Fixed Points of (α, φ_K) - Geraghty contractions in metric like spaces

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Abstract- In this paper, we define (α, φ_k)- generalized Geraghty contraction maps in metric-like spaces where α is an admissible function and φ is an altering distance function, and prove the existence of fixed points. Our results extend the some of the known results. We provide examples in support of our results.
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I. Introduction

Banach contraction principle is one of the fundamental results in fixed point theory for which several authors generalized and extended it both in terms of considering a more general contraction condition and a more general ambient space. In 2012, Amini-Harindi[1] proved some fixed point results in metric-like spaces. Aydi, Karipinar [2] proved some fixed point results in metric-like spaces with (α-Ψ) contractions. Recently O.Acar and Ishak Altuin [4] proved a fixed point theorem for Ψ k- Geraghty contraction in metric-like spaces. Khan, Swaleh and Sessa [12] studied the existence of fixed points in metric spaces by using altering distance functions.

Definition 1.1 [1] . Let X be a nonempty set. A function σ : X xX → $[0, \infty)$ is said to be a metric-like space on X if for any x, y in X the following conditions are satisfied :

(i) $\sigma\ (x, y) = 0\ \Rightarrow$ x=y
(ii) $\sigma\ (x, y) = \sigma\ (y, x)$, and
(iii) $\sigma\ (x, y) = \sigma\ (x, z) + \sigma\ (z, y)$.

The pair (X, σ) is called a metric-like space.

Each metric-like σ on X generates a τ_0 on X which has a base consisting of the family of open σ - balls Bσ(x, ε) : x ∈ X, ϵ >0 ,
where Bσ(x, ε) = y ∈ X: | $\sigma\ (x, y) - \sigma\ (x, x)$|<ϵ for all x ∈ X and ϵ >0 .

Definition 1.2 [1] (i) A sequence $\{x_n\}$ in a metric-like space (X, σ) converges to a point x ∈ X if and only if $\sigma(x,x) = \lim_{n\to\infty} \sigma(x,x_n)$.

(ii) A sequence $\{x_n\}$ in a metric-like space (X, σ) is called a Cauchy sequence $\lim_{n,m\to\infty}(x_n,x_m)$ exists (and is finite).

(iii)A metric –like space (X, σ) is said to be complete if every Cauchy sequence $\{x_n\}$ in converges , with respect to τ_p , to a point x ∈ X

such that $\lim_{n\to\infty} \sigma(x,x_n) = \sigma(x,x) = \lim_{n,m\to\infty}(x_n,x_m)$.

Definition 1.3[1] Let (X, σ) is called a metric-like space. A mapping T: (X, σ) →(X, σ) is continuous if for any sequence $\{x_n\}$ in X such that $\sigma(x_n,x) \to \sigma(x,x)$ as $n \to \infty$,

we have $\sigma(Tx_n,Tx) \to \sigma(Tx,Tx)$ as $n \to \infty$.

Lemma 1.4 [10]. Let (X, σ) be a metric-like space. Let $\{x_n\}$ be a sequence in X such that $x_n \to x$, where x ∈ X and $\sigma(x,x) = 0$.

Then for all y ∈ X ,

we have $\lim_{n\to\infty} \sigma(x_n,y) = \sigma(x,y)$.

Definition 1.5 ([12]) A function : $R^+ \to R^+$, $R^+ = [0, \infty)$ is said to be an *altering distance function* if the following conditions hold:

(i) φ is continuous,

(ii) φ is non-decreasing, and

(iii) $\varphi(t) = 0$ if and only if $t = 0$.

In 1973, Geraghty [8] introduced a new contractive mapping in which the contraction constant was replaced by a function having some specific properties taken from the class of functions S, where $S = \{\beta : [0,\infty) \to [0,1) / \beta(t_n) \to 1 \Rightarrow t_n \to 0\}$

Definition 1.6 . [13] Let T: XxX be a self map and α: XxX → R be a function . Then T is said to be α – admissible function if α(x, y) ≥1 implies α(Tx, Ty) ≥1.

In 2015 Karipinar. E, Alsulami H.H., Noorwali M.,[11] proved the following Geraghty theorem in metric-like space.

Theorem1.7. [11] . Let (X, σ) be a complete metric – like space and T: X → X be a mapping. Suppose that there exists β ∈ S such that $\sigma(Tx,Ty) \le \beta(\sigma(x,y))\sigma(x,y)$ for all x, y in X. Then T has a unique point u ∈ X with $\sigma(u,u) = 0$.

