-33M (-3M+204)/11 (-M+187)/10 170/9	x_3 s_3 s_2 *	A_3 A_2 A_1 *	(3,3) (6,2) (5,1) *	Surplus(3) Artificial(3) variables	No No No Yes	No	No

3.4.1 Conclusions:

In this Case, The following conclusions are obtained.

(i). One Slack variable, two Surplus Variables & two Artificial Variables are introduced in the Constraints.

(ii).In the first iteration: Entering Variable- x_3,Leaving Variable- $A_{3;}$

In the Second iteration: Entering Variable- s_3,Leaving Variable- A_2;

In the Third iteration: Entering Variable- $s_{2,}$,Leaving Variable- A_1

(iii).The Pivot position moves from first iteration (3,3) to second iteration (6,2) & from second iteration (6,2) to third iteration (5,1).

(iv).No optimum basic feasible solution is obtained.It has an unbounded Solution.

4.Over all Conclusions:

Case	Optimum Basic Feasible Solution (X)	The value of the objective function (Z)	An Alternative Solution (if exists)
I	$x_1 = x_2 = 0,$ $x_3 = (n+5)/(n+2)$	$Min\ Z = (n+5)(2n+1)/(n+2)$	**No**
II	**Not satisfied**	*	*
III	**Not satisfied**	*	*
IV	**Unbounded Solution**	*	*

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