Some properties of contra #αrg- continuous functions.

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ABSTRACT

In this paper we introduce and investigate some classes of generalized-functions called contra #αrg-continuous functions. We get several characterizations and some of their properties. Also we investigate its relationship with other types of functions.

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1.INTRODUCTION

In 1996, Dontchev [2] presented a new notion of continuous function called contra-continuity. This notion is a stronger from of LC-continuity. The purpose of this present paper is to define a new class of generalized continuous functions called contra #αrg-continuous and investigate some of their properties.

2. PRELIMINARIES

Throughout the paper X and Y denote the topological spaces (X,τ) and (Y,σ) respectively and on which no separation axioms are assumed under otherwise explicitly stated. For any subset A of a space (X,τ), the closure of A, interior of A and the complement of A are denoted by cl (A), int(A) and A^c or X\A respectively. (X,τ) will be replaced by X if there is no chance of confusion. Let us recall the following definitions as pre requesters.

Definition 2.1. A subset A of a space X is called
1) a preopen set [8] if A⊆intcl(A) and a preclosed set if clint (A) ⊆A.
2) a semi open set [5] if A⊆clint(A) and a semi closed set if intcl (A) ⊆A.
3) a regular open set [8] if A=intcl (A) and a regular closed set if A = clint (A).
4) a regular semi open [1] if there is a regular open U such U⊆A⊆cl(U).
5) an α-open set [13] if A ⊆ int(cl(int(A))) and an α-closed set if cl(int(cl(A))) ⊆A.

Definition 2.2. A subset A of (X,τ) is called
1) rw-closed [14] if cl A ⊆U whenever A ⊆U and U is regular semi open.
2) #rg-closed [10] if cl(A)⊆U whenever A⊆U and U is rw-open.
3) #αrg-closed [13] if αcl(A)⊆U whenever A⊆U and U is rw-open.

Definition 2.3. A space X is said to be strongly S-closed[2] if every closed cover of X has a finite subcover.

Definition 2.4. A topological space X is said to be ultra normal[7]if each pair of nonempty disjoint closed sets can be separated by disjoint clopen sets.

Definition 2.5. A topological space X is said to be ultra Hausdorff[7] if for each pair of distinct points x and y in X there exist clopen sets A and B containing x and y respectively such that A∩B=ϕ In this chapter, we study some of their properties of contra #αrg – continuousfunction.

3. Properties of contra #αrg- continuous functions.

Defintion 3.1. The #αrg-frontier of a subset A of a space X is given by #αrg-fr(A)= #αrg-cl(A)∩ #αrg-cl(X\A).

Theorem 3.2. Let the collection of all #αrg-closed sets of space (X,τ) be closed under arbitrary intersections. The set of all points x∈X at which a function f: (X,τ) ⟶(Y,σ) is not contra #αrg-continuous is identical with the union of #αrg-frontier of the inverse images of closed sets containing f(x).

Proof: Necessity: Suppose that f is not contra #αrg-continuous at x∈X. Then there exists a closed set A of Y containing f(x) such that f(U) is not contained in A for every U∈#αRGO(X) containing x. Then U∩(X\f^{-1}(A))≠ ϕ for every U∈#αRGO(X) f^{-1}(A)⊂#αrg-cl(f^{-1}(A)) and hence x∈#αrg-fr(f^{-1}(A)).

Sufficiency: Suppose that f is contra #αrg-continuous at x∈X, and let A be a closed set of Y containing f(x). Then there exists U∈#αRGO(X) containing x such that U⊂f^{-1}(A): hence x∈#αrg-int(f^{-1}(A)). Therefore, x∉#αrg-fr(f^{-1}(A)) for each closed set A of Y containg f(x). This completes the proof.

Corollary 3.3. Let #αRGC(X) be closed under arbitary intersections. A function f:X⟶Y is not contra #αrg-continuous at x if and only if x∈#αrg-fr(f^{-1}(F)) for some F∈C(Y,f(x)).

Definition 3.4.A space X is said to be #αrg-T_1 if for each pair of distinct points x and y in X, there exist #αrg-open sets U and V containing x and y, respectively, such that y∉U and x∉V.

Definition 3.5.A space X is said to be #αrg-T_2 if for each pair of distinct points x and y in X, there exists U∈#αRGO(X,x) and V∈#αRGO(X,y) such that U∩V=ϕ.

Theorem 3.6. Let X and Y be topological spaces. If

(1) For each pair of distinct points x and y in X, there exists a function f of X into Y such that f(x)≠f(y)
(2) Y is an Urysohn space and f is contra #αrg-continuous at x and y, then X is #αrg-T_2.