In 2017 Aydi H., Felhi A., and Sahmim S [3] considered a new type of Geraghty contractions in the class of metric-like spaces and proved the existence of fixed points for the following contractive map.

Theorem 1.8. [3] Let (X, σ) be a complete metric-like space and T: X → X be a mapping. Suppose that there exists β ∈ S such that $\sigma(Tx,Ty) \le \beta(F(x,y))F(x,y)$ for all x, y in X ,

where $F(x,y) = \sigma(x,y) + |\sigma(x,Tx) - \sigma(y,Ty)|$. Then T has a unique fixed point u ∈ X with $\sigma(u,u) = 0$.

Now, we define φk - generalized Geraghty contraction in metric-like spaces.

Definition 1.9. Let (X, σ) be a metric-like space and let T: X → X be a self map. If there exists β ∈ S such that $(\varphi(\sigma(Tx,Ty)) \le \beta(\varphi(K(x,y)))\varphi(K(x,y))$

Where$K(x,y)=\max\{\sigma(x, T(x)), \sigma(y, Ty), \frac{\sigma(x,Tx)+\sigma(y,Ty)}{2}, \sigma(x,y) + |\sigma(x,Tx) - \sigma(y,Ty)|\}$

for all x , y ∈ X then we call T is a φ_k - generalized Geraghty contraction in metric-like spaces.

Now we define (α , φ_k) - generalized Geraghty contraction maps in metric-like spaces where α is an admissible function and φ is an altering distance function.

Definition 1.10. Let (X, σ) be a metric-like space and let T: X → X be a self map. If there exists β ∈ S such that $\alpha(x, y) \; (\varphi(\sigma(Tx,Ty)) \le \beta(\varphi(K(x,y)))\varphi(K(x,y))$ Where$K(x,y)=\max\{\sigma(x, T(x)), \sigma(y, Ty), \frac{\sigma(x,Tx)+\sigma(y,Ty)}{2}, \sigma(x,y) + |\sigma(x,Tx) - \sigma(y,Ty)|\}$ for all x , y ∈ X then we call T is a (α ,φ_k) - generalized Geraghty contraction in metric-like spaces.

Lemma 1.10. [2] Let (*X, d*) be metric space. Let $\{x_n\}$ be a sequence in *X* such that $d(x_{n+1}, x_n) \to 0$ as $n \to \infty$. If $x_n\}$ is not a Cauchy sequence then there exist an $\epsilon > 0$ and sequences of positive integers $\{m(k)\}$ and $\{n(k)\}$ with $n(k) > m(k) > k$ and

$d(x_{m(k)}, x_{n(k)}) \geq \epsilon$. For each $k > 0$, corresponding to m(k), we can choose n(k) to be the smallest integer such that $d(x_{m(k)}, x_{n(k)}) \geq \epsilon$ and $d(x_{m(k)}, x_{n(k)\ 1}) < \epsilon$. It can be shown that the following identities are satisfied.

(i) $\lim_{k\to\infty} d(x_{n(k)}, x_{m(k)}) = \varepsilon$

(ii) $\lim_{k\to\infty} d(x_{n(k)-1}, x_{m(k)+1}) = \varepsilon$,

(iii) $\lim_{k\to\infty} d(x_{n(k)-1}, x_{m(k)}) = \varepsilon$,

(iv) $\lim_{k\to\infty} d(x_{n(k)}, x_{m(k)+1}) = \varepsilon$.

Now, we prove the existence of fixed points of (α, φ_k) generalized Geraghty contraction maps in metric-like spaces .

2. MAIN RESULTS

Theorem 2.1. . *Let (X, σ) be a complete metric-like space. Let $T: X \to X$ be a (α, φ_k) generalized Geraghty contraction. Suppose that*

(i) T is α admissible;

(ii) there exists x_0 ⍰ X such that $\alpha(x_0, Tx_0) \geq 1$;

(ii) T is continuous.

Then there exists a u ⍰ X such that $\alpha(u; u) = 0$. Assume that in addition that (H1) if $\sigma(x, x) = 0$ for some x in X , then $\alpha(x; x) \geq 1$. Then such u is a fixed point of T.

Proof. Let $x_0 \in X$ be such that $\alpha(x_0, Tx_0) \geq 1$. We define $\{x_n\}$ in X by $x_n = Tx_{n-1}$ for each n.

