Dependence of Blood and Plasma viscosities on the flow of blood though small vessels in the presence of axially symmetric stenosis

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Abstract-*Taken in to consideration, the viscosity of blood and plasma in small blood vessels, a mathematical model is developed in this chapter for study of blood flow through a blood vessel in the existence of stenosis. Non-Newtonian aspect of blood has been considered by supposing the constitutive equation of blood as Casson fluid. Analytic expressions for velocity, plug velocity, volumetric flow rate and shear stress have been received. It is observed that dependence of viscosity bears the potential to influence the velocity and plug velocity distribution of blood to an outstanding level. The analytical results that we have obtained in this study has been shown numerically and graphically to give apparent insight of this problem.*

Keywords –plug velocity, shear stress, plasma viscosity, hematocrit level

I .INTRODUCTION

It is well known that under diseased environment, an unusual and unnatural expansion develops in the lumen at different place in the cardiovascular system. Arteriosclerosis or stenosis is one of the most wide spread diseases. It has been pointed out that the deposits of cholesterol, pearly substance on the arterial wall and proliferation of connective tissues in the arterial wall form plaques which grow inwards and obstruct blood flow. To understand the influence of stenosis in the lumen many researchers have studied blood flow in stenosed arteries by assuming that blood reacts like a Newtonian fluid. However, it has been observed that blood being a suspension of red cells in plasma, behaves like a non-Newtonian fluid at lower shear rate in smaller diameter tubes. It is also observed that for low shear rates the yield stress for blood is non-zero. **Akbar and Butt (2015)** wrote about the MHD response intended for Cu nanoparticles for blood flow during composite stenosed arteries having permeable wall. They noticed that velocity of blood shows decreasing trends for copper water instead of taking pure water as it will create flexible environment in arteries so that velocity becomes deliberate. **Shit and Majee (2018)** constructed a mathematical model of transient flow of blood having heat transfer properties by a sinusoidally fluctuating arterial section in the influence of magnetic field by assuming indulgence of energy because of magnetic field intensity and viscosity of blood. They observed that potency of vortices reduces when MHD effect grows. For calculating hemodynamics of non-Newtonian flow of blood, **Ghigo et al. (2018)** presented a time dependent one dimension model which is helpful to explain local space-time deviation of viscous performance of blood. They have compared the theoretical and experimental data in this concern.**Mustapha et al. (2009)** has been discussed electrically conducting flow of blood through arteries having multiple stenoses asymmetrical in shape in the uniform transverse magnetic field surroundings. The elasticity of the arterial wall has also been described for in the present exploration.**Ghasemi**

et al. (2016) discussed the blood flow under periodic body acceleration by assuming blood as third degree non-Newtonian fluid. They Crank-Nicholson method and Differential Quadrature method to solve PDE arising in their model. **Zaman and Sajid (2014)** presented a mathematical analysis of transient pulsatile flow of blood during a time dependent tapered stenotic artery by considering the blood rheology as in Sisko model. They have considered moving wall of artery in place of rigid wall and used finite difference method to solve governing equations. **Singh and Singh (2013)** intended a scientific model of blood stream What's more watched that impact of hematocrit. Their study demonstrates though those rate of hematocrit increments the divider shear stress declines.**Verma and Parihar (2010)** encountered with urban decay because of deindustrialization, engineering concocted, government lodgi functioned once a scientific model about decreased conduit with stenosis and gained that those deepest membranes from claiming an artery developed because of those secondary divider shear stress. At last those graphical outcomes describing the result from claiming hematocrit for wall shear stress Also imperviousness should stream.

II .MATHEMATICAL FORMULATION

Let us consider an artery with stenosis symmetrical about the axis but non-symmetrical with respect to radial co-ordinates. The mathematical expression for geometry is given by

$$\frac{R}{R_0} = 1 - A\left[{l_0}^{m-1}(z-d) - (z-d)^m\right]; d \le z \le d + l_0$$

$$= 1; Or\ else \qquad (1)$$

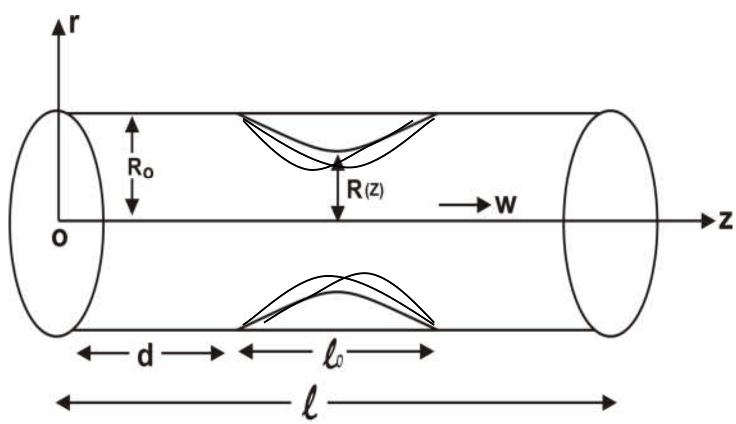


Fig 4.1: Geometry of an axially symmetric and radially non-symmetric stenosed artery

