On Soft Multi Bitopological Space

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Abstract. *The aim of this paper is to introduce and study the notions of soft multi bitopological spaces which are defined over an initial universe with a fixed set of parameters. In this multi bitopological space the notion of pairwise soft multi open sets, closed sets, pairwise soft multi interior and closure operators are introduced. Moreover, some basic properties and fundamental theorems, examples and counter examples regarding to these notions are obtained.The importance of these notions is that it is considered to be a generalization of the notions of soft bitopological spaces, soft multi open sets, soft multi closed sets, soft multi interior and soft multi closure in soft multi bitopological spaces.*

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1. INTRODUCTION

In the year 1999, Russian researcher Molodtsov [1], initiated the concept of soft sets as a new mathematical tool to deal with uncertainties while modeling problems in engineering physics, computer science, economics, social sciences and medical sciences. In 2003, Maji, Biswas and Roy [6], studied the theory of soft sets initiated by Molodtsov. They defined equality of two soft sets, subset and super set of a soft set, complement of a soft set, null soft set and absolute soft set with examples. Soft binary operations like AND, OR and also the operations of union and intersection were also defined. In 2005, D. Chen [7], presented a new definition of soft set parametrization reduction and a comparison of it with attribute reduction in rough set theory.

Topological structures of soft set have been studied by some authors in recent years. In 2011, M. Shabir et al. and Naim Cagman et al. initiated the study of soft topology and soft topological spaces independently. M. Shabir and M. Naz [2], introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters and showed that a soft topological space gives a parameterized family of topological spaces. They introduced the definitions of soft open sets, soft closed sets, soft interior, soft closure and soft separation axioms. Also they obtained some interesting results for soft separation axioms which are really valuable for research in this field. N. Cagman, S. Karatas and S. Enginoglu [8], defined the soft topology on a soft set and presented its related properties and foundations of the theory of soft topological spaces. In 1963, J. C. Kelly [9], first initiated the concept of bitopological spaces. He defined a bitopological space (X,τ_1,τ_2) to be a set X equipped with two topologies τ_1 and τ_2 on X and initiated the systematic study of bitopological space. Later work done by C. W. Patty [10], I. L. Reilly [11] and others. Reilly discussed separation axioms properties in bitopological spaces.

The main purpose of the present paper is to introduce and study the notions of soft multi bitopological spaces which are defined over an initial universe with a fixed set of parameters and also generalize the notions of soft topological spaces such as open soft sets, closed soft sets, soft interior, soft closure in a soft multi bitopological spaces. Also the notions of pairwise open (closed) soft multi sets, pairwise soft multi interior (multi closure) operators are introduced in a soft multi bitopological space (U,τ_1,τ_2,E). The properties of these notions and some important results related to it are obtained. The following definitions which are prerequisites for present study are considered unless otherwise stated.

2. PRELIMINERIES:

In this chapter, we remind some basic concepts of soft multi sets. Let $\{U_i : i\in I\}$ be a collection of universes such that $\cap_{i\in I} U_i = \phi$ and let $\{E_{Ui} : i\in K\}$ be a collection of sets of parameters. Let $U= \Pi_{i\in I} P(U_i)$ where $P(U_i)$ denotes the power set of U_i, $E= \Pi_{i\in I} E_{Ui}$. We denote the family of these soft multi sets by $SMS(U_E)$.

Definition 2.1 [1]. Let X be an initial universe and E be a set of parameters. Let $P(X)$ denotes the power set of X and A be a non-empty subset of E. A pair (F,A) is called a soft set over X, where F is a mapping given by $F: A\to P(X)$ defined by $F(e)\in P(X)\ \forall e\in A$. In other words, a soft set over X is a parameterized family of subsets of the universe X. For $e\in A$, F(e) may be considered as the set of e-approximate elements of the soft set (F,A).

Definition 2.2 [2]: Let τ be the collection of soft sets over X, then τ is said to be a soft topology on X if it satisfies the following axioms.

(1) $\tilde{\phi}$, $\tilde{X}$ belong to τ.

(2) The union of any number of soft sets in τ belongs to τ.

(3) The intersection of any two soft sets in τ belongs to τ.

The triplet (X,τ,E) is called a soft topological space over X. Let (X,τ,E) be a soft topological space over X, then the members of τ are said to be soft open sets in X. A

soft set (F,A) over X is said to be a soft closed set in X, if its relative complement $(F,A)^c$ belongs to τ.

Definition 2.3. [3]. Let $\{U_i : i\in I\}$ be a collection of universes such that $\cap_{i\in I} U_i = \phi$ and let $\{E_{Ui} : i\in K\}$ be a collection of sets of parameters. Let $U= \Pi_{i\in I} P(U_i)$ where $P(U_i)$ denotes the power set of U_i, $E= \Pi_{i\in I} E_{Ui}$ and $A \subseteq E$. A pair (F, A) is called a soft multi set over U, where F is a mapping given by $F: A\to U$.