If $x_n = x_{n+1}$ for some $n \in N$, then $x_n = Tx_n$ and hence x_n is a fixed point of T. Hence, without loss of generality, we assume that $x_n \neq x_{n+1}$ for all $n \in N$.

(i) Since T is α admissible, we have

$\alpha(x_0, x_1) = \alpha(x_0, Tx_0) \geq 1$ implies $\alpha(Tx_0, Tx_1) = \alpha(x_1, x_2) \geq 1$.

By mathematical induction,

it is easy to see that $\alpha(x_n, x_{n+1}) \geq 1$ for all $n \in N$.

We consider

$$\varphi(\sigma(x_{n+1}, x_{n+2})) = \varphi(\sigma(Tx_n, Tx_{n+1}))$$

$$\leq \alpha(x_n, x_{n+1})\, \varphi(Tx_n, Tx_{n+1})$$

$$\leq \beta(\varphi(K(x_n, x_{n+1})))(\varphi(K(x_n, x_{n+1})) \qquad (2.1.1)$$

Now

$$K(x_n, x_{n+1}) = \max\{\sigma(x_n, Tx_n), \sigma(x_{n+1}, Tx_{n+1}), \frac{\sigma(x_n, Tx_n), +\sigma(x_{n+1}, Tx_{n+1})}{2}, \sigma(x_n, x_{n+1}) + |\sigma(x_n, Tx_n) - \sigma(x_{n+1}, Tx_{n+1})|\}$$

$$K(x_n, x_{n+1}) = \max\{\sigma(x_n, x_{n+1}), \sigma(x_{n+1}, x_{n+2}), \frac{\sigma(x_n, x_{n+1},), +\sigma(x_{n+1}, x_{n+2})}{2}, \sigma(x_n, x_{n+1}) + |\sigma(x_n, x_{n+1}) - \sigma(x_{n+1}, x_{n+2})|\}$$

Suppose that $\sigma(x_n, x_{n+1}) \leq \sigma(x_{n+1}, x_{n+2})$

$$K(x_n, x_{n+1}) = \max\{\sigma(x_n, x_{n+1}), \sigma(x_{n+1}, x_{n+2}), \frac{\sigma(x_n, x_{n+1},), +\sigma(x_{n+1}, x_{n+2})}{2}, \sigma(x_n, x_{n+1}) + (\sigma(x_{n+1}, x_{n+2}) - \sigma(x_n, x_{n+1})\}$$

Suppose that $\sigma(x_{n+1}, x_{n+2}) \leq \sigma(x_n, x_{n+1})$

$$K(x_n, x_{n+1}) = \max\{\sigma(x_n, x_{n+1}), \sigma(x_{n+1}, x_{n+2}), \frac{\sigma(x_n, x_{n+1},), +\sigma(x_{n+1}, x_{n+2})}{2}, \sigma(x_n, x_{n+1}) + (\sigma(x_{n+1}, x_{n+2}) - \sigma(x_n, x_{n+1})\}$$

$$K(x_n, x_{n+1}) = \max\{\sigma(x_n, x_{n+1}), \sigma(x_{n+1}, x_{n+2})\}$$

If $\max\{\sigma(x_n, x_{n+1}), \sigma(x_{n+1}, x_{n+2})\} = \sigma(x_{n+1}, x_{n+2})$

then from (2.1.1), we have

$$\varphi(\sigma(x_{n+1}, x_{n+2})) \leq \beta(\varphi(K(x_n, x_{n+1})))(\varphi(K(x_n, x_{n+1}))$$

$$\leq \beta(\varphi(K(x_n, x_{n+1})))(\varphi(\sigma(x_n, x_{n+1}))$$

$< \varphi(\sigma(x_{n+1}, x_{n+2}))$, a contradiction.

So that we have $\max\{\sigma(x_n, x_{n+1}), \sigma(x_{n+1}, x_{n+2})\} = \sigma(x_n, x_{n+1})$, and hence

$$\varphi(\sigma(x_{n+1}, x_{n+2})) \leq \beta(\varphi(K(x_n, x_{n+1})))(\varphi(K(x_n, x_{n+1})) < \varphi(\sigma(x_n, x_{n+1})) \text{ for all n.}$$

Thus it follows that $\{\varphi(\sigma(x_n, x_{n+1}))\}$ is a decreasing sequence of non negative reals and so $\lim_{n\to\infty} \varphi(\sigma(x_n, x_{n+1}))$ exists and it is r(say). i.e., $\lim_{n\to\infty} \varphi(\sigma(x_n, x_{n+1})) = r \geq 0$.

