Stress Concentration Factors for Various Specimen with ANSYS

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Abstract

In 1953 R.E.Peterson presented the stress concentration factors for various standard specimens in the form of charts. So far the design community is fully dependent on these charts to find out the stress concentration factors. But these charts are confined to standard 2D and 3D specimens. The stress concentration factors for complex 2Dand 3D structures are not available in the form of charts. This paper deals with validation R.E.Peterson's contribution in the field of stress concentration factors, with latest technologies like Computer Aided Design (CAD) and Finite Element Analysis (FEA) and extended to present stress concentration factors in the form of charts for complex 2D and 3D structures.

The analysis of discontinuities of standard specimens is carried out in ANSYS 14 which is a general and popular finite element analysis program. The elements considered for standard specimens are 2D plane element, 2D Axi-symmetric element and 3D tetrahedron element. The results obtained from ANSYS are verified with graphical results of R.E.Peterson. Two numerical examples are considered for the extension of the present work to complex objects. Stepped cantilever beam with a circular hole and nozzle junction of a heat exchanger are solved for stress concentration factors with ANSYS. The results are presented in the form of graphs which can be further utilized.

Keywords: Stress concentration, geometric discontinuities, structural analysis, FEA, ANSYS

1. Introduction

In mechanical engineering design strength of the material places a major role. The strength of the material can be obtained theoretically as well as experimentally. The strength of the material depends on various factors such as loading conditions, structure of the material i.e. linear or nonlinear. It also depends on the continuity and discontinuity of the material. The strength of continuous materials can be found from design first principles. The materials with discontinuities such as holes, notches, grooves and sudden change in cross sections cause severe effect on the property of the material. To find the effect of discontinuities in the material, stress concentration factors are utilized [1] [2].

The standard specimens considered to verify the stress concentration factors obtained by R.E. Peterson [3] are as follows.

1.1 Rectangular plate with a transverse circular hole subjected to tensile load

The geometry is as shown in Fig.1.The stress concentration factors for various geometrical conditions like change in 'd/w' ratio are to be obtained. The 'd/w' ratio's considered are 0.1, 0.2, 0.3, 0.4, 0.5 0.6.

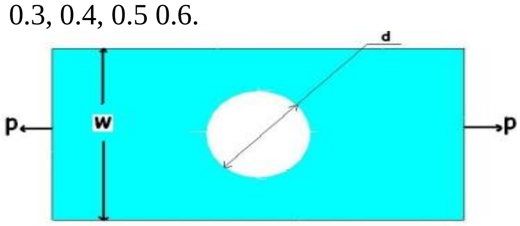


Fig-1

1.2 Flat plate with a shoulder fillet in Tension

The stress concentration factors for various geometrical conditions like ‘D/d’ and ‘r/d’ ratios are to be obtained. D/d ratios considered in this case are 1.1, 1.2, 1.5 and 2. r/d ratios considered in the range from 0.03 to 0.3 as shown in Fig 1.2.

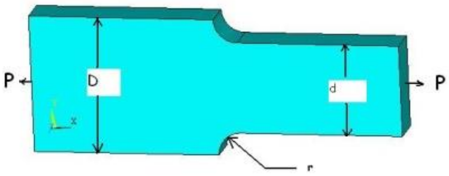


Fig-1.2

1.3. Circular Shaft with Shoulder fillet Subjected to Tension

The stress concentration factors for various geometrical conditions like ‘D/d’ and ‘r/d’ ratios are to be obtained. D/d ratios considered in this case are 1.1, 1.2, 1.5 and 2. r/d ratios considered in the range from 0.03 to 0.3. The geometry is as shown in Fig.1.3.

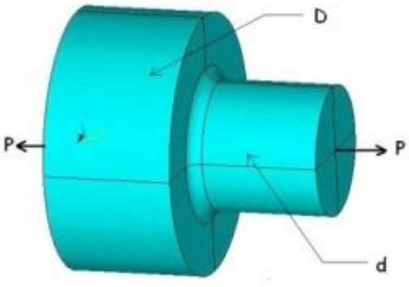


Fig-1.3

1.4. Circular Shaft with Shoulder fillet Subjected to Moment

The stress concentration factors for various geometrical conditions like ‘D/d’ and ‘r/d’ ratios are to be obtained. D/d ratios considered in this case are 1.1, 1.2, 1.5 and 2. r/d ratios considered in the range from 0.03 to 0.3. The geometry is as shown in Fig.1.4.

