

# Numerical Studies on Supersonic Flow over a Cavity Relating Velocity, Coefficient of Pressure and OASPL Expending LES

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## Abstract

An appropriate numerical model is established to assess supersonic flow past a 3D open cavity. The studies implicate supersonic flow over the 3D open cavity having length-to-depth ratio of 2, including the supersonic free-stream Mach number of 2 in addition to the flow Reynolds number of  $10^5$ . The numerical simulation has been conducted by utilizing the Large Eddy Simulation (LES) method. The results acquired have been compared with both experimental as well as numerical simulation predictions reported in the literature. The results have been shown in the form of both coefficient of pressure ( $C_p$ ) and overall sound pressure level (OASPL) at the centreline of the aft wall of the open cavity. The coefficient of pressure appears to match both qualitatively and quantitatively with the existing experimental and numerical results available from the other investigators. Nevertheless, the overall sound pressure level at the centreline of the aft wall of the open cavity is over-predicted by nearly 30-40 dB. In addition, very large recirculation is also witnessed within the cavity as observed from the velocity vectors and therefore these require to be suppressed for reducing OASPL.

**Keywords:** Numerical Simulation, Cavity, LES, Velocity Vector, Coefficient of Pressure, OASPL.

## 1. Introduction

The challenging flow physics pertaining to the flow over a cavity has spellbound the researchers around the world for more and more investigations and remains as a front-line area of investigation. Heller et al. [1] illustrated on flow-induced pressure oscillations in shallow cavities. Tam and Block [2] studied on the tones and pressure oscillations induced by flow over rectangular cavities. Kaufman et al. [3] reported on Mach 0.6 to 3.0 flows over rectangular cavities. Sweby [4] applied high resolution schemes using flux limiters on hyperbolic conservation laws. Rizzetta [5] performed numerical simulation on supersonic flow over a three-dimensional cavity. Anderson and Wendt [6] described about the fundamentals of computational fluid dynamics. Piomelli [7] demonstrated on achievements and challenges of large-eddy simulation. Hamed et al. [8] conducted numerical simulations of fluidic control for transonic cavity flows. Li et al. [9] carried out LES study of feedback-loop mechanism of supersonic open cavity flows. Vijayakrishnan [10] executed a validation study on unsteady RANS computations of supersonic flow over two dimensional cavity. It is apprehended that a general study on cavity flow has been done both experimentally and numerically for enhancing the aerodynamic effect.

Despite the fact, the flow over a cavity has been studied experimentally/numerically by many investigators, nevertheless, complete modelling of both large and small scales of motions at a time, not yet done which is one of the leading inadequacies. But, Large Eddy Simulation (LES) is the technique which resolves the large eddies as it is and models the small eddies that can provide sensibly more convincing results too. The impotunity of this investigation work is to study the flow physics and modes of oscillations in a 3D open cavity supersonic flow. It includes details about the governing equations and the establishment together with the implementation of the LES method counting the sub-grid scale modelling. The discretization trials have also been discussed. The simulation predictions have been provided in the form of pressure flow field, coefficient of pressure ( $C_p$ ) and overall sound pressure level (OASPL) at

the centreline of the aft wall of the open cavity. The numerical results of supersonic flow past an open cavity have also been compared with the experimental as well as numerical results reported in the literature. In overall, very good agreement between the above said results is also witnessed from the current studies. Nonetheless, the investigations relating to the usage of passive control methods/devices for the reduction of recirculation within the open cavity is planned for the future. As these devices perform over a broad range of parameters, definitely it will influence the flow physics of incoming boundary layer for offering equally efficient flow circumstances.

## 2. Description of Physical Problem

Supersonic flow past a three-dimensional cavity is studied numerically. The streamwise length, depth and spanwise length of the cavity are 20 mm, 10 mm, and 10 mm, respectively. The length-to-depth ratio ( $L/D$ ) for the cavity is 2. The width-to-depth ratio ( $W/D$ ) is 1. The cavity is three-dimensional with streamwise length-to-spanwise length ratio ( $L/W$ )  $> 1$ . In addition, the Mach number of the free-stream along with the Reynolds number based on the cavity depth are taken as 2 and  $10^5$ , respectively, for setting the inflow conditions.

### A. Geometric model

The computational domain of the cavity used in the present simulation is shown in figure 1. The size of the computational domain, as mentioned earlier, is  $2D \times D \times D$  (length  $\times$  breadth  $\times$  width). The inlet boundary is located at a distance of  $D$  upstream from the leading edge of the cavity. The outlet boundary is located at a distance of  $4D$  downstream from the trailing edge of the cavity. The upper boundary is also located at a distance of  $4D$  above the cavity.