We now show that $r = 0$.

If r>0, then from (2.1.1) we have

$$\begin{aligned}\varphi(\sigma(x_{n+1}, x_{n+2})) &\leq \varphi(Tx_n, Tx_{n+1})) \\ &\leq \beta(\varphi(K(x_n, x_{n+1})) \ \varphi(K(x_n, x_{n+1})) \\ &\leq \beta(\varphi(K(x_n, x_{n+1})) \ \varphi(\sigma(x_n, x_{n+1})), \text{ and hence}\end{aligned}$$

$$\frac{\varphi(\sigma(x_{n+1}, x_{n+2}))}{\varphi(\sigma(x_n, x_{n+1}))} \leq \beta(\varphi(K(x_n, x_{n+1})) < 1 \text{ for each } n \geq 1.$$

Now on letting $n \to \infty$, we get

$$1 = \lim_{n\to\infty} \frac{\varphi(\sigma(x_{n+1}, x_{n+2}))}{\varphi\sigma(x_n, x_{n+1})} \leq \lim_{n\to\infty} \beta(\varphi(K(x_n, x_{n+1}))) \leq 1$$

So that $\beta(\varphi K(x_n, x_{n+1}))) \to 1 \ as \ n \to \infty$.

This implies that $\lim_{n\to\infty}(\varphi(K(x_n, x_{n+1}))) = 0$.

Since $\varphi(\sigma(x_n, x_{n+1})) \leq \varphi(K(x_n, x_{n+1}))$ for all n, we have

$$\lim_{n\to\infty}\left(\varphi(\sigma(x_n, x_{n+1}))\right) \leq \lim_{n\to\infty}\left(\varphi(K(x_n, x_{n+1}))\right) = 0.$$

Hence $\lim_{n\to\infty} \varphi(\sigma(x_n, x_n + 1)) = 0. i.e., r = 0$.

Suppose that $\{x\}$ is not a Cauchy sequence. Then by Lemma 1.11,

There exist an $\in < 0$ and sequences of positive integers $\{m(k)\}$ and $\{n(k)\}$ with m(k) > n(k) > k and (i), (ii), (iii) and (iv) of Lemma 1.11 hold.

By taking $x = x_{n(k)}, \ y = x_{m(k)-1} \ in \ (2.1.1), it \ follows \ that$

$$\begin{aligned}\varphi(\sigma(x_{n(k)+1}, x_{m(k)}) &= \varphi(\sigma(T(x_{n(k)}), Tx_{m(k)-1}))) \\ &\leq \beta\left(\varphi\left(K(x_{n(k)}, x_{m(k)-1})\right)\right)\varphi\left(K(x_{n(k)}, x_{m(k)-1})\right)\end{aligned}$$

(2.1.2)

Where

$$\begin{aligned}K(x_{n(k)}, x_{m(k)-1}) &= \max\{\sigma(x_{n(k)}, Tx_{n(k)}), \sigma(x_{m(k)-1}, Tx_{m(k)-1}), \\ &\frac{\sigma(x_{n(k)}, Tx_{n(k)}) + \sigma(x_{m(k)-1}, Tx_{m(k)-1})}{2} \sigma(x_{n(k)}, x_{m(k)-1}) + |\sigma(x_{n(k)}, Tx_{n(k)}) \\ &- \sigma(x_{m(k)-1}, Tx_{m(k)-1})|\}\end{aligned}$$

$$\begin{aligned}K(x_{n(k)}, x_{m(k)-1}) &= \max\{\sigma(x_{n(k)}, x_{n(k)+1}), \sigma(x_{m(k)-1}, x_{m(k)}), \\ &\frac{\sigma(x_{n(k)}, Tx_{n(k)+1}) + \sigma(x_{m(k)-1}, x_{m(k)})}{2}, \sigma(x_{n(k)}, x_{m(k)-1}) + |\sigma(x_{n(k)}, x_{n(k)+1}) \\ &- \sigma(x_{m(k)-1}, x_{m(k)})\}\end{aligned}$$