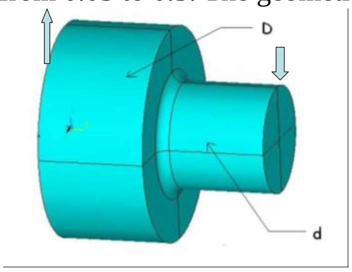


Fig-1.4

1.5. Circular Shaft with Shoulder fillet Subjected to Torsion

The stress concentration factors for various geometrical conditions like ‘D/d’ and ‘r/d’ ratios are to be obtained. D/d ratios considered in this case are 1.1, 1.2, 1.5 and 2. r/d ratios considered in the range from 0.03 to 0.3. The geometry is as shown in Fig.1.5.

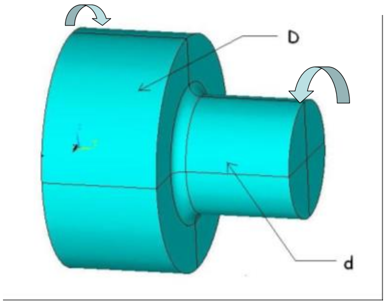


Fig-1.5

The material considered for all the above standard specimens is steel with young's modules E=2 x 10. and Poisson's ratio=0.3.

1.6. Numerical Example- 1. Stepped Cantilever beam with a circular hole

The geometry loading and boundary conditions are as shown in Fig.3.6. The stress concentration factors for different geometrical conditions like **d/w** ratios are to be found. The **d/w** ratios considered for the study are 0.1, 0.2, 0.3, 0.4, 0.5 and 0.6. The geometry is as shown in Fig-1.6.

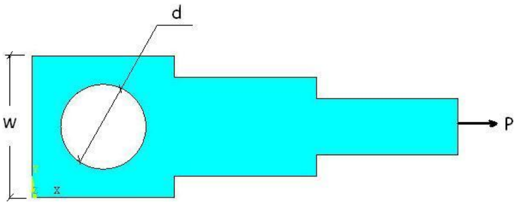


Fig-1.6

1.7. Numerical Example-2: Nozzle junction of a pressure vessel.

Design Data for Nozzle Junction of a Pressure vessel (Fig.1.7).

Pressure vessel shell inner dia = 1000mm,
Pressure vessel shell outer dia = 1046mm
Pressure vessel shell length considered = 750mm
Nozzle inner dia =250m,

Nozzle outer dia = 278m
Nozzle length considered = 300mm,
The Internal pressure = 2.27 N/mm^2

The stress concentration factors from different geometrical variations like D/d ratios are to be obtained. The D/d ratios considered are 4, 4, 5, 5, 5.5 and 6.0.

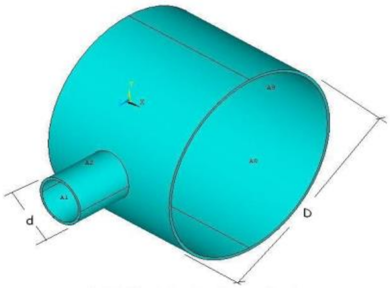


Fig-1.7

found. The **d/w** ratios considered for the study are 0.1, 0.2, 0.3, 0.4, 0.5 and 0.6. The geometry is as shown in Fig-1.7.

2. Theoretical Analysis

The nominal stresses induced in standard specimens and numerical examples considered are obtained from 'design first principles'. The loading conditions for various standard specimens and complex 3D and 2D problems considered are three types.

1. Direct tensile load, P
2. Moment, M at the free end and
3. Torque at the Free End, T

The sample calculations of different variations of geometry and loading are presented below.

Case 1: Plate with circular hole subjected to direct tensile load, P

Model Calculation for d/W = 0.4

Nominal stress, σt = P / (D-d)X t

= 500/ (500-200)X1

= 1.66 N/mm^2

Case 2: Flat plate with shoulder fillet subjected to direct tensile load, P.

Model Calculation for d/W = 0.4

σt = P / (dX t)

= 500/ (500X1)

= 1.0 N/mm^2

Case 3: Shaft with shoulder fillet Subjected to Tensile loading.