Proof: Let x and y be distinct points in X. Then, there exists a Urysohn space Y and a function f:X⟶Y such that f(x) ≠f(y) and f is contra #αrg-continuous at x and y. Let z=f(x) and v=f(y). Then z≠v. We have to prove X is #αrg-T_2 space. Since Y is Urysohn, there exist open sets V and W containing z and v, respectively such that cl(V)∩cl(W)=ϕ. Since f is contra #αrg-continuous at x and y, then there exists #αrg-open sets A and B containg x and y, respectively such that, f(A)⊂cl(V) and f(B) ⊂cl(W). We have A∩B=ϕ, since cl(V) ∩cl(W)=ϕ. Hence X is #αrg-T_2.

Corollary 3.7. Let f:X⟶Y be a contra #αrg-continuous injection. If Y is an Urysohn space, then it is #αrg-T_2.

Theorem 3.8. For a topological space X, the following properties are equivalent:

(1) X is #αrg-connected.
(2) The only subsets of X which are both #αrg-open and #αrg-closed are the empty set ϕ and X.
(3) Each contra #αrg - continuous function of X into a discrete space Y with at least two points is a constant function.

Proof. (1)⟹(2) Suppose A⊂X is a proper subset which is both #αrg-open and #αrg-closed. Then its complement X\A is also #αrg-open and #αrg-closed. Then X=A∪(X\A) is a disjoint union of two nonempty #αrg-open sets which contradicts the fact that X is #αrg-connected. Hence, A=ϕ of X.

(2) ⟹(1) Suppose X=A∪B where A∩B=ϕ, A≠ ϕ, B≠ ϕ and A and B are #αrg-open. Since A=X\B, A is #αrg-closed. But by hypothesis A=ϕ, which is a contradiction. Hence (1) holds.

(2) ⟹(3) Let f:X⟶Y be a contra #αrg-continuous function where Y is a discrete space with atleast two points. Then f^{-1}({y}) is #αrg-closed and #αrg-open for each y∈Y and X=∪{f^{-1}({y}):y∈Y}. By hypothesis, f^{-1}({y})=ϕ or X. If f^{-1}({y}=ϕ for all y∈Y, f is not a function. Also there connot exist more than y∈Y such that f^{-1}({y})=X. Hence there exists only one y∈Y such that

f^{-1}({y}) = X and f^{-1}({y_1})=ϕ where y≠y_1∈Y. This shows that f is a constant function.

(3) ⟹(2) Let P be both #αrg-open and #αrg-closed in X. Suppose P≠ ϕ. Let f:X⟶Y be a contra #αrg-continuous function defined by f(P)={a} and f(X\P)={b} where a≠b and a, b∈Y. By hypothesis, f is constant. Therefore, P=X.

Theorem 3.9. If f is a contra #αrg-continuous function from a #αrg-connected space X onto any space Y, then Y is not a discrete space.

Proof: Suppose that Y is discrete. Let A be a proper nonempty clopen subset of Y. Then f^{-1}(A) is a proper nonempty #αrg-clopen subset of X, which is a contradiction to the fact that X is #αrg-connected.

Theorem 3.10. A space X is #αrg-connected if every contra #αrg-continuous function from a space X into any T_0-space Y is constant.

Proof: Suppose that X is not #αrg-connected and that every contra #αrg-continuous function from X into Y is constant. Since X is not #αrg-connected, there exists proper nonempty #αrg-clopen subset A of X. Let Y={a,b} and τ={Y, ϕ,{a},{b}} be a topology for Y. Let f:X⟶Y be a function such that f(A)={a} and f(X\A)={b}. Then f is non-constant and contra #αrg – continuous such that Y is T_0 which is a contradiction. Hence, X must be #αrg-connected.

Theorem: 3.11. If f:X⟶Y is a contra #αrg-continuous surjection and X is #αrg-connected, then Y is connected.

Proof: Suppose that Y is not a connected space. There exists nonempty disjoint open sets V_1 and V_2 such that Y=V_1∪V_2. Therefore, V_1 and V_2 are clopen in Y. Since f is contra #αrg-continuous, f^{-1}(V_1) and f^{-1}(V_2) are #αrg-open in X. Moreover, f^{-1}(V_1) and f

$^{1}(V_2)$ are nonempty disjoint and X= $f^{-1}(V_1) \cup f^{-1}(V_2)$. This shows that X is not #αrg-connected. This contradicts that Y is not connected assumed. Hence, Y is connected.

Theorem: 3.12. Let p: X x Y⟶X be a projection. If A is #αrg-closed subset of X, then $p^{-1}(A)$=A x Y is #αrg-closed subset of X x Y.