Definition 2.4. [3] For two soft multi sets F_A and G_B over U, F_A is called a soft multi subset of G_B if

(1) $A \subseteq B$ and,

(2) $e_{Ui,j} \in a_K$, $(e_{Ui,j}, Fe_{Ui,j}) \tilde{\subseteq} (e_{Ui,j}, Ge_{Ui,j})$.

where, $a_K\in A$, K = {1,2, 3, …… n}, i={1, 2, 3,…….,m} and j = {1,2,3,…..,r}.

This relationship is denoted by $F_A \tilde{\subseteq} G_B$. In this case, G_B is called a soft multi super set of F_A.

Definition 2.5. [3] Two soft multi sets F_A and G_B over U are said to be equal if F_A is a soft multi subset of G_B and G_B is a soft multi subset of F_A.

Definition 2.6 [3]. A soft multi set (F, A) over U is called a null soft multi set denoted by $\tilde{\phi}$ if for all $a\in A$, $F(a)=\phi$.

Definition 2.7 [3]. A soft multi set (F, A) over U is called an absolute soft multi set denoted by $\tilde{A}$, if for all a∈A, F(a)=U.

Definition 2.8 [3]. For any soft multi set (F, A), a pair ($e_{Ui,j}$,$F_{eUi,j}$) is called a U_i- soft multi part, ∀ $e_{Ui,j}$ ∈a_k, and $F_{eUi,j}$ ⊆ F(A) is an approximate value set, where, F(A) ⊆ U = $\Pi_{i\in I}$ P(Ui),a_k∈A, k∈{1,2,3,..,n}, i∈{1,2,3,..,m} and j∈{1,2,3,..,r}.

Definition 2.9 [3]. Union of two soft multi sets (F, A) and (G, B) over U denoted by (F,A) $\tilde{\cup}$

(G, B) is the soft multi set (H, C), where C= A∪B and ∀e∈C,

$$H(e)= \begin{cases} F(e), if e \in A-B \\ G(e), if e \in B-A \\ F(e) \cup G(e), if e \in A \cap B \end{cases}$$

Definition 2.10 [3]. Intersection of two soft multi sets (F, A) and (G, B) over U denoted by (F,A) $\tilde{\cap}$ (G, B) is the soft multi set (H, C), where C= A∪B and ∀e∈C,

$$H(e)= \begin{cases} F(e), if e \in A-B \\ G(e), if e \in B-A \\ F(e) \cap G(e), if e \in A \cap B \end{cases}$$

Definition 2.11[4]. The relative complement of a soft multi set (F, A) over (U,E) is denoted by $(F,A)^c$ and is defined by $(F, A)^c = (F^c, A)$, where F^c: A→U is a mapping given by $F^c(e)$=U-F(e), ∀e∈E.

Definition 2.12[4]. A soft multi set (F,A)∈SMS(U, E) is called a soft multi point in (U, E), denoted by $e_{(F,A)}$, if for the element e∈A, F(e) ≠ ϕ and ∀e′∈A-{e}, F(e′) = ϕ.

Definition 2.13 [4]. A soft multi point $e_{(F,A)}$ is said to be in the soft multi set (G,B), denoted by $e_{(F,A)}$ $\tilde{\in}$ (G, B), if (F, A) $\tilde{\subseteq}$ (G, B).

Definition 2.14[4]. A sub family τ of SMS(U_E), is called soft multi set topology on (U, E), if the following axioms are satisfied

[P1]. $\tilde{\phi}$,$\tilde{U}$ ∈τ

[P2]. The union of any number of soft multi sets in τ belongs to τ, i.e. for any {(F^k_{Ak})| k∈Λ, index set} ⊆ τ ⇒ $\tilde{\cup}_{k\in\Lambda}(F^k_{Ak})$∈τ.

[P3]. If (F_A), (G_B)∈τ, then F_A $\tilde{\cap}$ G_B ∈τ.

Then the pair ((U, E), τ) is called soft multi topological space. The members of τ are called soft multi open sets (or τ-open soft multi sets or simply open sets) and the conditions [P1], [P2] and [P3] are called the axioms for soft multi open sets.

Definition 2.15. [5] Let (U,τ, E) be a soft multi topological space on U and F_A be a soft multi set in U. Then the intersection of all soft multi closed set containing F_A is called the closure of F_A and is denoted by Int(F_A) and denoted by Int(F_A) = $\tilde{\cup}${G_B : G_B is a soft multi open set contained in F_A}.

Definition 2.16. [5] Let (U,τ, E) be a soft multi topological space on U and F_A be a soft multi set in U.Then the intersection of all soft multi closed set containing F_A is called the closure of F_A and is denoted by Cl(F_A) and denoted by Cl(F_A) =$\tilde{\cap}${G_B:G_B is a soft multi closed set containing F_A}.

Proposition 2.17. [12] Let I be an arbitrary set and { F_E^i : i∈Λ, indexing set} $\tilde{\subseteq}$ SMS(U,τ_1,τ_2,E). Then, the following statements are true:

(a) $F_E^i \tilde{\subseteq} \tilde{\bigcup}_{i\in\Lambda}$ { F_E^i : $i\in\Lambda$}, for every i∈Λ. **(b)** $\tilde{\bigcap}_{i\in\Lambda}$ { F_E^i : $i\in\Lambda$} $\tilde{\subseteq}$ F_E^i, for every $i\in\Lambda$.