Model Calculation for D/d = 1.5 , r/d = 0.1

Nominal stress , σt = P / ($\pi d^2/4$)

= 500/ ($\pi 500^2/4$)

= 0.00254 N/mm^2

Case 4: Shaft with shoulder fillet Subjected to Moment, M.

Model Calculation for D/d = 1.5 , r/d = 0.1

Maximum Bending Moment,

Mb = P X L

= 500 X 500 Nmm

Y = d/2 = 500/2 = 250 mm

I = ($\pi d^4/64$)

= ($\pi 500^4/64$)

Maximum Bending Stress,

σb = Mb Y / I

= 0.02037 N/mm^2

Case 5: Shaft with shoulder fillet Subjected to Torque, T.

Model Calculation for D/d = 1.5 , r/d = 0.1

Torque , T = P X L

= (500X2) X (500X2) mm^2

r = d/2 = 500/2 = 250 mm

Polar Moment of Inertia, J = ($\pi d^4/32$)

= ($\pi 500^4/32$)

Maximum Shear Stress,

τ_{max} = **T X r / J**

= 0.040743 N/mm^2

Numerical Example 1: Stepped Cantilever with circular hole

Model Calculation for d/W = 0.5

Nominal Stress , σt = P / (D-d)X t

= 500/ (500-250)X1

= 2.0 N/mm^2

Numerical Example 2: Nozzle Junction of a Pressure Vessel

Model Calculation for D/d = 4.0

Hoop Stress, σh = P D / 2 Xt

= 2.27X 1000 / 2X 23

= **49.34 N/mm^2**

3. Modeling and Meshing

3.1 Modeling

All the standard specimens and numerical examples considered are modelled in ANSYS pre-processor. 2D and 3D modelling techniques are utilized as per the problems of geometrical complexity. The standard specimens like plate with circular hole, flat plate with shoulder fillet, and circular shaft subjected to Tension, are modelled in 2D . But in case of standard specimens like circular shaft subjected to Bending and Torsion, 3D modelling followed. Because of the geometrical complexity the nozzle junction of a pressure vessel is also modelled in 3D. Whereas the stepped cantilever with a circular hole is presented in 2D. They are as shown in Fig. 3.1 to 3.6

3.2 Meshing

The standard specimens like plate with circular hole, flat plate with shoulder fillet and stepped cantilever beam with circular hole are meshed with 2D plane element.

The remaining cases, i.e. Circular shaft subjected to Bending and Torsion and Nozzle Junction of a pressure vessel are meshed with 3D tetrahedron elements. All the meshed models along with boundary conditions are discussed and presented in the next article.

3.3 Loading and Boundary conditions

Case1. Plate with circular hole subjected to Tension.

In this case, geometrical and loading symmetry about the vertical center line passing through hole is considered. So the left edge is applied with symmetry boundary condition on line. The tension load P=500 N is applied on the right edge (Distributed to all nodes on the edge) as shown in Fig.3.1

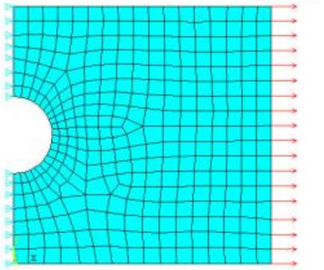


Fig-3.1

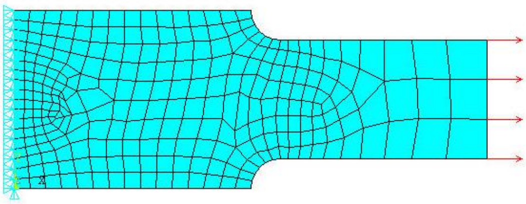
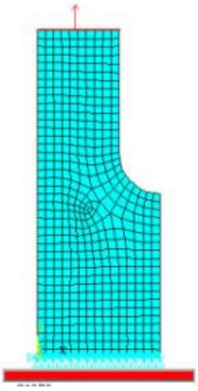


Fig-3.2

Case 2: Flat plate with shoulder fillet subjected to Tension.

This case also, symmetry boundary conditions are followed as the model is symmetrical about horizontal axis. The tension load P=500 N is applied as in case-1, is shown in Fig-3.2 .