Where R, R_0 are tube radius (with, without stenosis), $m \ge 2$is a shape parameter determining stenosis shape, $l_0$is stenosis length, d indicates its location.

$$A = \frac{\delta}{R_0}\frac{m^{m-1}}{{l_0}^m(m-1)} \qquad (2)$$

δ be the maximum height of the stenosis located at $z = d + \frac{l_0}{m^{\left(\frac{1}{m}\right)-1}}$

The constitutive equation for Casson fluid is given as follows:

$$\tau^{1/2} = {\tau_y}^{1/2} + \mu(\dot{\gamma})^{1/2}; \tau \ge \tau_y$$

$$\dot{\gamma} = 0; \tau \le \tau_y \qquad (3)$$

Equation of motion is given as

$$P = \frac{1}{r}\frac{\partial}{\partial r}(r\tau) \tag{4}$$

Where $P = -\frac{\partial p}{\partial z}$

On solving (5) to get

$$\tau = \frac{1}{2}Pr \tag{5}$$

$$P \times \pi {r_p}^2 = \tau_y \times 2\pi r_p \text{gives } \tau_y = \frac{1}{2}Pr_p \tag{6}$$

Equation (3) gives

$$\dot{\gamma} = \frac{(\sqrt{\tau}-\sqrt{\tau_y})^2}{\mu^2} \tag{7}$$

From (5), (6) and (7), we get the expression for velocity

$$\frac{dw}{dr} = \frac{P}{2\mu^2}\left(\sqrt{r}-\sqrt{r_p}\right)^2 = \frac{P}{2\mu^2}\left(r + r_p - 2\sqrt{rr_p}\right) \tag{8}$$

The boundary conditions are

$$w = 0 \;\; atr = R \tag{9}$$

Solving the differential equation (8) with boundary condition (9)

$$w = \frac{P}{2\mu^2}\left[\frac{1}{2}(r^2 - R^2) + r_p(r - R) - \frac{4}{3}\sqrt{r_p}(r^{\frac{3}{2}} - R^{3/2})\right]$$

$$w = \frac{PR^2}{2\mu^2}\left[\frac{1}{2}\left\{\left(\frac{r}{R}\right)^2 - 1\right\} + \beta\left(\frac{r}{R} - 1\right) - \frac{4}{3}\sqrt{\beta}\left\{\left(\frac{r}{R}\right)^{3/2} - 1\right\}\right] \tag{10}$$

Where $\beta = \frac{r_p}{R}$ (11)

The plug velocity can be find by substituting $r = \beta R$ implies that $\frac{r}{R} = \beta$

Expression for plug velocity is

$$w_p = \frac{P}{2\mu^2}\left(\frac{r}{\beta}\right)^2\left[\frac{1}{2}\{\beta^2 - 1\} + \beta(\beta - 1) - \frac{4}{3}\sqrt{\beta}\{\beta^{3/2} - 1\}\right]$$

The volumetric flow rate is given by

$$Q = 2\pi\int_0^{r_p} rw_p dr + 2\pi\int_{r_p}^{R} rwdr = 2\pi\left(Q_p + Q_1\right)$$

$$Q_p = \frac{PR^2}{2\mu^2}\left[\begin{matrix}\frac{1}{8}{r_p}^2\{\beta^2 - 2\} + \frac{1}{12}{r_p}^2\{3\beta^2 - 4\beta\} \\ -\frac{2}{15}{r_p}^2\{5(\beta)^2 - 8\sqrt{\beta}\}\end{matrix}\right]$$

And $Q_1 = \frac{PR^2}{2\mu^2}\left[-\frac{R^2}{8} + \frac{\beta R^2}{6} + \frac{2}{7}\sqrt{\beta}R^2 - \frac{1}{8}{r_p}^2\{\beta^2 - 2\} - {r_p}^2(2\beta^2 - 3\beta) + \frac{2}{21}{r_p}^2(4\beta^2 - 7\sqrt{\beta})\right]$