(c) [$\tilde{\bigcup}_{i\in\Lambda}$ { F_E^i : $i\in\Lambda$}]^c = [$\tilde{\bigcap}_{i\in\Lambda}$ { F_E^{ic}: $i\in\Lambda$}]. **(d)** [$\tilde{\bigcap}_{i\in\Lambda}$ { F_E^i : $i\in\Lambda$}]^c = [$\tilde{\bigcup}_{i\in\Lambda}$ { F_E^{ic}: $i\in\Lambda$}].

Theorem 2.18[4]. *Let* (U,τ_1,τ_2,E) *be a soft multi bitopological space. Then,*

(1) $\tilde{\phi}_E$, X_E *are soft multi closed sets.*

(2) *The intersection of arbitrary number of soft multi closed sets is soft multi closed set.*

(3) *The union of any two soft multi closed sets is a soft multi closed set.*

3. SOFT MULTI BITOPOLOGICAL SPACES

In this section the notions of soft multi bitopological spaces are introduced which are defined over an initial universe with a fixed set of parameters.

Definition 3.1: Let U be an initial universe set and E be the non-empty set of parameters. Let, (U,τ_1,E) and (U,τ_2,E) be the two different soft multi topologies on U. Then (U,τ_1,τ_2,E) is called a soft multi bitopological space.

The two soft topologies (U,τ_1,E) and (U,τ_2,E) are independently satisfy the axioms of soft topology. The members of τ_1 will be called τ_1-soft multi open sets and the complements of τ_1-soft multi open sets are called τ_1-soft multi closed sets.

Similarly, the members of τ_2 will be called τ_2-soft multi open sets and the complements of τ_2-soft multi open sets are called τ_2-soft multi closed sets.

Throughout this paper (U,τ_1,τ_2,E) denote soft multi bitopological space over U on which no separation axioms are assumed unless explicitly stated.

Example 3.2: Let us suppose that there be two initial universes U_1 ={a_1, a_2, a_3} and U_2 = {b_1, b_2, b_3}. Let, E_{U1} = {$e_{U1,1}$, $e_{U1,2}$}, E_{U2} ={$e_{U2,1}$, $e_{U2,2}$}, U = U_1×U_2, E = E_{U1}×E_{U2}, τ_1 = {$\tilde{U}_E$, $\tilde{\phi}_E$, F_E^1, F_E^2, F_E^3, F_E^4, F_E^5, F_E^6, F_E^7, F_E^8, F_E^9, F_E^{10}, F_E^{11}, F_E^{12}} and τ_2 ={$\tilde{U}_E$, $\tilde{\phi}_E$, G_E^1, G_E^2, G_E^3, G_E^4, G_E^5, G_E^6, G_E^7, G_E^8, G_E^9, G_E^{10}, G_E^{11}, G_E^{12}, G_E^{13}, G_E^{14}, G_E^{15}, G_E^{16}} where, F_E^1, F_E^2, F_E^3, F_E^4, F_E^5, F_E^6, F_E^7, F_E^8, F_E^9, F_E^{10}, F_E^{11}, F_E^{12}, G_E^1, G_E^2, G_E^3, G_E^4, G_E^5, G_E^6, G_E^7, G_E^8, G_E^9, G_E^{10}, G_E^{11}, G_E^{12}, G_E^{13}, G_E^{14}, G_E^{15}, G_E^{16} are soft multi sets over U, defined as follows:

F_E^1 = {(($e_{U1,1}$, $e_{U2,1}$), ({ a_1, a_2},{ b_2})), (($e_{U1,2}$, $e_{U2,2}$),({ a_2, a_3},{ b_1, b_3}))}

F_E^2 = {(($e_{U1,1}$, $e_{U2,1}$), ({ a_1, a_3},{ b_1 ,b_2})), (($e_{U1,2}$, $e_{U2,2}$),({ a_3},{ b_2}))}

F_E^3 = {(($e_{U1,1}$, $e_{U2,1}$), ({ a_1},{ b_1})), (($e_{U1,2}$, $e_{U2,2}$),({ a_3},ϕ))}

F_E^4 = {(($e_{U1,1}$, $e_{U2,1}$), ({ a_1, a_2 ,a_3},{ b_1 ,b_2})), (($e_{U1,2}$, $e_{U2,2}$),({ a_2, a_3},{ b_1, b_2,b_3}))}

F_E^5 = {(($e_{U1,1}$, $e_{U2,1}$), ({ a_2},{ b_2})), (($e_{U1,2}$, $e_{U2,2}$),(ϕ, ϕ))}

F_E^6 = {(($e_{U1,1}$, $e_{U2,1}$), ({ a_1, a_2},{ b_1 ,b_2})), (($e_{U1,2}$, $e_{U2,2}$),({ a_2, a_3},{ b_1, b_3}))}

F_E^7 = {(($e_{U1,1}$, $e_{U2,1}$), ({ a_2},ϕ)), (($e_{U1,2}$, $e_{U2,2}$),(ϕ, ϕ))}

