Closed Form Solution forSchrodinger Equation using He's Variational Iteration method

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Abstract

Partial differential equations have been widely used to describe different physical models in the field of engineering and sciences. Schrodinger equation has a significant role to illustrate the behavior of the short pulses of optical solitons. This equation has a number of applications in optical ultra waves, optical devices – sources and detectors, quantum mechanics and solitons waves etc. In this manuscript, we have provided the closed form solution of linear partial Schrodinger differential equation via He's Variational Iteration Method (VIM).This method provides rapid convergent series solution and also circumvents round off errors. The proposed solution can be used to study the solitary wave phenomena of optical solitons.

Keywords: *Schrodinger Equation, He's Variational Iteration Method, Linear Partial Differential Equation, Lagrange Multiplier*

I. INTRODUCTION

Due to the rapid development inadvanced networks, there are enormous requirements of super communication systems to supplyheavy bandwidth for supportingbig data at higher speed. Optical networks guarantee the solution for huge transmission ability at larger transmission distance [1]. The fatalities for light propagate in fibers are incredibly small for modern single-mode silica fibers, so that many tons of kilometers can be bridged without amplifying the signals [2]. This equation was firstly derived in [3] and widely used in optics waveguide, light sources, ultrafast signal routing, telecommunication system, molecular biology, hydrodynamics etc. [4-6]. The Auxiliary Equation method [7], Darboux-Bäcklund Transformation method [8].

In this manuscript, we are addressing He's VariationalIterational Method (VIM) to obtain the solitary solution of nonlinear Schrodinger equations. A correctional function by a general Lagrange Multiplier identified by variational theory has been constructed to find the closed form solution of these equations without any unphysical restrictive assumptions.To the best of our knowledge, these types of equations have not been solved using VIM in the literature. It also provides the solution with faster rate of convergence, less storage and do not require unphysical restrictive assumptions including discretization of space & time variable.

II. BRIEF DESCRIPTION OF VIM

General non linear differential equation

$$ ły + ŋy = r(y) \qquad (1)$$

Where r(y) represents a non-homogeneous term, Ł and ɳ are linear and non linear operators respectively. Now, consider a correction functional as

$Y_{p+1}(y) = y_p(y) + \int_0^y \lambda\, Ł(y_p(\eta) + ɳ\, \widetilde{y_p}(\eta) - r(\eta))\, dy$ (2)

where λ is a Lagrange multiplier, p represents the p^{th} approximation, $\widetilde{y_p}$ can be shown as a restricted variation i.e. $\delta\, \widetilde{y_p} = 0$. The successive approximation y_{p+1}, $p \geq 0$ of the solution y can be readily obtained by the determined Lagrange multiplier and any selective $zero^{th}$ approximation y_0, consequently, the solution is given by

$y = \lim_{p\to\infty} y_p$ (3)

III. SOLITARY SOLUTION OF NONLINEAR SCHRODINGER EQUATION BY USING VIM

Let us consider the Nonlinear Schrödinger equation is

$$i\emptyset_t = -\frac{1}{2}\emptyset_{xx} + (|\emptyset|^2 + \cos^2 x)\emptyset = 0$$

Subject to condition $\emptyset(x,0) = \sin x$ (4)

In view of VIM, choose initial component$\emptyset_0 = sinx$, correctional functional will be:

$$\emptyset_{k+1}(x,t) = \emptyset_k(x,t) + \int_0^t \lambda\, [\frac{\partial}{\partial\xi}(\emptyset_k) + \frac{1}{2i}\frac{\partial^2}{\partial x^2}(\emptyset_k) - \frac{\emptyset_k}{i}(\cos^2 x) - \frac{1}{i}\widetilde{\emptyset_k}/\emptyset_k^2]\, d\xi$$

On taking variation w.r.t independent variables $\emptyset_k$ with $N(\widetilde{\emptyset_k}) = 0$ $\&\lambda = -1$

$$\emptyset_{k+1}(x,t) = \emptyset_k(x,t) - \int_0^t \frac{1}{2i}(-sinx) - \frac{sinx}{i}(\cos^2 x)\, d\xi \quad (5)$$

The solutions components as follows:

$$\emptyset_1 = sinx + [\frac{1}{2i}(sinx) + \frac{sinx}{i}(\cos^2 x)]t$$

$$= sinx\,[\,1 + \frac{1}{2i}t + (\cos^2 x)/it$$

$$= sinx\,[\,1 + \frac{1}{2i}t + (1 - \sin^2 x)/it$$

$$= sinx\; [1 + \frac{3}{2i}t] \text{ after neglecting the highest power terms} \quad (6)$$

Similarly$\emptyset_2 = sinx\,[\,1 - \frac{3}{2}it + \frac{(-\frac{3}{2}it)^2}{2!}]$

$$\emptyset_3 = sinx\,[\,1 - \frac{3}{2}it + \frac{(-\frac{3}{2}it)^2}{2!} + \frac{(-\frac{3}{2}it)^3}{3!}]$$

After taking limit $k \to \infty$, solution will be

$$\emptyset(x,t) = \sin x\, e^{-\frac{3}{2}it} \quad (7)$$

IV. CONCLUSION

In this manuscript Schrodinger equation has been solved using He's Variational Iteration Method (VIM). The proposed method gives the closed solution with fast convergence for solitary waves as compare to laborious calculations and great complexity of existing techniques. The same method can also extend for two, three and four dimensional solitary wave equations to demonstrate the exactness. This can also be extended to solve other physics and quantum mechanics equations in future

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