And $Q = \frac{\pi PR^2}{\mu^2}\left[-\frac{R^2}{8} + \frac{\beta R^2}{6} + \frac{2}{7}\sqrt{\beta}R^2 + {r_p}^2\left(-\frac{7}{4}\beta^2 + \frac{8}{3}\beta\right) + {r_p}^2\left(\frac{1}{21}\beta^2 - \frac{2}{15}\sqrt{\beta}\right)\right]$

$$Q = \frac{\pi PR^4}{\mu^2}\left[-\frac{1}{8} + \frac{\beta}{6} + \frac{2}{7}\sqrt{\beta} + \beta^2\left(-\frac{7}{4}\beta^2 + \frac{8}{3}\beta\right) + \beta^2\left(\frac{1}{21}\beta^2 - \frac{2}{15}\sqrt{\beta}\right)\right]$$

$$P = \frac{Q\mu^2}{\pi R^4 \theta} \quad (12)$$

Where $\theta = -\frac{1}{8} + \frac{\beta}{6} + \frac{2}{7}\sqrt{\beta} + \beta^2\left(-\frac{7}{4}\beta^2 + \frac{8}{3}\beta\right) + \beta^2\left(\frac{1}{21}\beta^2 - \frac{2}{15}\sqrt{\beta}\right)$

Expression for shear stress is as follows

$$\tau_R = \frac{1}{2}Pr = \frac{Q\mu^2 r}{2\pi R^4 \theta} \quad (13)$$

III . NUMERICAL RESULTS

Fig.2: Velocity vs Radius of obstructed tube for different viscosities

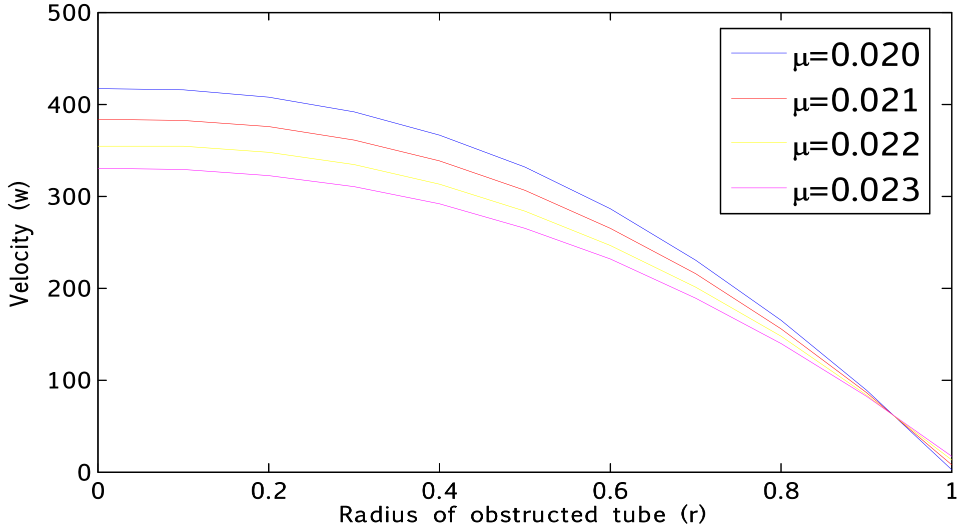
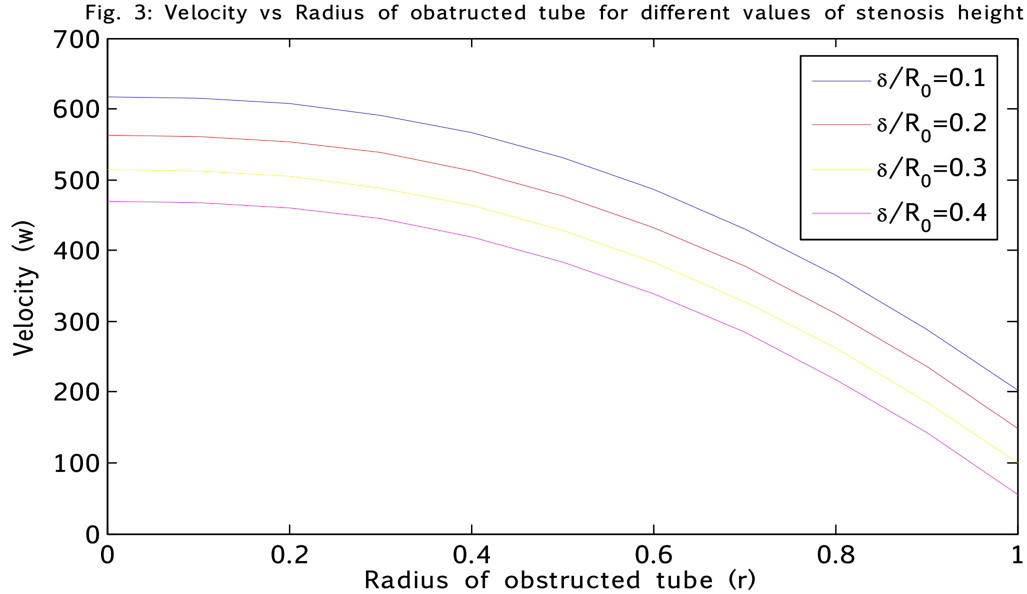