Case 3: Circular shaft with shoulder fillet subjected to Tension
The loading and boundary conditions in this case are as shown in Fig.3.3.



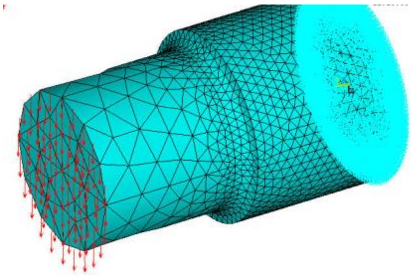
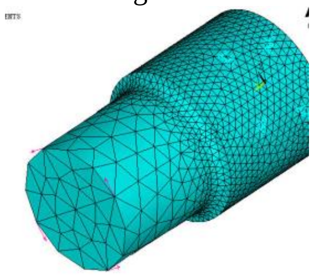


Fig-3.3 **Fig-3.4**

Case 4: Circular shaft with shoulder fillet subjected to Bending.
One end of the shaft is fixed and on the other free end a vertical load P =500 N is applied to provide Bending Movement to the shaft. This is presented in Fig.3.4

Case 5: Circular shaft with shoulder fillet subjected to Torque.
The boundary conditions are similar to previous case. The torque is applied as four equal (P=500N) and opposite loads applied circumferentially at the Free End of the shaft as shown in Fig.3.5



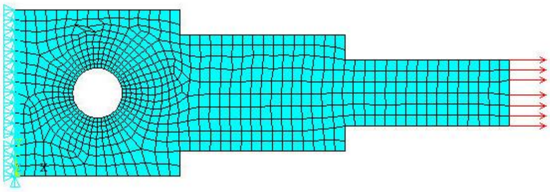


Fig-3.5 **Fig-3.6**

Case 6: Stepped Cantilever Beam subjected to Tension.
Since this is a cantilever beam, the left end is fixed in all degree of freedom. The tension load (P=500N) is applied at the free end is shown in Fig.3.6

4. Discussion on Results

The results are in the form of stress contours of various cases i.e. for standard specimens and considered numerical problem are obtained. Again in each case different variations like (r/d, D/d and w/D ratios) are observed to study the effect of discontinuity on the property of the material. All the results are obtained, only one in each case is presented (Fig. 4.1 to Fig.4.6) for want of space. All the results are tabulated as shown in Tables 1 to 6.

The results obtained with ANSYS are closely followed the standard design data presented by R.E.Peterson. Both results are plotted on graphs for clear observation and to understand the deviations. They are presented in Fig. 4.8 to Fig. 4.14 for the six cases considered for the study. The first five cases are for standard specimens and the remaining one is for complex object.

In the first case, i.e. Rectangular plate with a circular hole subjected to tensile load, presented in Fig. 4.8. The percentage of variation from Design Data Graph to ANSYS has come down with increase in d/W ratio. This is due to the fact that less d/W ratio means more complexity in Geometry hence more stress concentration factors.

The results for second case, i.e. Flat plate with shoulder fillet in tension are almost coincident with design data values. The Fig. 4.9 presents stress concentration factors for D/d ratio 1.2 and r/d ratios ranging from 0.02 to 0.10.

There is very little variation in the ANSYS results in case of shaft with shoulder fillet subjected to Tension and Bending Moment. This can be observed in Fig. 4.10 and Fig.4.11

But the shaft subjected to Torque has got varied results (Fig. 4.12). The variation is more when r/d ratio is 0.1 and is less with r/d ratio 0.3. This can be attributed to the loading conditions in ANSYS. The torque applied as four circumferential forces at the free end of the shaft. If the forces are more uniformly distributed over the circumference, the variation may come down.

Fig. 4.13 presents smooth curve between stress concentration factors and d/W ratios for a stepped cantilever beam with a circular hole. The higher the d/W ratio, the lesser Kt values observed. This is in line with the theory of stress concentrations.