On letting $k \to \infty$ and from the Lemma 1.11 we get

$$\lim_{n\to\infty} K(x_{n(k),}\, x_{m(k)-1} = \max\{0,0,0, \in +0-0\} = \in.$$

Now, we have

$$\varphi\left(\sigma\left(x_{n(k)+1,}\, x_{m(k)}\right)\right) \le \beta\left(\varphi\left(K\left(x_{n(k),}\, x_{m(k)-1}\right)\right)\right)\varphi(K(x_{n(k),}\, x_{m(k)-1}))$$
$$\le \beta\left(\varphi\left(K\left(x_{n(k),}\, x_{m(k)-1}\right)\right)\right)\varphi(K\left(x_{n(k),}\, x_{m(k)-1}\right))$$
$$\le \beta\left(\varphi\left(K\left(x_{n(k),}\, x_{m(k)-1}\right)\right)\right)\varphi(\sigma\left(x_{n(k),}\, x_{m(k)-1}\right))$$

And hence

$$\frac{\varphi(\sigma\left(x_{n(k)+1,}\, x_{m(k)}\right))}{\varphi(\sigma\left(x_{n(k),}\, x_{m(k)-1}\right)} \le \beta\left(\varphi\left(K\left(x_{n(k),}\, x_{m(k)-1}\right)\right)\right) < 1.$$

On letting $k \to \infty$ and from the Lemma 1.11, we get

$$1 = \frac{\varphi(\epsilon)}{\varphi(\epsilon)} \le \lim_{k\to\infty} \beta(\varphi(K(x_{n(k),}\, x_{m(k)-1}))) \le 1$$

So that $\beta\left(\varphi\left(K\left(x_{n(k),}\, x_{m(k)-1}\right)\right)\right) \to 1 \; as\; k \to \infty.$

Since $\beta \in S, \varphi\left(K\left(x_{n(k),}\, x_{m(k)-1}\right)\right) \to 0 \; as\; k \to \infty. i.e., \varphi(\epsilon) = 0,$

Since φ is continuous. Hence it follows that $\epsilon = 0,$ a contradiction.

Therefore $\{x_n\}$ is a Cauchy sequence in X, and since X is complete metric-like space, there exists $u \in X$ such that $\lim_{n\to\infty} x_n = u.$

$$\lim_{n\to\infty} \sigma(x_n, u) = \sigma(u,u) = \lim_{n\to\infty} \sigma(x_{n,}x_m) = 0. \qquad (2.1.3)$$

Now, we show that u is a fixed point of T.
First we assume that (iii) hold. i.e., T is continuous.
In this case, we have

$u = \lim_{n\to\infty} T^n(x_0) = \lim_{n\to\infty} T^{n+1}(x_0) = T\left(\lim_{n\to\infty} T^n(x_0)\right) = T(u)$. Therefore u is a fixed point of T in X.

Theorem 2.2. Let (X, d) be a complete metric-like space, $\alpha : X \times X \to R$ *be a function and let* $T: X \to X$ *be a* (α, φ_K) *generalized Geraghty contraction map. Suppose that the following conditions hold*

(i) $T\ is\ \alpha - admissible$;

(ii) *there exists x_0 ∈ X such that α (x_0, Tx_0) ≥1 and set* $x_n = Tx_{n-1}$ *for n= 1,2,3,…*

(iii) *If* $\{x_n\}$ *is a sequence in X such that α (x_n,* x_{n+1}*) ≥1 for all n and* $x_n \to x$ *as* $n \to \infty$ *then there exists a sub-sequence* $\{x_{n(k)}\}$ *of* $\{x_n\}$ *such that* $\alpha\left(x_{n(k),}x\right) \ge 1$ *for all k*

Then T has a fixed point u in X.

Proof. From the proof of the theorem2.1, we have the sequence $\{x_n\}$defined by $\{x_{n+1}\} = Tx_n$ for all $n \ge 0$is a Cauchy in (X, σ) and converges to to some $u \in X$. Also 2.1.3 holds, so is Cauchy sequence in (X, σ) and converges to some $u \in X$.

$$\lim_{n\to\infty} \sigma(x_n(k)+1, Tu) = \sigma(u, Tu).$$

Now we show that $Tu = u$

Suppose that $Tu \ne u$. i.e.,$\sigma(Tu, u) > 0.$

From condition (iii), we have that there exists a sub-sequence$\{x_{n(k)}\}$ of $\{x_n\} such\ that\ \alpha\left(x_{n(k),}u\right) \ge 1\ for\ all\ k.consider$

$$\sigma(x_n(k)+1, Tu) \le \alpha(x_n(k), Tu)\sigma(Tx_n(k), Tu)$$
$$\le \beta\left(\varphi\left(K(x_n(k), u)\right)\right)\varphi\,(K(x_n(k), u)) \qquad (2.2.1)$$