F_E^8 = {(($e_{U1,1}$, $e_{U2,1}$), ({ a_1,a_2,a_3},{ b_1 ,b_2})), (($e_{U1,2}$, $e_{U2,2}$),({ a_3},{ b_2}))}

F_E^9 = {(($e_{U1,1}$, $e_{U2,1}$), ({ ϕ,{ b_2})), (($e_{U1,2}$, $e_{U2,2}$),(ϕ, ϕ))}

F_E^{10} = {(($e_{U1,1}$, $e_{U2,1}$), ({ a_1, a_2},{ b_1,b_2})), (($e_{U1,2}$, $e_{U2,2}$), ({ a_3},ϕ))}

F_E^{11} = {(($e_{U1,1}$, $e_{U2,1}$), ({ a_1, a_2},{ b_1})), (($e_{U1,2}$, $e_{U2,2}$),({ a_3},ϕ))}

F_E^{12} = {(($e_{U1,1}$, $e_{U2,1}$), ({ a_1},{ b_1,b_2})), (($e_{U1,2}$, $e_{U2,2}$),({ a_3},ϕ))}

G_E^1 = {(($e_{U1,1}$, $e_{U2,1}$), ({ a_2, a_3},{ b_2})), (($e_{U1,2}$, $e_{U2,2}$),({ a_3},{ b_2}))}

G_E^2 = {(($e_{U1,1}$, $e_{U2,1}$), ({ a_1, a_2},{ b_2 ,b_3})), (($e_{U1,2}$, $e_{U2,2}$),({ a_1 ,a_3},{ b_1,b_3}))}

G_E^3 = {(($e_{U1,1}$, $e_{U2,1,2}$), ({ a_1,a_3},{ b_2})), (($e_{U1,2,1}$, $e_{U2,2}$),({ a_1,a_2},{b_1, b_2,b_3}))}

G_E^4 = {(($e_{U1,1}$, $e_{U2,1,2}$), ({ a_2},{ b_2})), (($e_{U1,2,1}$, $e_{U2,2}$),({ a_3}, ϕ))}

G_E^5 = {(($e_{U1,1}$, $e_{U2,1,2}$), ({ a_3},{ b_2})), (($e_{U1,2,1}$, $e_{U2,2}$),(ϕ,{b_2}))}

G_E^6 = {(($e_{U1,1}$, $e_{U2,1}$), ({ a_1},{ b_2})), (($e_{U1,2}$, $e_{U2,2}$),({ a_1},{ b_1, b_3}))}

G_E^7 = {(($e_{U1,1}$, $e_{U2,1}$), (ϕ,{b_2})), (($e_{U1,2}$, $e_{U2,2}$),(ϕ, ϕ))}

G_E^8 = {(($e_{U1,1}$, $e_{U2,1}$), ({a_1,a_2,a_3},{ b_2 ,b_3})), (($e_{U1,2}$, $e_{U2,2}$),({ a_1,a_3}, {b_1,b_2,b_3}))}

G_E^9 = {(($e_{U1,1}$, $e_{U2,1}$), ({a_1,a_2,a_3},{ b_2})), (($e_{U1,2}$, $e_{U2,2}$),({a_1,a_2,a_3},{ b_1,b_2,b_3}))}

G_E^{10} = {(($e_{U1,1}$, $e_{U2,1}$), ({ a_1, a_2 ,a_3},{ b_2,b_3})), (($e_{U1,2}$, $e_{U2,2}$),({ a_1},{b_1,b_2,b_3}))}

G_E^{11} = {(($e_{U1,1}$, $e_{U2,1}$), ({ a_1, a_2 ,a_3},{ b_2,b_3})), (($e_{U1,2}$, $e_{U2,2}$), ({a_1,a_2,a_3},{ b_1,b_2,b_3}))}

G_E^{12} = {(($e_{U1,1}$, $e_{U2,1,2}$), ({ a_1, a_2 ,a_3},{ b_2})), (($e_{U1,2,1}$, $e_{U2,2}$),({ a_1},{ b_1,b_2,b_3}))}

G_E^{13} = {(($e_{U1,1}$, $e_{U2,1,2}$), ({ a_2},{b_2})), (($e_{U1,2,1}$, $e_{U2,2}$),(ϕ,ϕ))}

G_E^{14} = {(($e_{U1,1}$, $e_{U2,1,2}$), ({ a_1,a_2},{b_2})), (($e_{U1,2,1}$, $e_{U2,2}$),({ a_1},{b_1,b_3}))}

G_E^{15} = {(($e_{U1,1}$, $e_{U2,1,2}$), ({ a_2 ,a_3},{ b_2})), (($e_{U1,2,1}$, $e_{U2,2}$), (ϕ,{b_2}))}

G_E^{16} = {(($e_{U1,1}$, $e_{U2,1,2}$), ({ a_1, a_2 ,a_3},{b_2})), (($e_{U1,2,1}$, $e_{U2,2}$), ({a_1,a_2},{b_1,b_2,b_3}))}

Then (U,τ_1,τ_2,E) is a soft multi bitopological space.

Example 3.3: Let U be an initial universe set and E be the nonempty set of parameters. Soft multi indiscrete topology τ_1 = {$\tilde{U}_E, \tilde{\phi}_E$} and soft multi discrete topology τ_2 is the collection of all soft multi sets defined over X. Then (U,τ_1,τ_2,E) is a soft bitopological space.