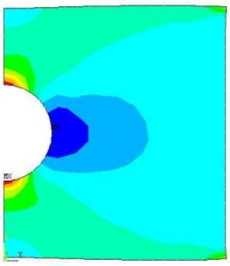


Fig-4.1

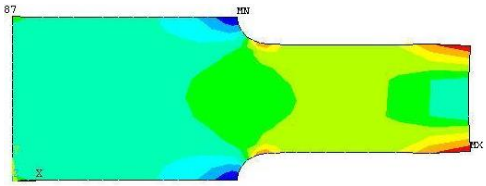


Fig-4.2

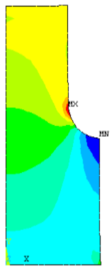


Fig-4.3

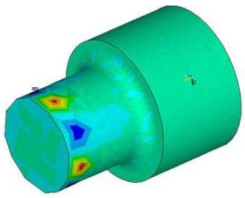


Fig4.4

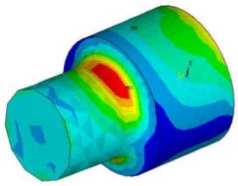


Fig-4.5

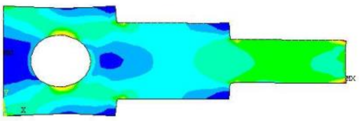


Fig-4.6

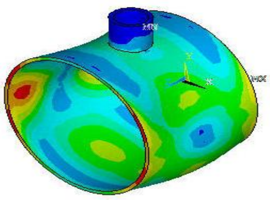


Fig-4.7

Table No. 1
Case1. Plate with circular hole subjected to Tension.

S.No.	d/W	Max Stress N/mm^2	Nominal Stress N/mm^2	Kt ANSYS	Kt GRAPH
1	0.1	2.87	1.11	2.58	2.7
2	0.2	3.03	1.25	2.42	2.51
3	0.3	3.26	1.48	2.30	2.36
4	0.4	3.81	1.66	2.30	2.25
5	0.5	4.24	2.0	2.12	2.16
6	0.6	5.18	2.5	2.07	2.08

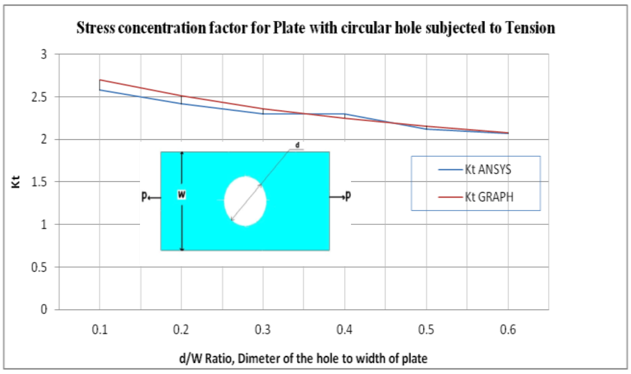


Fig-4.8

Table No. 2
Case 2: Flat plate with shoulder fillet subjected to Tension.

S.N	D/d	r/d	Max Stress N/mm^2	Nominal Stress N/mm^2	Kt ANSYS	Kt GRAPH
1	1.1	0.03	2.2167	1	2.2167	2.24
		0.04	2.0707	1	2.0707	2.08
		0.05	2.0606	1	2.0606	1.95
2	1.2	0.04	2.4219	1	2.4219	2.41
		0.06	2.1914	1	2.1914	2.12
		0.08	2.0673	1	2.0673	1.9
3	1.5	0.10	2.1918	1	2.1918	2.07
		0.15	1.9336	1	1.9336	1.84
		0.20	1.4527	1	1.4527	1.43
4	2.0	0.1	2.2911	1	2.2911	2.27
		0.2	1.8785	1	1.8785	1.89
		0.3	1.7093	1	1.7093	1.66

