Secure Regular Set Domination in Graphs

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ABSTRACT

A set D of vertices in a graph G=(V, E) is said to be a regular set dominating set if for every set I ⊆V− D there exists a non empty set S ⊆ D such that the induced graph <I∪S> is regular and for /I /=1, <I∪S> is 1- regular . A subset D of the vertices of graph G is called a secure dominating set of a graph G if for every vertex v ∈ V–D there exists u ∈ D such that v is adjacent to u and D_1 = (D− {u})∪{v} is a dominating set. A secure dominating set D is an SRSD- dominating set of G if D is also a regular set dominating set of G. The SRSD-domination number of G denoted by $\gamma_{rs}^{s}(G)$ is the minimum cardinality of an SRSD-dominating set. We start the study of SRSD – domination in graphs and obtain some bounds.

Keywords: Domination, Regular set domination, Secure domination, SRSD – domination.

INTRODUCTION

All graphs taken here are finite, undirected with neither loops nor multiple edges. Any undefined term in this paper may be found in T.W.Haynes, S.T.Hedetniemi, P.J.Slater [3].

Let G = (V, E) be a graph. The graph G has 'n' vertices and 'm' edges, thus |V(G)| = n and |E(G)| = m. The complement of G is a graph which has the same vertices as G and two vertices are adjacent if these vertices are not adjacent in G, it is denoted as $\overline{G}$.The open neighborhood of a vertex v of G defined as the set $N_G(v)$ = { u ∈ V(G) ; uv ∈ E(G)}. The degree of vertex v is the cardinality of open neighborhood denoted as $d_G(v)$ [3].

The path on p vertices is denoted as P_p and the cycle on p vertices is denoted by C_p. The wheel on p vertices is a graph which formed by connecting a vertex to remained vertices of a cycle C_{p-1} and is denoted as W_p, p≥4. We denote a complete bipartite graph with partite sets of cardinality m and n by $K_{m,n}$. The graph $K_{1,n}$ is a star [3].

A subset D ⊆ V(G) is a dominating set of G if for each vertex of V−D has a neighbor in D. The domination number is the minimum cardinality of a dominating set of G. For deep survey of domination in graph, see [2, 3].

Let G(V, E) be a connected graph and a set D is subset of V is a set – dominating set if for each set T ⊆ V− D, there exists an S ⊆ D which is non- empty such that the subgraph < S∪T> induced by S∪T is connected and the minimum cardinality of a set dominating set of G is the set domination number denoted by γ_s (G)[5].

In a graph G(V,E), the subset D of V is said to be regular set dominating set if for every set I ⊆ V\ D there exists a non empty set S⊆ D such that induced subgraph < I∪S> is regular and for | I | =1, < I ∪ S> is 1 –regular . The minimum cardinality of the regular set dominating set is regular set dominating number which is denoted by γ_{rs} (G) [4].

A subset D⊆ V(G) is called a secure dominating set of G if for every v ∈ V –D there exists u∈D such that uv∈ E and D_1 = (D –{u}) ∪{v} is a dominating set and the minimum cardinality of F is the secure dominating number which is denoted by γ^s(G) [1].

The concept of secure domination in graphs was introduced by E. J. Cockayne, P. J. P. Grobler, W. R. Gründlingh, J. Munganga, and J. H. van Vuuren [1] by considering the following situation, we want to place guards at selected positions of a museum in such a way that whole positions of museum are covered or adjacent. If one guard out of guards of selected positions changes his position to unselected position, then the new configuration of guards is also covers or adjacent each position of a museum.

We introduce a new variant of secure domination namely the secure regular set domination (SRSD –domination), and we initiate the study of this type of parameter. A secure dominating set D is an SRSF –dominating set of G if D is a regular set dominating set of G. The minimum cardinality of this set is SRSD –dominating number and is denoted by γ_{rs}^s .we consider all graphs with non –isolated vertices.

2. MAIN RESULTS

In this section, we study the secure regular set domination in various graphs i.e., complete graph, cyclic graph, path graph, wheel graph, complete bipartite graph. Moreover, the secure regular set dominating number γ_{rs}^s(G) ≤ n-1 for any graph G of order n. We initiate with the following straightforward observations.

Definition 2.1 Let G(V,E) be a graph. A secure dominating set D of G is called a secure regular set dominating set if for every set I ⊆ V\ D there exists a non empty set S ⊆ D such that induced sub graph < I∪S> induced by I∪S is regular. The minimum cardinality of secure regular set dominating set is called the secure domination number of G and is denoted by $\gamma_{rs}^s(G)$.

Clearly, a dominating set D is secure regular set dominating set if and only if D is secure as well as regular set dominating set.

Observation 2.2. For any connected graph G,

$\gamma(G) \le \gamma_s(G) \le \gamma_{rs}(G) \le \gamma_{rs}^s(G)$.

Observation 2.3. For any graph G without isolated vertices,

$\gamma(G) \le \gamma^s \le \gamma_{rs}^s(G)$.

The following Fig. 1 represents the relationship between the domination parameters.

Observation 2.4. If G is a cyclic graph on p vertices then

$$\gamma_{rs}^s(W_p) = \begin{cases} p-1 \\ 1, p=3 \end{cases}$$

Observation 2.5. If G is path graph on p vertices then

$\gamma_{rs}^s(P_p) = p-1$.

Observation 2.6. If G is complete graph on p vertices then

$\gamma_{rs}^s(K_p) = 1$.

Observation 2.7. If G = W_p is wheel graph on p- vertices, then

$\gamma_{rs}^{s}(W_p) = p-1$.