Definition 3.4: Let (U,τ_1,τ_2,E) be a soft multi bitopological space over U and Y_E be a non-empty soft multi subset of U_E. Then τ_{1Y} = {${}^YF_E$ = $Y_E \tilde{\cap} F_E$: $F_E \in \tau_1$} and τ_{2Y} = {${}^YG_E$ = $Y_E \tilde{\cap} G_E$: $G_E \in \tau_2$} are said to be the relative topologies on Y. Then (Y, τ_{1Y}, τ_{2Y}, E) is called a relative soft multi bitopological space of (U,τ_1,τ_2,E).

Theorem 3.5: If (U,τ_1,τ_2,E) is a soft multi bitopological space then $\tau = \tau_1 \tilde{\cap} \tau_2$ is a soft multi topology over U.

Proof: (1). $\tilde{U}_E, \tilde{\phi}_E$ belong to $\tau_1 \tilde{\cap} \tau_2 = \tau$.

(2). Let $\{F^{\alpha}_E : \alpha \in \Lambda$, an index set$\}$ be a family of soft sets in $\tau_1 \tilde{\cap} \tau_2$.

Then for each $\alpha \in \Lambda$, $F^{\alpha}_E \in \tau_1$ and $F^{\alpha}_E \in \tau_2$. This implies that $\tilde{\cup}_{\alpha \in \Lambda} F^{\alpha}_E \in \tau_1$ and $\tilde{\cup}_{\alpha \in \Lambda} F^{\alpha}_E \in \tau_2$. Thus, $\tilde{\cup}_{\alpha \in \Lambda} F^{\alpha}_E \in \tau_1 \tilde{\cap} \tau_2 = \tau$.

(3). Let $F_E, G_E \in \tau_1 \tilde{\cap} \tau_2 = \tau$. Then $F_E, G_E \in \tau_1$ and $F_E, G_E \in \tau_2$.

Since, $F_E \tilde{\cap} G_E \in \tau_1$ and $F_E \tilde{\cap} G_E \in \tau_2 \Rightarrow F_E \tilde{\cap} G_E \in \tau_1 \tilde{\cap} \tau_2 = \tau$.

Thus, $\tau_1 \tilde{\cap} \tau_2 = \tau$ defines a soft topology on X.

Remark 3.6: If (U,τ_1,τ_2,E) is a soft multi bitopological space, then $\tau_1 \tilde{\cup} \tau_2$ is not in general soft multi topology on U which can be seen from the following example.

Example 3.7: In *example 3.2*, we find that (U,τ_1,τ_2,E) is a soft multi bitopological space but $\tau = \tau_1 \tilde{\cup} \tau_2 = \{\tilde{U}_E, \tilde{\phi}_E, F_E^1, F_E^2, F_E^3, F_E^4, F_E^5, F_E^6, F_E^7, F_E^8, F_E^9, F_E^{10}, F_E^{11}, F_E^{12}, G_E^1, G_E^2, G_E^3, G_E^4, G_E^5, G_E^6, G_E^7, G_E^8, G_E^9, G_E^{10}, G_E^{11}, G_E^{12}, G_E^{13}, G_E^{14}, G_E^{15}, G_E^{16}\}$ is not a soft multi topology on U since, $F_E^1 \tilde{\cup} G_E^1 = \{((e_{U1,1}, e_{U2,1}), (\{a_1,a_2,a_3\},\{b_1,b_2\})), ((e_{U1,2}, e_{U2,2}),(\{a_2,a_3\},\{b_1,b_2,b_3\}))\} \notin \tau_1 \tilde{\cup} \tau_2$.

Example 3.8: Consider the universes $U_1 = \{a_1, a_2, a_3\}$ and $U_2 = \{b_1, b_2, b_3\}$. Let, $E_{U1} = \{e_{U1,1}, e_{U1,2}\}$, $E_{U2} = \{e_{U2,1}, e_{U2,2}\}$, $U = U_1 \times U_2$, $E = E_{U1} \times E_{U2}$, $\tau_1 = \{\tilde{U}_E, \tilde{\phi}_E, F_E^1, F_E^2, F_E^3\}$ and $\tau_2 = \{\tilde{U}_E, \tilde{\phi}_E, G_E^1, G_E^2\}$ where, F_E^1, F_E^2, F_E^3, G_E^1 and G_E^2 are soft multi sets over U, defined as follows:

$F_E^1 = \{((e_{U1,1}, e_{U2,1}), (\{a_1\},\{b_2\})), ((e_{U1,2}, e_{U2,2}),(\{a_3\},\{b_3\}))\}$

$F_E^2 = \{((e_{U1,1}, e_{U2,1}), (\{a_3\},\{b_3\})), ((e_{U1,2}, e_{U2,2}),(\{a_1\},\{b_2\}))\}$

$F_E^3 = \{((e_{U1,1}, e_{U2,1}), (\{a_1,a_3\},\{b_2,b_3\})), ((e_{U1,2}, e_{U2,2}), (\{a_1,a_3\},\{b_2,b_3\}))\}$