Proof: Let A x Y ⊂ U and U be rw-open subset of X x Y. Then U = V x Y for some rw-open set of X. Since A is #αrg-closed in X, αcl(A)⊂V and so αcl(A) x Y⊂VxY=U, ie., αcl(AxY) ⊂U. Hence, A x Y is #αrg-closed subset of X x Y.

Proposition: 3.13. If f:X⟶Y is a #αrg-irresolute surjection and X is #αrg-connected then Y is #αrg-connected

Propostion: 3.14. If the product space of two nonempty topological spaces is #αrg-connected, then each factor space is #αrg-connected.

Proof: Let X x Y be the product space of the nonempty spaces X and Y and X x Y be #αrg-connected. The projection p: X x Y⟶ X is #αrg-irresolute and then p(XxY)=X is #αrg-connected. The proof for the space Y is similar to the case of X.

Definition 3.15. A space X is said to be

(i) #αrg-compact if every #αrg-open cover of X has a finite subcover,
(ii) Countable #αrg-compact if every countable cover of X by #αrg-open sets has a finite subcover.
(iii) #αrg-Lindelof if every #αrg-open cover of X has a countable subcover.

Theorem 3.16. If f:X⟶Y is contra #αrg-continuous and A is #αrg-compact relative to X, then f(A) is strongly S-closed in Y.

Proof: Let $\{V_i : \in I\}$ be any cover of f(A) by closed sets of the subspace f(A). For each i∈I, there exists a closed set A_i of Y such that $V_i=A_i \cap f(A)$. For each x∈A, there exists i(x) ∈ I such that $f(x) \in A_{i(x)}$ and there exists $U_x \in \#\alpha RGO(X,x)$ such that $f(U_x) \subseteq A_{i(x).}$ Since the family $\{U_x: x \in A\}$ is a cover of A by #αrg-open sets of X, there exists a finite subset A_0 of A such that $A \subseteq \cup\{U_x : x \in A_0\}$. Hence we obtain $f(A) \subseteq \cup\{f(U_x) : x \in A_0\}$ which is a subset of $\cup(A_{i(x)} : x \in A_o\}$. Thus $f(A)=\cup\{V_{i(x)} : x \in A_o\}$ and hence f(A) is strongly S-closed.

Corollary 3.17. If f:X⟶Y is contra #αrg-continuous surjection and X is #αrg-compact, then Y is strongly S-closed.

Theorem 3.18. If the product space of two nonempty topological spaces is #αrg-compact, then the factor space is #αrg-compact.

Proof: Let X x Y be the product space of the nonempty spaces X and Y and X x Y be #αrg – compact. The projection p: X x Y⟶X is #αrg-irresolute and then p(XxY)=X is #αrg-compact. The Proof for the space Y is similar to the case of X.

Theorem: 3.19. The contra #αrg-continuous images of #αrg-lindelof (resp. countably #αrg-compact) spaces are strongly S-lindelof (respectively strongly countable S-closed).

Proof: Let f:X⟶Y be a contra #αrg – continuous surjection. Let $\{V_i : i \in I\}$ be any closed cover of Y. Since f is contra #αrg-continuous, then $\{f^{-1}(V_i) : i \in I\}$ is a #αrg – open cover of X and hence there exists a countable subset I_0 of I such that $X=\cup\{f^{-1}(V_i) : i \in I_0\}$. Therefore, we have $Y= \cup\{V_i : i \in I_0\}$ and Y is strongly S-Lindelof.

Definition: 3.20. The graph G(f) of a function f: X⟶Y is said to be contra #αrg-graph if for each (x,y) ∈ (X xY)\G(f), there exist a #αrg-open set A in X containing x and a closed set B in Y containing y such that (A x B) ∩G(f) = ϕ.

Proposition:3.21. The following properties are equivalent for the graph G(f) of a function f:

(i) G(f) is contra #αrg-graph;
(ii) for each (x,y) ∈ (X x Y)\G(f), there exists a #αrg-open set A in X and a closed set B in Y containing y such that f(A)∩B=ϕ.

Theorem: 3.22. If f: X⟶Y is contra #αrg-continuous and Y is Uryshon, G(f) is contra #αrg-graph in X x Y.

Proof: Let (x,y) ∈ (X x Y)\G(f). It follows that f(x)≠y. Since Y is Urysohn, there exists open sets B and C such that f(x) ∈B, y∈C and cl(B)∩cl(C)= ϕ. Since f is contra #αrg-continuous, there exists a #αrg-open set A in X containing x such that f(A)⊆cl(B). Therefore, f(A) ∩cl(C)= ϕ and G(f) is contra #αrg-graph in X x Y.