Fig. 3: Velocity vs Radius of obatructed tube for different values of stenosis height



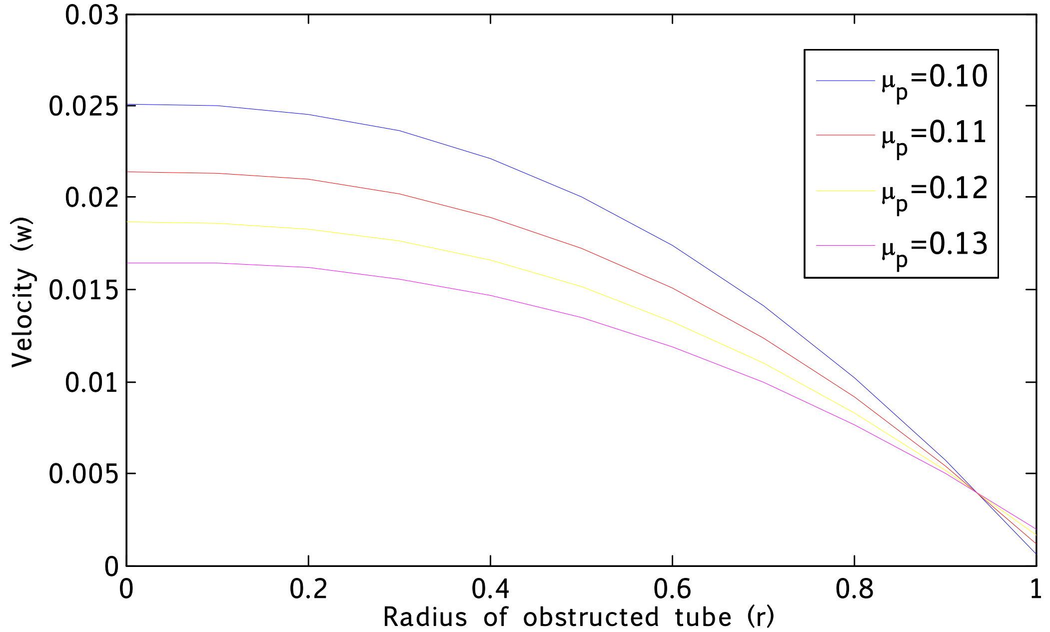


Fig. 4: Velocity vs Radius of obatructed tube for different values of plasma viscosities

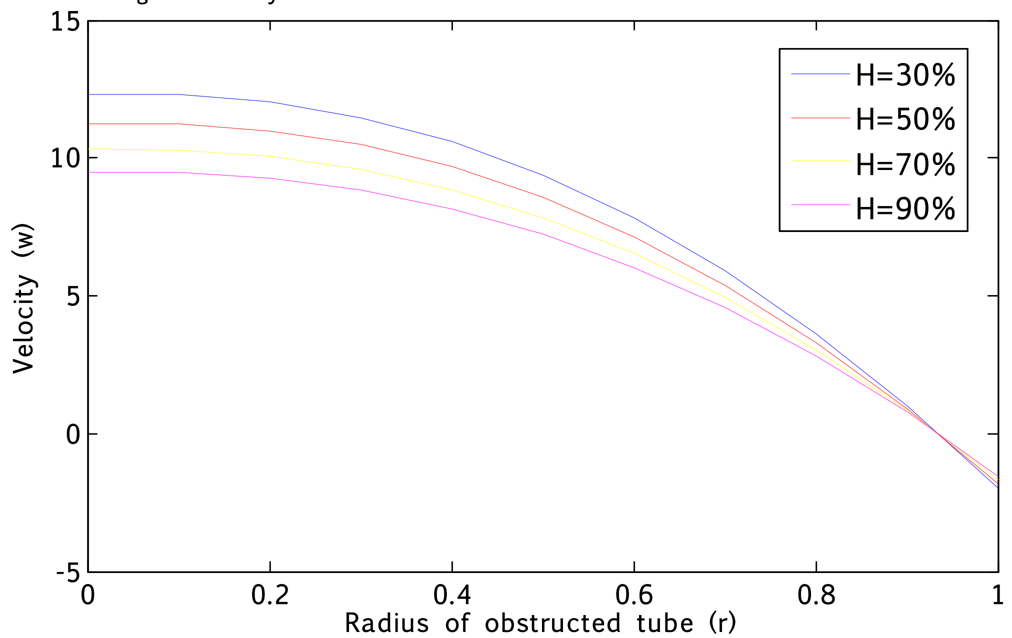


Fig. 5: velocity vs Radius of obatructed tube for different hematocrit level

Fig. 6: Plug velocity vs radius of obstructed tube for different values of blood viscosities