$G_E^1 = \{((e_{U1,1}, e_{U2,1}), (\{a_2, a_2\},\{b_1,b_2\})), ((e_{U1,2}, e_{U2,2}),(\{a_2,a_3\},\{b_2,b_3\}))\}$

$G_E^2 = \{((e_{U1,1}, e_{U2,1}), (\{a_1\},\{b_1\})), ((e_{U1,2}, e_{U2,2}),(\{a_2\},\{b_2\}))\}$

Then (U,τ_1,τ_2,E) is a soft multi bitopological space but $\tau = \tau_1 \tilde{\cup} \tau_2 = \{\tilde{U}_E, \tilde{\phi}_E, F_E^1, F_E^2, F_E^3, G_E^1, G_E^2\}$ is not a soft multi topology on U since $F_E^3 \tilde{\cup} G_E^1 = \{((e_{U1,1}, e_{U2,1}), (\{a_1,a_2,a_3\},\{b_1,b_2,b_3\})), ((e_{U1,2}, e_{U2,2}),(\{a_1,a_2,a_3\},\{b_2,b_3\}))\} \notin \tau_1 \tilde{\cup} \tau_2$.

4. Some properties of soft multi bitopological spaces

In this section, the notions of pairwise open (closed) soft sets, pairwise soft interior (respectively, closure, kernel) operators are introduced and studied in a soft multi bitopological space (U,τ_1,τ_2,E). Moreover, the basic properties and fundamental theorems of these notions are presented. Finally, relationships between these soft sets are obtained.

Defnition 4.1. Let (U,τ_1,τ_2,E) be a soft multi bitopological space. A soft multi set (G,E) over U is said to be a pairwise open soft multi set in (U,τ_1,τ_2,E) (p-open soft multi set, for short) if there exist an open soft multi set G_E^1 in τ_1 and an open soft multi set G_E^2 in τ_2 such that $(G;E) = G_E^1 \tilde{\cup} G_E^2$.

Defnition 4.2. Let (U,τ_1,τ_2,E) be a soft multi bitopological space. A soft set (*F*,*E*) over *U* is said to be a pairwise closed soft multi set in (U,τ_1,τ_2,E) (p-closed soft multi set, for short) if its complement is a p-open soft multi set in (U,τ_1,τ_2,E).

Clearly, a soft multi set (*F*,*E*) over *U* is a p-closed soft multi set in (U,τ_1,τ_2,E) if there exist a closed soft multi set F_E^1 in τ_1^c and a closed soft multi set F_E^2 in τ_2^c such that (*F*,*E*) = F_E^1 $\tilde{\cap}$ F_E^1, where, τ_i^c={$G_E \in SMS(U_E)$: $G_E \in \tau_i$}, i=1,2.

The family of all p-open (closed) soft multi sets in (U,τ_1,τ_2,E) is denoted by *PSOS*(U,τ_1,τ_2,E) (*PSCS*(U,τ_1,τ_2,E).

The following theorem studies the main properties of p-open (closed) soft multi sets.

Theorem 4.4. *Let* (U,τ_1,τ_2,E) *be a soft multi bitopological space. Then,*

(a) $\tilde{\phi}_E, \tilde{U}_E$ *are p-open soft multi sets and p-closed soft multi sets.*

(b) *An arbitrary union of p-open soft multi sets is a p-open soft multi set.*

(c) *An arbitrary intersection of p-closed soft multi sets is a p-closed soft multi set.*

(d) *If* $G_E \in \tau_1 \tilde{\cap} \tau_2$ *and* $H_E \in PSMO$(U,τ_1,τ_2,E), *then* $G_E \tilde{\cap} H_E \in PSMO$(U,$\tau_1$,$\tau_2$,E).

Proof. **(a)** Since $\tilde{U}_E \in \tau_1, \tau_2$, and $\tilde{U}_E = \tilde{U}_E \tilde{\cup} \tilde{U}_E$, then U_E is a p-open soft *multi* set. Similarly, $\tilde{\phi}_E$ is a p-open soft *multi* set.

(b) Let { F_E^i : $i \in \Lambda$, indexing set} $\tilde{\subseteq}$ *SPOS*(U,τ_1,τ_2,E). Then, F_E^i is a p-open soft multi set for all $i \in \Lambda$, therefore there exist $F_{1E}^i \in \tau_1$ and $F_{2E}^i \in \tau_2$ such that $F_E^i = F_{1E}^i \tilde{\cup} F_{2E}^i$ for all $i \in \Lambda$, which implies that $\tilde{\bigcup}_{i\in\Lambda} F_E^i = \tilde{\bigcup}_{i\in\Lambda} [F_{1E}^i \tilde{\cup} F_{2E}^i] = [\tilde{\bigcup}_{i\in\Lambda} F_{1E}^i] \tilde{\cup} [\tilde{\bigcup}_{i\in\Lambda} F_{2E}^i]$. Now, since τ_1 and τ_2 are soft multi topologies, then$[\tilde{\bigcup}_{i\in\Lambda} F_{1E}^i] \in \tau_1$ and $[\tilde{\bigcup}_{i\in\Lambda} F_{2E}^i] \in \tau_2$. Consequently, $\tilde{\bigcup}_{i\in\Lambda} F_E^i$ is a p-open soft multi set.