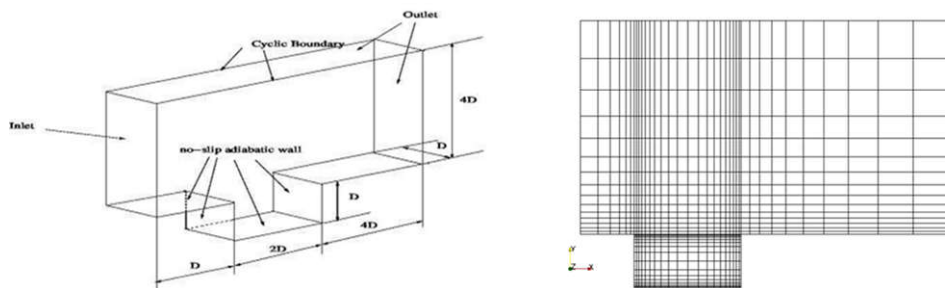


Fig 1. Computational domain of cavity. Fig 2. Computational grid in X-Y Plane.

### B. Initial and boundary conditions

The inflow boundary conditions are initialized with free-stream conditions of  $M_\infty = 2$ ,  $P_\infty = 101.325$  kPa, and  $T_\infty = 300$  K. The Reynolds number of the flow used in the simulation is  $10^5$ , which is based on the cavity depth. No-slip adiabatic wall boundary conditions is applied at the wall boundaries. Zero-gradient condition is applied at all the outflow boundaries. Periodical boundary condition is applied in the spanwise direction of the cavity.

## 3. Mathematical Formulation and Numerical Procedures

### A. Generalized governing transport equations

The most generalized governing transport equations of mass, momentum and energy for turbulent and compressible flow are as mentioned below.

$$\text{Continuity: } \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{U}) = 0 \quad (1)$$

$$\text{Momentum: } \frac{\partial (\rho \mathbf{U})}{\partial t} + \nabla \cdot (\rho \mathbf{U} \cdot \mathbf{U}) - \nabla \cdot \nabla (\mu \mathbf{U}) = -\nabla p \quad (2)$$

$$\text{Energy: } \frac{\partial (\rho e)}{\partial t} + \nabla \cdot (\rho e \mathbf{U}) - \nabla \cdot \nabla (\mu e) = -p(\nabla \cdot \mathbf{U}) + \mu \left[ \frac{1}{2} (\nabla \mathbf{U} + \nabla \mathbf{U}^T) \right]^2 \quad (3)$$

Where,  $\mathbf{U}$  = velocity vector =  $u\mathbf{i} + v\mathbf{j} + w\mathbf{k}$

$$\frac{1}{2} (\nabla U + \nabla U^T) = \text{strain rate tensor.}$$

It is assumed that the gas is a perfect gas denoted by the equation,  $p = \rho RT$  (4)

Where,  $R =$  specific gas constant  $= C_p - C_v$  (5)

For a calorically perfect gas (constant specific heats), the caloric equation of state is,  $e =$  internal energy per unit mass  $= C_v T$  (6)

**B. LES turbulence modelling**

The LES turbulence model is introduced by splitting the time and space varying flow variables into two constituents, the resolved one  $\bar{f}$  and  $f'$ , the unresolved part:

$$(x, t) = \bar{f}(x, t) + f'(x, t) \tag{7}$$

The filtering operation when applied to the Navier-Stokes equation gives:

$$\frac{\partial \bar{\rho}}{\partial t} + \nabla \cdot (\bar{\rho} \bar{U}) = 0 \tag{8}$$

$$\frac{\partial (\bar{\rho} \bar{U})}{\partial t} + \nabla \cdot (\bar{\rho} \bar{U} \bar{U}) - \nabla \cdot \nabla (\bar{\mu} \bar{U}) = -\nabla \bar{p} \tag{9}$$

$$\frac{\partial (\bar{\rho} \bar{e})}{\partial t} + \nabla \cdot (\bar{\rho} \bar{U} \bar{e}) - \nabla \cdot \nabla (\bar{\mu} \bar{e}) = -\bar{p} (\nabla \cdot \bar{U}) + \bar{\mu} \left[ \frac{1}{2} (\nabla \bar{U} + \nabla \bar{U}^T) \right]^2 \tag{10}$$

The eddy viscosity models relating the subgrid-scale stresses ( $\tau_{ij}$ ) and the resolved-scale rate of strain-tensor ( $S_{ij}$ ) is denoted as,  $\tau_{ij} - (\delta_{ij}/3) = -2\nu_T S_{ij}$  (11)

Where,  $S_{ij}$  is the resolved-scale rate of strain tensor  $= (\partial \bar{u}_i / \partial x_j + \partial \bar{u}_j / \partial x_i) / 2$ .

The algebraic model for the eddy viscosity:  $\mu_{sgs} = \rho C \Delta^2 |S| S_{ij}$ ,  $|S| = (2S_{ij} S_{ij})^{1/2}$  (12)

**C. Numerical techniques**

The transformed governing transport equations are solved by expending pressure based coupled framework relating to finite volume method (FVM) using the SIMPLER algorithm. Figure 3 shows the grid of the computational domain. As an outcome of this test, the optimum number of grid points used for the final simulation, in the upper cavity region as  $360 \times 150 \times 1$  and those of in the inside cavity region as  $200 \times 150 \times 1$ . Thus, the total number of grid points is 84000. The values of  $\Delta X^+$ ,  $\Delta Y^+$  and  $\Delta Z^+$  at the leading edge of the cavity are 5, 12.5 and 1.0, respectively. Corresponding time step taken in the simulation is 0.000001 seconds.