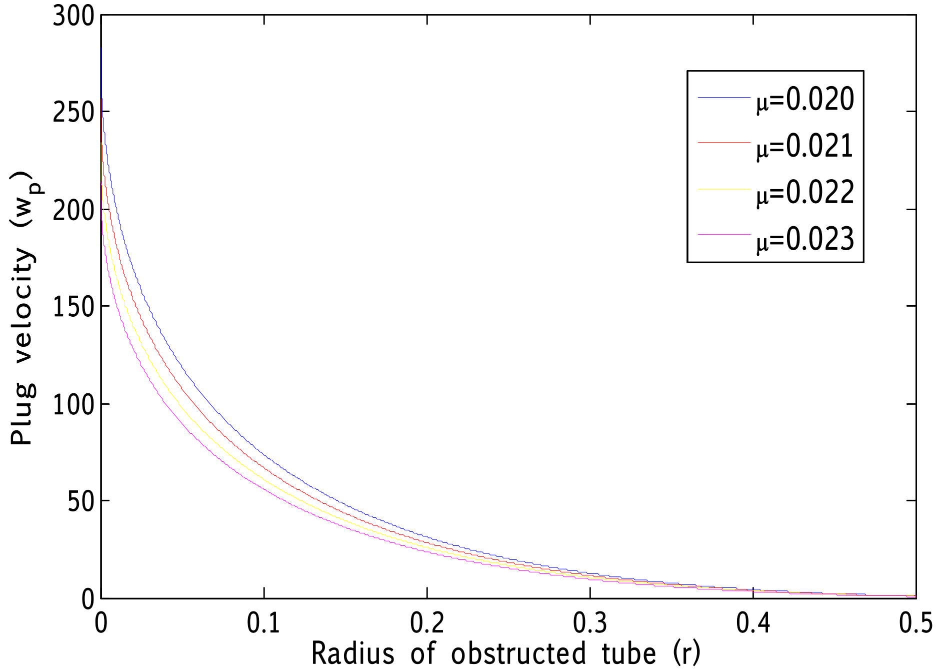


Fig. 7: Plug velocity vs Radius of obatructed tube for different stenosis heights

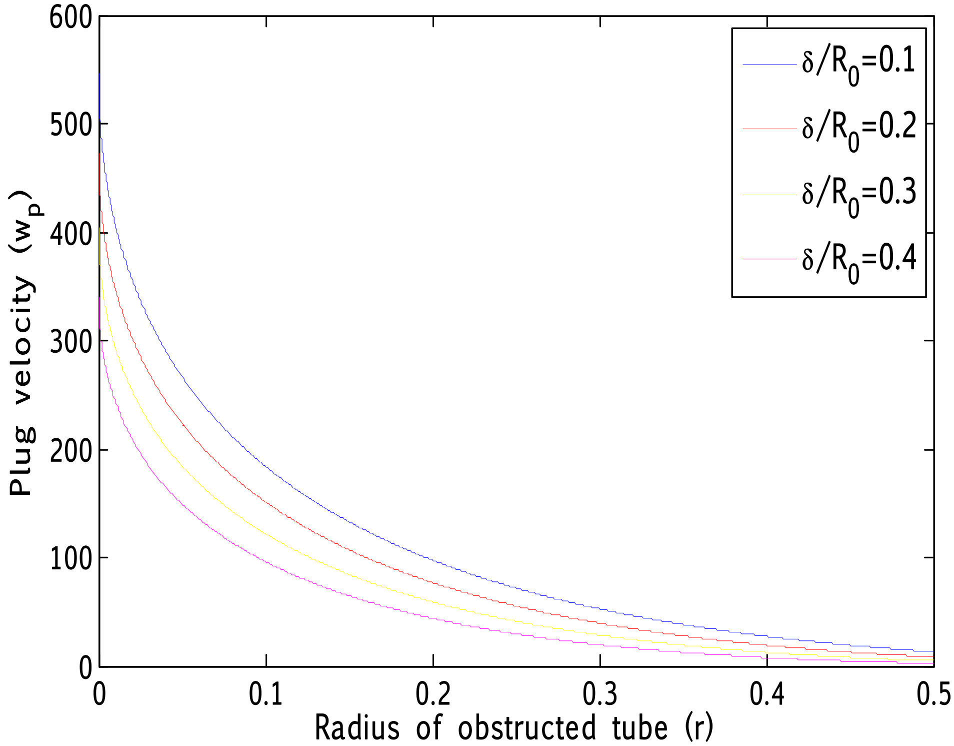


Fig. 8: Plug velocity vs Radius of obstructed tube for different plasma viscosities

