Mathematical Modelling of Love-Waves

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ABSTRACT: *The propagation of Love waves due to a point source in a medium of fibre-reinforced types has been investigated in this paper. The upper layer has been considered to be homogeneous fibre-inforced medium. In case of homogeneous isotropic case, it has been observed that wave number ranging from 0.91 to 5.41 for variation of phase velocity from 1.03 to 1.19. In case of non-homogeneous fibre-reinforced medium a substantial decrease has been observed. The Greens function technique has been used to solve the problem. All the results are compared with GA-SVM approach.*

Keywords: SH-Wave, SVM, GA-SVM and Fiber-Reinforced medium.

1. INTRODUCTION

The propagation of Love waves due to a point source in a homogeneous layer overlying a semi-infinite homogeneous substratum has been discussed by Sezawa [1935]. Sato [1952] studied the propagation of Love saves in a double superficial layer on a heterogeneous medium, the heterogeneity being due to variation in rigidity[1-4]. Ghosh [1970] studied the propagation of Love waves due to a point source in the interface between an upper layer and semi-infinite substratum, one medium having a slow linear variation in the rigidity. Chattopadhyay and Kar [1977] have discussed the propagation of Love waves due to a point source in an isotropic lattice medium under initial stress. Chattopadhyaya et.al., [1984] have studied propagation of SH waves due to a point source in a homogeneous medium lying over an inhomogeneous substratum [5-8].

Toughness of fibre reinforced cement based materials can be controlled by fibre reinforcement and that is the main role of fibres. Materials such as resins reinforced by string aligned fibres exhibit highly anisotropic elastic moduli for extension in the fibre direction are frequently of the order 50 or more times greater than their elastic moduli in transverse extension or in shear [9-14]. An idealization of this property is to be assumed that the material inextensible in the fibre direction[15-18]. The fibre can exhibit a strong resonance even at very low frequencies. This resonance was not predicted by the quasistatic theory[19-21].

The propagation of shear waves due to a point source in a medium of fibre-reinforced type has been investigated in this paper. The Green's function technique has been used to solve the problem [22-24].

2. SOLUTION OF THE PROBLEM

Let us suppopse that horizontal fibre-reinforced medium of thickness H is overlying semi-infinite inhomogeneous fibre-reinforced medium. The origin Oof coordinates is chosen along the face free surface with z-axis vertically downwards. In order to simplify the problem the point-source of disturbance is taken at S, where the z-axis intetrsects the intetrface[25].

Equation governing the propagation of small elastic disturbance in fibre-reinforced medium are[26-27]

$$\tau_{ij,j} = \rho \frac{\partial^2 u}{\partial t^2} \quad (1)$$

For wave propagation in the x-direction and causing displacement in the y-direction only, we shall assume that[28-29]

$$u_1 = u_3 = 0 \;\; and \;\; u_2 = v(x, z, t) \quad (2)$$

The fibre-reinforced vector has components (a1,0,a3). Using (2) in (1) we have,

$$\frac{\partial \tau_{21}}{\partial x} + \frac{\partial \tau_{23}}{\partial z} = \rho \frac{\partial^2 v}{\partial t^2} \qquad (3)$$

$$2a_1a_2 + \frac{\partial \tau_{21}}{\partial x} + \frac{\partial \tau_{23}}{\partial z} = \rho \frac{\partial^2 v}{\partial t^2} \qquad (4)$$

If $\tau(r,t)$ is the force density distribution in the upper layer due to the source, the equation of motion for shear wave propagation along $x-$axis is

$$\frac{\partial \tau_{23}}{\partial z} = \rho \frac{\partial^2 v}{\partial t^2} \quad (5)$$

the above equation may be written as

$$\frac{\partial^2 v_1}{\partial z^2} + m_1 \frac{\partial v}{\partial z} = \frac{2}{b}\delta \quad (6)$$

For lower inhomogeneous medium, the inhomogeneity is taken in the form

$$\frac{\partial^2 v_1}{\partial z^2} + m_1 \frac{\partial v}{\partial z} + n^2 v = \frac{2}{b}\delta + 4\prod \delta \quad (7)$$

substituting equation (7) in equation (6) we will get

$$\frac{\partial^2 v_1}{\partial z^2} + m_1 \frac{\partial v}{\partial z} + n^2 v = 4\prod \delta \quad (8)$$

So that the displacement function v in the lower medium may be determined by assuming the medium to be homogeneous, isotropic and having source density distribution.

Substituting v1 in equation(6), we get

$$\frac{\partial^2 v_2}{\partial z^2} + n^2 v = 4\prod \delta \quad (9)$$

similarly substituting v in equation(8), we get

$$\frac{\partial^2 v'_2}{\partial z^2} + n^2 v = 4\prod \delta + \frac{1}{4} m - n^2 \quad (9)$$

The boundary conditions of determination v2 and v’ from equations (9) and (10) are

$$\tau_{23}' = 0,\ z = 0;\ v_1 = v,\ z = H \quad (10)$$

If G1 is the Green’s function from the upper medium satisfying the condition

$$\frac{\partial G1}{\partial z} = 0 \quad for\ z = 0\ and\ z = H$$

$$\frac{\partial^2 G1}{\partial z^2} - \alpha^2 G1 = \delta \quad (11)$$

Where z0 is a point in the upper medium.