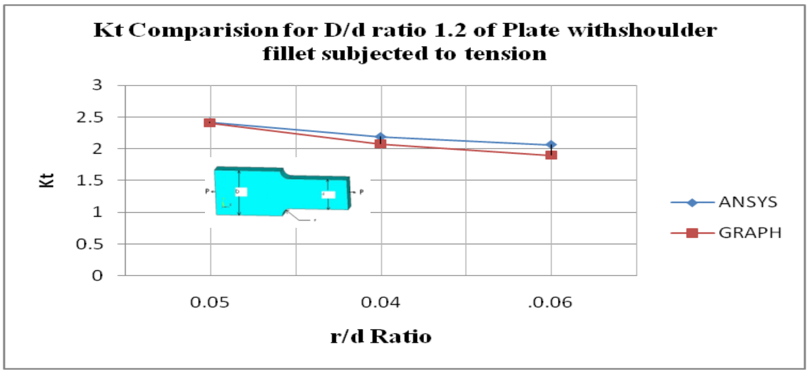


Fig-4.9

Table No. 3

Case 3: Circular shaft with shoulder fillet subjected to Tension

S. No.	D/d	r/d	Maximum Stress N/mm^2	Nominal Stress N/mm^2	Kt	
					ANSYS	Graph
1	1.1	0.03	0.005541	0.00254	2.18	2.05
		0.04	0.004828	0.00254	1.90	1.91
		0.05	0.004653	0.00254	1.83	1.85
2	1.2	0.04	0.005612	0.00254	2.20	2.22
		0.06	0.004991	0.00254	1.96	1.98
		0.08	0.00467	0.00254	1.83	1.85
3	1.5	0.10	0.004838	0.00254	1.90	1.95
		0.15	0.004283	0.00254	1.68	1.75
		0.20	0.003936	0.00254	1.54	1.59
4	2.0	0.1	0.005031	0.00254	1.98	2.11
		0.2	0.004167	0.00254	1.64	1.7
		0.3	0.00376	0.00254	1.48	1.48

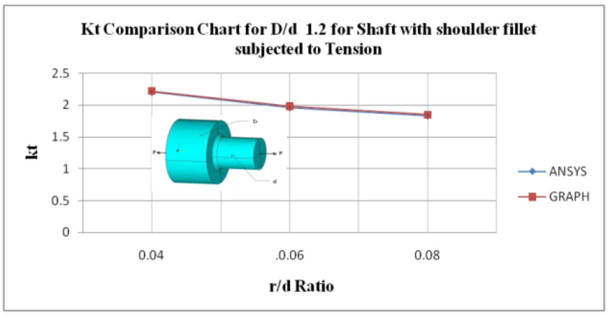


Fig-4.10

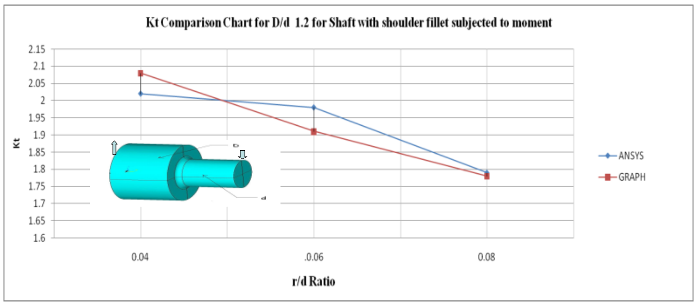


Fig-4.11

Table No. 4
Case 4: Circular shaft with shoulder fillet subjected to Bending

S. No.	D/d	r/d	Maximum Stress N/mm^2	Nominal Stress N/mm^2	Kt	
					ANSYS	Graph
1	1.1	0.03	0.040537	0.0203	1.99	2.05
		0.04	0.039316	0.0203	1.93	1.92
		0.05	0.037531	0.0203	1.84	1.85
2	1.2	0.04	0.041147	0.0203	2.02	2.08
		.0.06	0.040429	0.0203	1.98	1.91
		0.08	0.036626	0.0203	1.79	1.78
3	1.5	0.10	0.034014	0.0203	1.66	1.71
		0.15	0.032123	0.0203	1.57	1.55
		0.20	0.029079	0.0203	1.42	1.44
4	2.0	0.1	0.035744	0.0203	1.75	1.76
		0.2	0.028923	0.0203	1.42	1.47
		0.3	0.027545	0.0203	1.35	1.34

Table No. 5
Case 5: Circular shaft with shoulder fillet subjected to Torque.

S. No	D/d	r/d	Maximum Stress N/mm^2	Nominal Stress N/mm^2	K_t	
					ANSYS	Graph
1	1.1	0.03	0.049787	0.04074	1.22	1.46
		0.04	0.043413	0.04074	1.06	1.40
		0.05	0.053112	0.04074	1.30	1.37
2	1.2	0.04	0.043974	0.04074	1.07	1.69
		0.06	0.053522	0.04074	1.31	1.54
		0.08	0.054267	0.04074	1.33	1.45
3	1.5	0.10	0.046452	0.04074	1.14	1.46
		0.15	0.047677	0.04074	1.17	1.33
		0.20	0.049141	0.04074	1.20	1.26
4	2.0	0.10	0.055716	0.04074	1.36	1.51
		0.20	0.048041	0.04074	1.17	1.31
		0.30	0.057428	0.04074	1.40	1.18