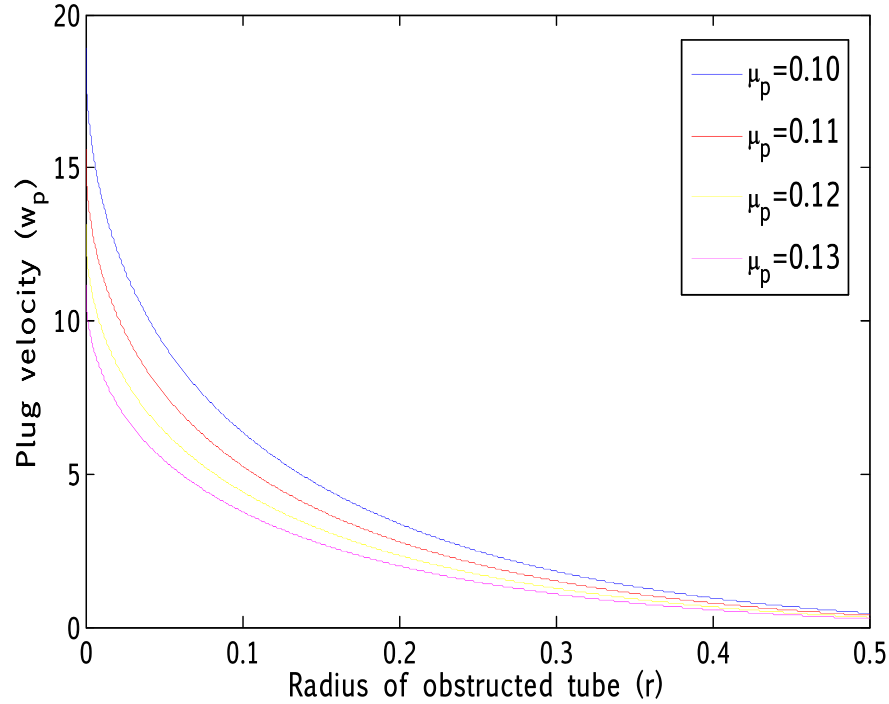


Fig. 9: Plug velocity vs Radius of obstructed tube for different hematocrit level

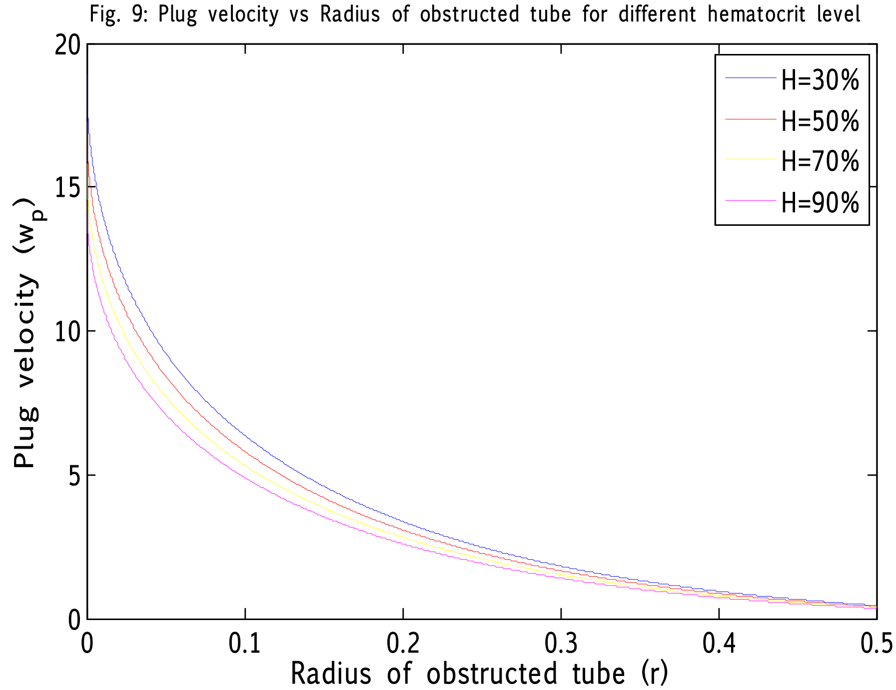


Fig. 10: Shear stress vs radius of obstructed tube for different blood viscosities