(c) It is immediate from the Definition 4.2, Propositions 2.17 and 2.18.

(d) It is obvious.

Corollary 4.5. Suppose that (U,τ_1,τ_2,E) be a soft multi bitopological space. Then, the family of all p-open soft multi sets is a supra soft multi topology on X. This supra soft multi topology we denoted by $\boldsymbol{\tau}_{12}$, i.e., $\boldsymbol{\tau}_{12}$= POS(U,τ_1,τ_2,E)=G_E={ $G_E^1 \tilde{\cup} G_E^2 : G_E^i \in \tau_i$,i = 1,2} and the triple (U,$\boldsymbol{\tau}_{12}$,E) is the supra soft topological space associated to the soft multi bitopological space (U,τ_1,τ_2,E). The following example shows that:

(a) $\boldsymbol{\tau}_{12}$ is not soft multi topology in general.

(b) The finite intersection of p-open soft multi sets need not be a p-open soft multi set.

(c) The arbitrary union of p-closed soft multi sets need not be a p-closed soft multi set.

Example 4.6. Consider the universes U_1={a_1, a_2, a_3} and U_2={b_1, b_2, b_3}. Let, E_{U1} ={$e_{U1,1}$, $e_{U1,2}$}, E_{U2} ={$e_{U2,1}$, $e_{U2,2}$}, U= $U_1 \times U_2$, E =$E_{U1} \times E_{U2}$, e^1=($e_{U1,1}$,$e_{U1,2}$), e^2=($e_{U2,1}$,$e_{U2,2}$), τ_1={$\tilde{U}_E$, $\tilde{\phi}_E$, F_E^1, F_E^2, F_E^3} and τ_2={$\tilde{U}_E$, $\tilde{\phi}_E$, G_E^1, G_E^2} where, $F_E^1, F_E^2, F_E^3, G_E^1$ and G_E^2 are soft multi sets over U, defined as follows:

F_E^1 = {(($e_{U1,1}$,$e_{U2,1}$), ({ a_1},{ b_2})), (($e_{U1,2}$, $e_{U2,2}$),({ a_3},{b_3}))}

F_E^2 = {(($e_{U1,1}$, $e_{U2,1}$), ({ a_3},{ b_3})), (($e_{U1,2}$, $e_{U2,2}$),({ a_1},{ b_2}))}

F_E^3 = {((e$_{U1,1}$, e$_{U2,1}$), ({a$_1$,a$_3$},{ b$_2$,b$_3$})), ((e$_{U1,2}$, e$_{U2,2}$), ({a$_1$,a$_3$},{ b$_2$,b$_3$}))}

G_E^1 = {((e$_{U1,1}$, e$_{U2,1}$), ({ a$_2$, a$_2$},{ b$_1$,b$_2$})), ((e$_{U1,2}$, e$_{U2,2}$),({ a$_2$,a$_3$},{b$_2$,b$_3$}))}

G_E^2 = {((e$_{U1,1}$, e$_{U2,1}$), ({ a$_1$},{ b$_1$})), ((e$_{U1,2}$, e$_{U2,2}$),({ a$_2$},{b$_2$}))}

Then (U,τ_1,τ_2,E) is a soft multi bitopological space.

Clearly, τ_{12}={$\tilde{U}_E, \tilde{\phi}_E, F_E^1, F_E^2, F_E^3, G_E^1, G_E^2, H_E^1, H_E^2, H_E^3$} where,

$H_E^1 = F_E^2 \tilde{\cup} G_E^1$ ={((e$_{U1,1}$, e$_{U2,1}$), ({a$_1$,a$_2$,a$_3$},{b$_1$,b$_2$,b$_3$})), ((e$_{U1,2}$, e$_{U2,2}$), ({a$_1$, a$_2$, a$_3$},{b$_2$,b$_3$}))}

$H_E^2 = F_E^1 \tilde{\cup} G_E^2$={((e$_{U1,1}$, e$_{U2,1}$), ({a$_1$},{b$_1$,b$_2$})), ((e$_{U1,2}$, e$_{U2,2}$), ({a$_2$, a$_3$},{b$_2$,b$_3$}))}

$H_E^3 = F_E^2 \tilde{\cup} G_E^2$={((e$_{U1,1}$, e$_{U2,1}$), ({a$_1$,a$_3$},{b$_1$,b$_3$})), ((e$_{U1,2}$, e$_{U2,2}$), ({a$_2$, a$_3$},{ b$_2$}))} is not a soft multi topology on U since $H_E^2 \tilde{\cup} H_E^3$={((e$_{U1,1}$, e$_{U2,1}$), ({a$_1$,a$_3$},{b$_1$,b$_2$,b$_3$})), ((e$_{U1,2}$, e$_{U2,2}$), ({a$_2$, a$_3$},{b$_2$,b$_3$}))} and $H_E^2 \tilde{\cap} H_E^3$={((e$_{U1,1}$, e$_{U2,1}$), ({a$_1$},{b$_1$})), ((e$_{U1,2}$, e$_{U2,2}$), ({a$_2$, a$_3$},{b$_2$}))}$\notin \tau_{12}$.