Observation 2.8. If G = $K_{p,q}$ is complete bipartite graph, then

$\gamma_{rs}^{s}(K_{p,q}) = p+q-1$.

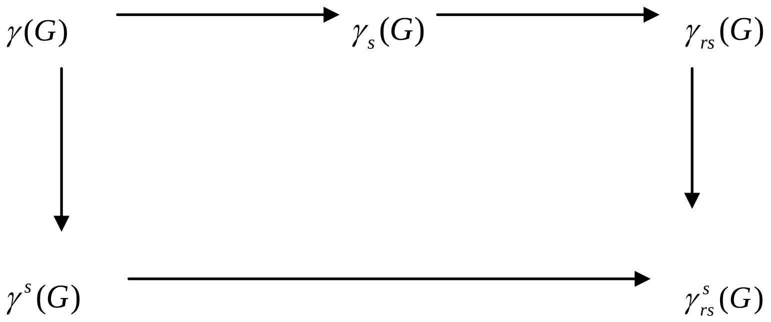

Fig. 1. Domination parameters relationship

We have the following properties for $\gamma_s(G)$, $\gamma_{rs}(G)$ and $\gamma_{rs}^{s}(G)$.

Theorem 2.9. for complete bipartite graph G =$K_{m,n}$,

$\gamma_s(G) = \gamma_{rs}(G) = \gamma_{rs}^{s}(G)$.

Proof. Let D be a regular set dominating set of G. Since, G is a complete bipartite graph implies any regular set dominating set is also a secure regular set dominating set of G. Hence, $\gamma_{rs}(G) = |D| = \gamma_{rs}^{s}(G)$.

Now, we show that D is also set dominating set for G. Clearly, G is connected. Assume that there exists two non –adjacent vertices u and v belong to V\D. Then there exists a vertex w ∈ V\D such that w is adjacent to u or v, say u. Thus, for the set I = V\D –{v} there exists no non –empty set S⊆D such that induced subgraph <I ∪ S> induced by I∪S is regular, which is a contradiction. So, there exists a set S⊆D such that <{u ,v}∪S> is a connected. Hence, D is a set dominating set of G and $\gamma_s(G) = |D| = \gamma_{rs}^{s}(G)$.

Theorem 2.10. A secure regular set dominating set D of G is minimal if and only if for each vertex u∈D one of the following condition is satisfied.

(i) Either N(u) ∩D = ϕ or D_1= (D-{u})∪{v} is not dominating set for all N(u)∩D.

(ii) there exists a set I ⊆ V-D such that every induced regular subgraph induced by set I and vertices of D contains u and for every v ∈ V –D there exists a vertex u ∈ D such that D_1= (D-{u})∪{v} is a dominating set.

(iii) there exists a set I ⊆ V-D such that every induced regular subgraph induced by set I and vertices of D does not contain u and for every v ∈ V –D there exists a vertex u ∈ D such that D_1= (D-{u})∪{v} is a dominating set.

Proof. Let D be a minimal secure regular set dominating set of G. Suppose (i) does not hold, then for some u∈ D there exists a vertex N(u)∩D such that D_1 is dominating set and for every v∈ V –D either N(v)∩D ≠ {u} or D_1 is not dominating set. Therefore D –{u} is SRSD-dominating set of G, a contradiction to the minimality of D. Hence (i) holds.

Also by the condition (ii) and (iii) , for every set I′ ⊆ V- D′ there exists a set S′ ⊆ D′ such that <I′ ∪S′> is regular and for every vertex v∈ V –D, there exist a vertex u∈ D such that

D_1 is dominating set . This implies that D′ is SRSD set of G, a contradiction. Hence, each u∈D satisfies one of the given conditions. The converse is trivial.

Theorem 2.11. For any non –trivial graph G, $1\le \gamma_{rs}^{s}(G)\le$ p-1. Further, equality of lower bound holds if and only if G = K_p ; p≥2 and equality of an upper bound holds if and only if G satisfies the following conditions.

(i) There exists a vertex u∈ V-D such that N(u)∩D={v} where v∈D and (D –{v})∪{u} is dominating set or

(ii) If $N(u)\cap D=\bigcup_{i=1}^{p-1} v_i$, where each v_i∈D then <v_i> $1 \le i \le$ p-1 is independent and there exists a vertex v_i∈D $1 \le i \le$ p-1 such that D_i = (D –{v_i})∪{u} is dominating set.

Proof. Let D be any secure regular secure dominating set of G. Firstly, we consider lower bound.

Let |D| =|{v}| = $\gamma_{rs}^{s}(G)$ =1.Suppose there exists two non -adjacent vertices u, w∈ V –D. Then <{u, v, w}> is not regular, contradiction. Hence every two vertices in V –D are adjacent . This implies that G is complete. Conversely, suppose G = K_p, then by Observation 2.6, $\gamma_{rs}^{s}(G)$=1.

Now consider the upper bound.

Suppose $\gamma_{rs}^{s}(G)$=p-1, D satisfies none of the above conditions. Then there exists at least adjacent vertices v_1 ,v_2∈D such that N[v_1] = {v_1 ,u} for u∈ V\D and for every I⊆ V – D, there does not exist a subset S ⊆ D such that < I∪S > which is regular, which is a contradiction that D is a secure regular set dominating set. Hence D must satisfy either of the above conditions. Converse is trivial.

3. CONCLUSION AND SCOPE

we have defined a new variant of the parameter of domination namely secure regular set domination number of various graphs and we have found the equality relationship of the selected number of domination parameters in case of complete bipartite graph $K_{m,n}$. Also, we have observed that the secure regular set domination number is always less than of its order.

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