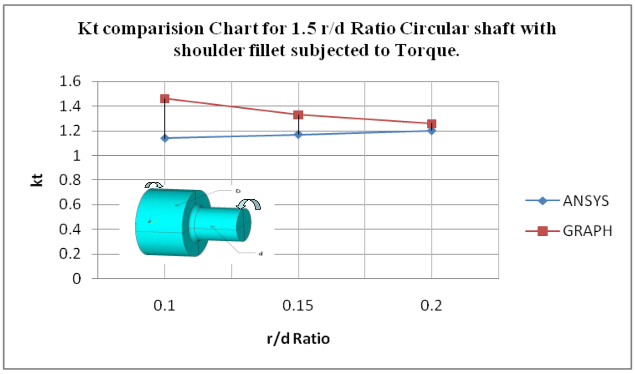


Fig-4.12

Table No. 6
Case 6: Stepped Cantilever Beam subjected to Tension

S.No	D/d	Maximum Stress N/mm^2	Nominal Stress N/mm^2	Kt With ANSYS
1	0.1	4.47864	1.11	4.03
2	0.2	4.47649	1.25	3.58
3	0.3	4.47651	1.42	3.15
4	0.4	4.47865	1.66	2.69
5	0.5	5.10727	2.00	2.55

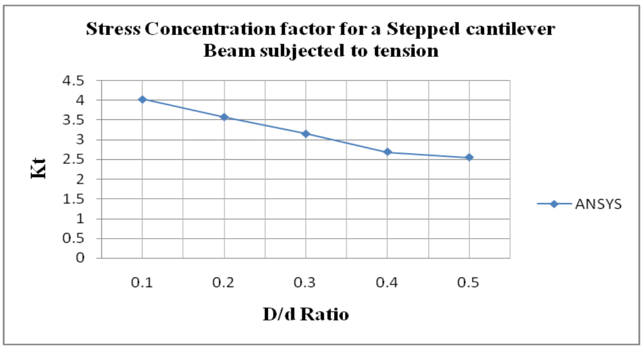
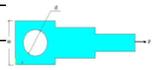


Fig-4.13

Table No. 7
Case 7: Nozzle Junction of a Pressure Vessel subjected to internal pressure

S.No	D/d	Maximum Stress N/mm^2	Nominal Stress N/mm^2	Kt With ANSYS
1	4.0	108.099	49.34	2.19
2	4.5	 [illegible]	49.34	2.149
3	5.0	[illegible]	49.34	2.144
4	5.5	105.123	49.34	2.13
5	6.0	102.64	49.34	2.08

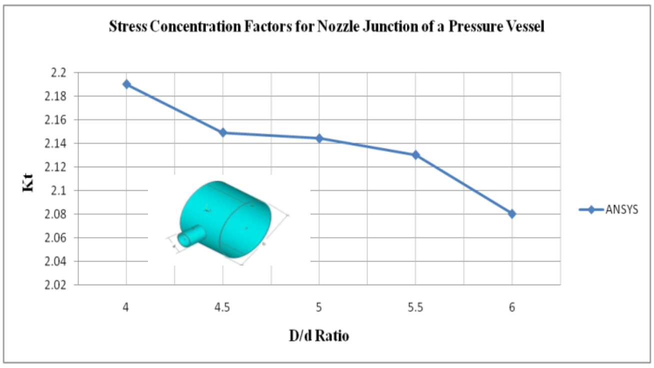


Fig-4.14

5. Conclusion

The observations and studies with ANSYS Finite Element Analysis approach to find stress concentration factors can be seriously considered for any analysis, since the values obtained with ANSYS are in line with Standard Design Data tables and Graphs.
This method can be applied to find out stress concentration factors of any Geometrically Complex objects. This method is reliable, feasible and also economical.

6. Future Scope of Work

This work can be extended considering the thickness variation of the plates and shells which has not been considered in the present study owing to time constraints. The study of stress concentration factors for different materials with same geometry discontinuities can be undertaken. The work on shaft subjected to Torque can be better studied to obtain good results.

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