Where

$$K(x_n(k),u) = \max\{\sigma(x_n,Tx_n),\sigma(u,Tu),\sigma(x_n,u) + |\sigma(x_n,Tx_n) - \sigma(u,Tu)|\}$$

$$\lim_{n\to\infty} K(x_n(k),u) = \lim_{n\to\infty} \max\{\sigma(x_n,x_{n+1}),\sigma(u,Tu),\sigma(x_n,u) + |\sigma(x_n,x_{n+1}) - \sigma(u,Tu)|\} = \sigma(u,Tu)$$

Letting k→ ∞ in (2.2.1)

$$\sigma(u,Tu) \leq \beta\left(\varphi\big(K(x_n(k),u)\big)\right)\varphi\big(\sigma(u,Tu)\big) < \sigma(u,Tu), which\ iscontradiction.\ so\ that\ u\ is\ a\ fixed\ pointof\ T.$$

3. COROLLARIES AND EXAMPLES

In the theorem 2.1, if φ_K is the identity map we have the following corollary.

Corollary 3.1. Let (X,σ) *be a complete metric-like space. Let* $T : X \to X$ *be a* α *generalized Geraghty contraction. Suppose that*

(i) T is α *admissible;*

(ii) there exists $x_0 \in X$ *such that α* $(x_0, T x_0) \geq 1$

(iii) T *is continuous.*

Then there exists $a\ u \in X$ *such that* $\sigma(u;\ u) = 0$.

Proof: Assume that in addition that (H1)

if $\sigma(x,x) = 0 for\ some\ x \in X, then\ \alpha(x,x) \geq 1.$

Then such u is a fixed point of T.

In the theorem 2.1 , if $\alpha = 1$ is the identity map we have the following corollary.

Corollary 3.2. Let (X, σ) be a complete metric-like space. Let $T : X \to X$ be a (φ) generalized Geraghty contraction. Suppose that

(i) T is α admissible;

(ii) there exists $x_0 \in X$ such that α $(x_0, Tx_0) \geq 1$

(ii) T is continuous.

Then there exists a u ∊ X such that α (u; u) = 0. Assume that in addition that (H1) if σ (x, x) = 0 for some x ∊ X , then α (x; x) ≥ 1.Then such u is a fixed point of T.

The following is an example in support of the theorem 2.1.

Example:3.3

$. Let\ X = [0,\infty) and\ \sigma(x,y) = x + y. Then\ (X,\sigma) is\ a$ complete metric-like space.

We define $T: X \times X\ by\ T(x) = \begin{cases} \frac{x^2}{2} & if\ x \in [0,1] \\ 6x - \frac{11}{2} & otherwise. \end{cases}$

We define $\varphi : [0,\infty) \to [0,\infty) by\ \varphi(t) = \frac{t}{2} and$

$$\alpha : X \times X \to [0,\infty) as\ \alpha(x,y) = \begin{cases} 1 & if\ x = \frac{3}{4}, y = \frac{1}{2} \\ 0 & otherwise \end{cases}$$

Now we verify the inequality 2.1.1 when $x = \frac{3}{4}, y = \frac{1}{2}$

$$\alpha\left(\frac{3}{4}\ \ \frac{1}{2}\right)\varphi\left(\sigma\left(T\frac{3}{4},T\frac{1}{2}\right)\right) = \varphi\left(\frac{12}{31}\right) = \frac{13}{64}$$

$$K\left(\frac{3}{4}\ \ \frac{1}{2}\right) = \frac{53}{32}, \varphi\left(K\left(\frac{3}{4},\frac{1}{2}\right)\right) = \left(\frac{53}{64}\right), \beta\left(\varphi\left(K\left(\frac{3}{4},\frac{1}{2}\right)\right)\right) = \frac{64}{117}$$

$$\alpha\left(\frac{3}{4}\ \ \frac{1}{2}\right)\varphi\left(\sigma\left(T\frac{3}{4},T\frac{1}{2}\right)\right) = \frac{13}{64} \leq \left(\frac{54}{117}\right).\left(\frac{53}{64}\right) = \beta\left(\varphi\left(K\left(\frac{3}{4}\ \ \frac{1}{2}\right)\right)\right).\varphi\left(K\left(\frac{3}{4}\ \ \frac{1}{2}\right)\right).$$

Therefore T satisfy all the conditions of the hypothesis Theorem 2.1 and T has a unique fixed point 0.

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