Theorem: 3.23. Let f: X⟶Y be a function and g: X⟶X x Y the graph function of f, defined by g(x)=(x,f(x)) for every x∈X. If g is contra #αrg-continuous, then f is contra #αrg-continuous.

Proof: Let U be an open set in Y, then X x U is an open set in X x Y. It follows that $f^{-1}(U)=g^{-1}(X \times U)$ ∈/#αRGC(x). Thus f is contra #αrg-continuous.

Theorem: 3.24. If f: X⟶Y and g: X⟶Y are contra #αrg-continuous and Y is Urysohn, then E={x∈X:f(x) = g (x) is #αrg-closed in X.

Proof: Let x∈X\E. Then f(x) ≠ g(x). Since Y is Urysohn, there exist open sets V and C such that f(x) ∈V, g(x) ∈C and cl(V))∩cl(C)= ϕ. Since f and g are contra #αrg-continuous, f^{-1}(cl(V) ∈#αRGO(X) and g^{-1}(cl(C)) ∈#αRGO(X). Let U=f^{-1}(cl(V)) and G=g^{-1}(cl(C)). Then U and V contain x. Set A=U∩G. A is #αrg-open in X. Hence f(A) ∩g(A)= ϕ and x∉#αrg-cl(E). Thus E is #αrg-closed in X.

Definition: 3.25.A subset A of a topological space X is said to be #αrg-dense in X if #αrg-cl(A)=X.

Theorem 3.26. Let f:X⟶Y and g: X⟶Y be a functions. If

(i) Y is Urysohn,
(ii) f and g are contra #αrg-continuous.
(iii) f=g on #αrg-dense set A⊂X, then f=g on X.

Proof: Since f and g are contra #αrg-continuous and Y is Urysohn, by the previous theorem E={x∈X :f(x)=g(x)} is #αrg-closed in X. We have f=g on #αrg-dense set A⊂X. Since A⊂E and A is #αrg-dense set in X, then X = #αrg-cl(A) ⊂#αrg-cl(E)=E. Hence, f= g on X.

Definition:3.27. A space X is said to be weakly Hausdorff[9] if each element of X is an intersection of regular closed sets.

Theorem: 3.28. If f: X⟶Y is a contra #αrg- continuous injection and Y is weakly Hausdorff, then X is #αrg-T_1.

Proof: Suppose that Y is weakly Hausdorff. For any distinct points x and y in X, there exists regular closed sets A,B in Y such that f(x)∈A, f(y) ∉A, f(x) ∉ B and f(y) ∈B. Since f is contra #αrg- continuous, $f^{-1}(A)$ and $f^{-1}(B)$ are #αrg-open subsets of X such that $x\in f^{-1}(A)$, $y\notin f^{-1}(A)$, $x\notin f^{-1}(B)$, $y\in f^{-1}(B)$. This shows that X is #αrg-T_1.

Theorem: 3.29. Let f: X⟶Y have a contra #αrg-graph. If f is injective, then X is #αrg-T_1.

Proof: Let x and y be any two distinct points of X. Then, we have $(x,f(y)) \in (X \times Y)\setminus G(f)$. Then, there exist a #$\alpha$rg-open set U in X containing x and $F\in C(Y,f(y))$ such that $f(U)\cap F=\phi$. Hence $U\cap f^{-1}(F)=\phi$. Therefore we have $y\notin U$. This implies that X is #αrg-T_1.

Theorem: 3.30. Let f: X⟶Y be a contra #αrg-continuous injection. If Y is an ultra Hausdorff space, then X is #αrg-T_2.

Proof: Let x and y be any two distinct points in X. Then, $f(x)\neq f(y)$ and there exist clopen sets A and B containing f(x) and f(y), respectively such that $A\cap B=\phi$. Since f is contra #αrg-continuous, then $f^{-1}(A) \in \#\alpha RGO(X)$ and $f^{-1}(B) \in \#\alpha RGO(X)$ such that $f^{-1}(A) \cap f^{-1}(B) = \phi$. Hence, X is #$\alpha$rg-$T_2$.

Theorem: 3.31. If f: X⟶Y is a contra #αrg-continuous closed injection and Y is ultra normal, then X is #αrg-normal.

Proof: Let A and B be disjoint closed subsets of X. Since f is a closed injection, f(A) and f(B) are disjoint and closed in Y. Since Y is ultra normal, f(A) and f(B) are separated by disjoint clopen sets C and D, respectively. Thus $A\subseteq f^{-1}(C)$, $B\subseteq f^{-1}(D) \in \#\alpha RGO(X)$ and $f^{-1}(C) \cap f^{-1}(D)=\phi$. Hence, X is #$\alpha$rg-normal.

The complements of the above mentioned closed sets are their respective open sets.

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