Multiplying the equation (9) by G1 and the equation (11) by v2, substracting and integrating with respect to z from z=0 to z= H we obtain,

$$v_2(z) = G1(z/H)[\frac{dv_2}{dz}] \quad (13)$$

Therefore,

$$v_2(z) = G1(z/H)[\frac{dv_2}{dz} + \frac{m_1 v_1}{2}] \quad (14)$$

Again, let G be the Green’s function for the lower medium which as argued previously, may be assumed to be homogeneous. We assume that G which is the solution of the equation

$$\frac{d^2G}{dz^2} - \beta^2 G = \delta \quad (15)$$

where z0 is a point in the lower medium, satisfy the condition derivative G approaches to 0 as z approaches to infinity and

$$\frac{dG}{dz} = 0 \;\; at \;\; z = H.$$

Multiplying the equation (10) by G and (15) by v' , substracting and integrating with respect to z from z=H to z= infinity, we get

$$\frac{dG}{dz} = 0 \;\; at \;\; z = H.\int_H^\infty \delta(Z - Z0)dz \quad (16)$$

Interchanging z and z0 in the integral on the right hand side of the equation(16), the value of the v' at any point z at the lower medium is

$$\frac{dG}{dz} = 0 \;\; at \;\; z = H.\int_H^\infty \delta(Z - Z0)dz \; + G(x,t) \; (17)$$

Since G(H/z)=G(z/H) and G(z/z0)=G(z0/z), using boundary conditions and substituting value of (16) in (17) we get

$$\int_H^\infty G(H/z_0)F(z_0)dz_0 \quad (18)$$

$$\int_H^\infty G(H/z_0)F(z_0)dz_0 \; + \int_H^0 G(H/z_0)F(z_0)dz_0 (19)$$

Now v(z) is to be determined from (19) by the method of successive approximations. The value of v(z) derived from (19) when substituted in (18) gives the value of v1(z) . Since we are interested in the value of v1(z), which will give the displacement at any point in the upper layer and since higher powers of parameter is neglected. As a first approximation we take ,

$$v(z)=\frac{e^{m/2}NV(3)}{D'N(1)} \quad (20)$$

which will obviously give the displacement at any point in the lower medium if it is taken to be homogeneous.

Substituting the value of v(z) from (20) in (18) for v1(z), we get

$$NV(5)=2G_1(z/H)d1+(z_0-H) \quad (21)$$

$$\frac{d^2v}{dz^2}-\alpha^2v=0 \quad (22)$$

vanishing at z= infinity

$$\frac{d^2v}{dz^2}-\alpha^2v+2G_1(z/H)d1+(z_0-H)=0 \quad (23)$$

$$G_1(z/H)=\frac{2G_1(z/H)d1+(z_0-H)}{\frac{d^2v}{dz^2}-\alpha^2v} \qquad (24)$$

$$G_1(H/H)=\frac{2G_1(z/H)d1+(z_0-H)}{\frac{d^2v}{dz^2}-\alpha^2v}+(z_0-H) \qquad (25)$$

$$G_1(H/z_0)=\frac{2G_1(z/H)d1}{\frac{d^2v}{dz^2}-\alpha^2v}+(z_0-H) \qquad (26)$$

$$G_1(H/H)=-\frac{1}{\beta} \qquad (27)$$

Substituting the values of G_1(z/H), G_1(H/H), G(H/z0) and G(H/H) from the above relations in equation (19) and neglecting the terms containing powers of parameter and higher than the first, the expression for v1(z) may be approximated.

The corresponding displacement v1(z,x) at point in the upper medium is obtained from the above by taking the Fourier transform as

$$v_1(z,x)=\int_{-\infty}^{+\infty}v_1(f,z)e^{-ifz}df \quad (28)$$

The denominator of the integral (28) when equated to zero gives the dispersion equation for SH waves.

3. NUMERICAL CALCULATIONS AND DISCUSSIONS

The material constants for fibre-reinforced medium have been considered due to Markham

$$\mu_T = 2.46X10^9 \ \ N/m^2 \ \ and \quad \mu_L = 5.66X10^9 \ \ N/m^2$$

$$\lambda = 5.65X10^9 \ \ N/m^2 \ \ and \ \ \alpha = -1.28X10^9 \ \ N/m^2$$

In case of homogeneous isotropicmedium, it is observed from figure thay kH is ranging from 0.91 to 5.41 for variation of phase velocity from 1.03 to 1.19. In case of non-homogeneous fibre-reinforced medium a substantial decrease has been observed. So it can be concluded that fibre-reinforced medium has effect of lowering the phase velocity for a fixed Kh.

4. REFERENCES

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