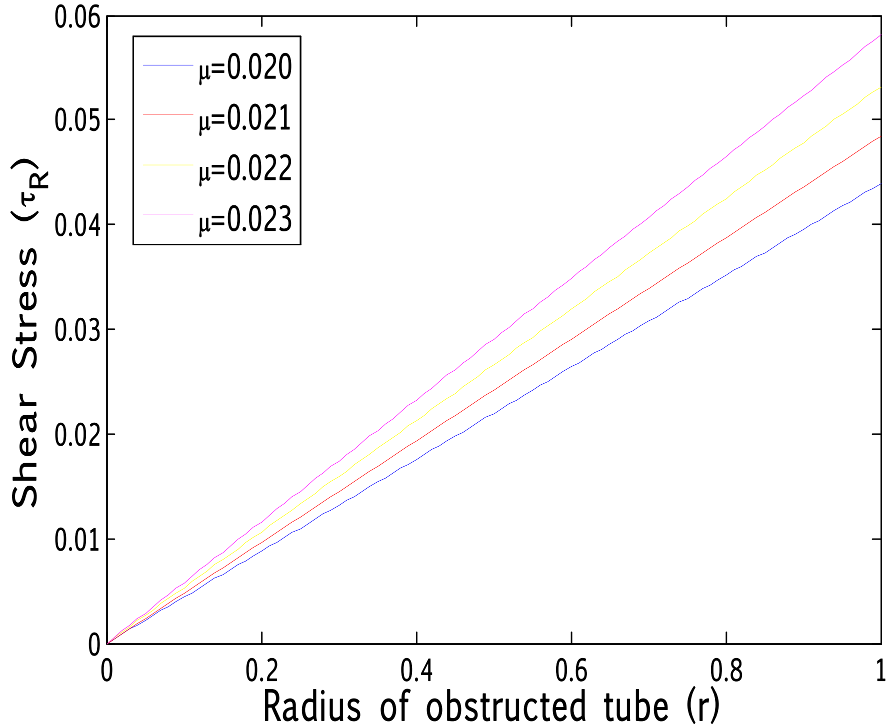


Fig. 11: Shear Stress vs Radius of obatructed tube for different stenosis heights

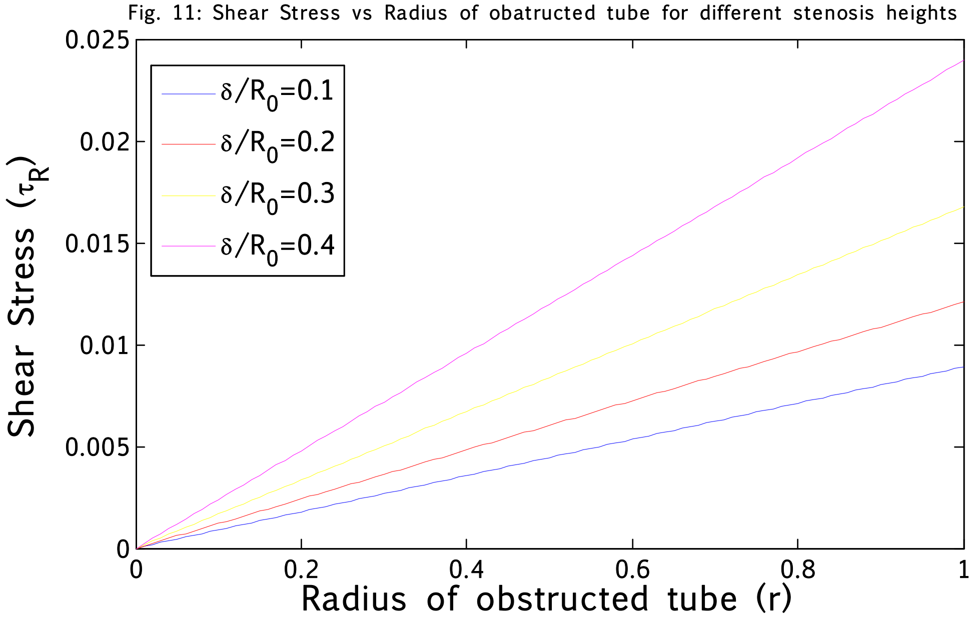


Fig. 12: Shear Stress vs Radius of obstructed tube for different plasma viscosities