**4. Results and Discussions**

**A. Velocity distributions**

Figures 3 and 4 illustrate the velocity vectors at two different instants of times like  $t = 0.1$  sec as well as  $t = 0.3$  sec, respectively. The tenacity of the velocity vector is to display the recirculation zone within the open cavity.

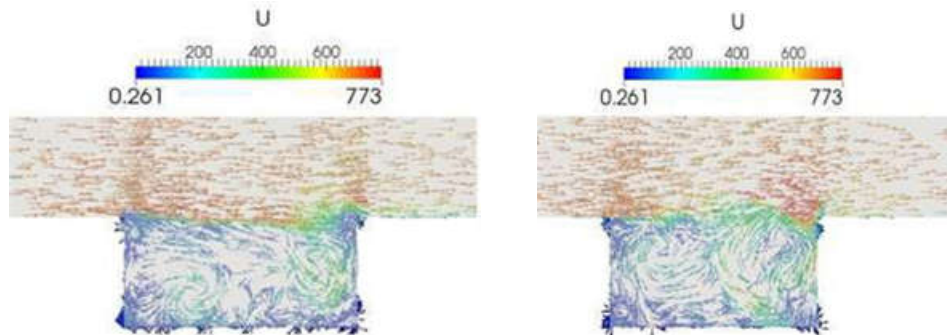
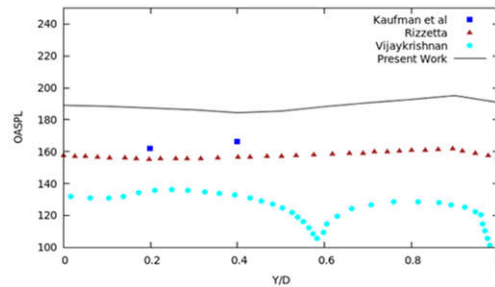
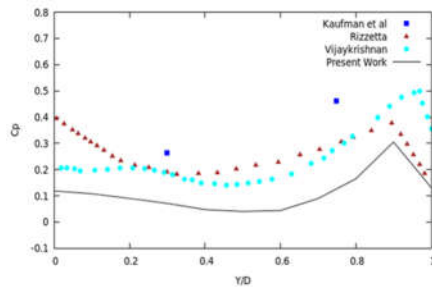


Fig 3. Velocity vector at time,  $t = 0.1s$  Fig 4. Velocity vector at time,  $t = 0.3s$

**B. Comparison of coefficient of pressure ( $C_p$ )**

The comparison of coefficient of pressure has been accomplished at the centreline of the aft wall of the open cavity which is exemplified in figure 5. Furthermore, the coefficient of pressure at the centreline of the stated open cavity wall is observed to be in both qualitative and quantitative agreement with the experimental along with the numerical results reported in the literature. The

deviation in the results from the experimental data of Kaufman et al. is caused by the numerical errors occurred throughout the simulation practices. In addition, the variation from the Vijaykrishnan research work is owing to the three-dimensionality effect and also the variation from the Rizzetta research work is because of the difference in the Mach number.



**Fig 5.  $C_p$  at the aft wall centreline. Fig 6. OASPL at the aft wall centreline.**

### C. Comparison of overall sound pressure level (OASPL)

The OASPL (Overall Sound Pressure Level) distribution at the centreline of the aft wall of the open cavity is depicted in figure 6. It is estimated to be about 25 dB higher than the experimental results (of Kaufman et al.). Certainly, the trend of results are comparable to the research works done by the other investigators. Nonetheless, the trend of result is very similar to the Kaufman et al. and Rizzetta research works rather than Vijaykrishnan research work. The sound pressure level observed at the aft wall is almost 5 dB higher than the front wall. The OASPL distribution at the centreline of the aft wall of the open cavity is over-predicted (from both Kaufman et al. and Rizzetta research results) by nearly 30-40 dB between aft and front walls. Conversely, the over-prediction from Vijaykrishnan research work is about 50-90 dB between aft and front walls. Furthermore, from the current research, the observed sound pressure level at the centreline of the aft wall of the open cavity seems to be nearly uniform throughout.

## 5. Conclusion

In the current research, the numerical simulations have been performed for supersonic flow past a 3D open cavity by means of LES. The numerical simulation predictions are done in the form of both cavity flow-field as well as aeroacoustic analyses. In addition, the aeroacoustic analysis is also done in the form of both coefficient of pressure ( $C_p$ ) and overall sound pressure level (OASPL) at the centreline of the aft wall of the open cavity. The present simulation results are compared with both experimental and numerical results exist in the literature. The LES model enables to foresee all the central flow behaviours of the open cavity. In addition, there also exists both qualitative and quantitative agreement of the coefficient of pressure with the experimental along with the simulation results available in the literature by the other researchers for supersonic flow past the said open cavity. Instead, the overall sound pressure level at the centreline of the aft wall of the open cavity is over-predicted by 30-40 dB. Besides, very large recirculation is observed within the open cavity as witnessed from velocity vectors and hence these must be reduced to suppress OASPL.

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