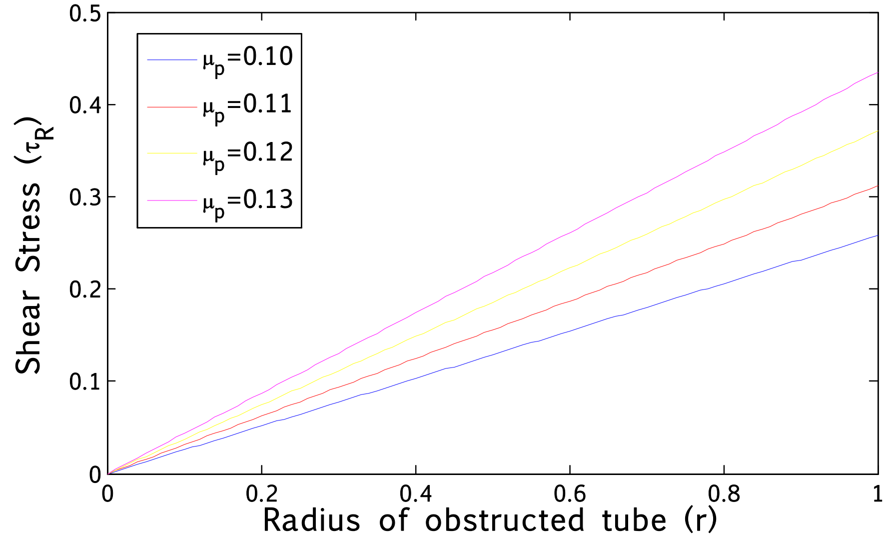
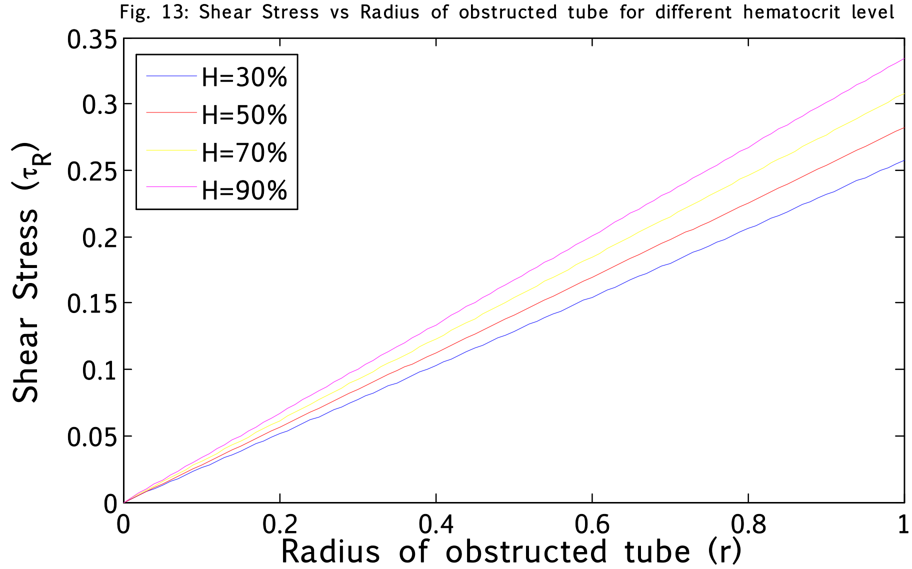


Fig. 13: Shear Stress vs Radius of obstructed tube for different hematocrit level



The analytical expressions derived in the earlier section have been computed numerically for different blood and plasma viscosities as well as stenosis heights and hematocrit level. The major reason of this computational work is to quantify the influence of these flow parameters at the wall on the velocity distribution. The computation has been carried out for

$z = 0.8, d = 0.5, P = 1, r_{p=}0.02, l = 2, l_0 = 1, Q = 10.$

Figures 2-5 illustrated variation of the non-dimensional axial velocity of blood flow in the stenosed arterial section with the radius of obstructed tube r. Similar variations for plug velocity and shear stress have been illustrated in the figures from 5-13. As cumulative results, from these figures it is obvious that higher viscosity of blood and plasma as well as hematocrit level may help to reduce axial velocity and plug velocity in diseased condition. On the other hand increased viscosities of plasma and blood in addition to hematocrit level give a positive potential to shear stress. It is also notice that non-Newtonian behavior of blood has amazing role to reduce shear stress in comparison Newtonian characteristic. The results that we have obtained are very close to previous researchers who are working in this direction. They have derived these results by considering the blood as Herschel-Bulkley fluid and also for particular case of Newtonian fluid. In our work inclusion of hematocrit level and stenosis height makes the results more valuable for upcoming researchers.

IV .CONCLUSIONS

At hand progress of the mathematical model of blood flow through a stenosed segment of an artery bear the potential to reveal may important characteristics of the problem. Stents and catheter are two medical devices that are useful to widen the stenosed arteries. Present investigation may be helpful to locate appropriate point in the obstructed tube to transplant catheter and stents in the tube. I so as to this work may be helpful to such medical devices proving a new diagnosis and curing tools to the patients suffering from various cardiovascular diseases.

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