Theorem 4.7. Let (U,τ_1,τ_2,E) be a soft multi bitopological space. Then,

(a) Every τ_i-open soft multi set is a p-open soft multi set, i = 1,2, i.e., $\tau_1 \tilde{\cup} \tau_2 \tilde{\subseteq} \tau_{12}$.

(b) Every τ_i-closed soft multi set is a p-closed soft multi set, i = 1, 2, i.e., $\tau^c_1 \tilde{\cup} \tau^c_2 \tilde{\subseteq} \tau^c_{12}$ (=PCSMS(U,τ_1,τ_2,E)).

(c) If $\tau_1 \tilde{\subseteq} \tau_2$ then, $\tau_{12}=\tau_2$ and $\tau^c_{12}=\tau^c_2$.

Proof. Straight forward.

Lemma 4.8. Let (U,τ_1,τ_2,E) be a soft multi bitopological space. A soft multi set G$_E$ over U is a p-open soft multi set if and only if for all soft multi point e$_{(F,A)}\in$G$_E$ there exists $G_{E1}^{e(F,A)} \in \tau_1$ such that $G_{E1}^{e(F,A)} \tilde{\subseteq}$ G$_E$ or there exists $G_{E2}^{e(F,A)} \in \tau_2$ such that $G_{E2}^{e(F,A)} \tilde{\subseteq}$ G$_E$, where $G_{Ei}^{e(F,A)}$ denote a τ_i-open soft multi set containing e$_{(F,A)}$, i = 1, 2.

Proof. Let G$_E$ be a p-open soft multi set in (U,τ_1,τ_2,E). Then, there exist $G_E^1 \in \tau_1$ and an open soft multi set $G_E^2 \in \tau_2$ such that G$_E$= $G_E^1 \tilde{\cup} G_E^2$. Now, let e$_{(F,A)}\in$G$_E$. Then, e$_{(F,A)}\in G_E^1 \tilde{\cup} G_E^2$ which implies that e$_{(F,A)}\in G_E^1$ or e$_{(F,A)}\in G_E^2$.Then we get, e$_{(F,A)}\in G_E^1 \tilde{\subseteq}$ G$_E$ or e$_{(F,A)}\in G_E^2 \tilde{\subseteq}$ G$_E$. Hence, for every e$_{(F,A)}\in$G$_E$, there exists $G_{E1}^{e(F,A)} = G_E^1 \in \tau_1$ such that $G_{E1}^{e(F,A)} \tilde{\subseteq}$ G$_E$ or there exists $G_{E2}^{e(F,A)} = G_E^2 \in \tau_2$ such that $G_{E2}^{e(F,A)} \tilde{\subseteq}$ G$_E$.

Conversely, suppose that for every e$_{(F,A)}\in$G$_E$, there exists $G_{E1}^{e(F,A)} \in \tau_1$ such that $G_{E1}^{e(F,A)} \tilde{\subseteq}$ G$_E$ or there exists $G_{E2}^{e(F,A)} \in \tau_2$ such that $G_{E2}^{e(F,A)} \tilde{\subseteq}G_E$. We shall prove that G$_E\in\tau_{12}$. From hypothesis, we set Σ_1 = $\tilde{\bigcup}_{e(F,A)\in GE}$ { $G_{E1}^{e(F,A)} \tilde{\subseteq}$ G$_E$: $G_{E1}^{e(F,A)} \in \tau_1$} and Σ_2 = $\tilde{\bigcup}_{e(F,A)\in GE}$ { $G_{E2}^{e(F,A)} \tilde{\subseteq}G_E$: $G_{E2}^{e(F,A)} \in \tau_2$}. Then, $\Sigma_1 \tilde{\cup} \Sigma_2$ = G$_E$. Since, $\Sigma_1\in\tau_1$ and $\Sigma_2\in\tau_2$, thus G$_E$ $\in\tau_{12}$.

Definition 4.9. Let (U,τ_1,τ_2,E) be a soft multi bitopological space and let G$_E\in$*SMS(U$_E$)*. The pairwise soft multi closure of G$_E$, denoted by *smcl$_p$(G$_E$)*, is the intersection of all p-closed soft multi super sets of (G$_E$), i.e., *smcl$_p$(G$_E$)*= $\tilde{\cap}$ {F$_E$ $\in \tau_{12}^c$: G$_E$ $\tilde{\subseteq}$F$_E$ }.

Clearly, *smcl$_p$(G$_E$)* is the smallest p-closed soft set containing G$_E$.

Theorem 4.10. Let (U,τ_1,τ_2,E) be a soft multi bitopological space and let G_E, $H_E \in SMS(\tilde{U}_E)$. Then, **(a)** $smcl_p(\tilde{\phi}_E) = \tilde{\phi}_E$ and $smcl_p(\tilde{U}_E) = \tilde{U}_E$.

(b) $G_E \tilde{\subseteq} smcl_p(G_E)$.

(c) G_E is a p-closed soft set if and only if $G_E = smcl_p(G_E)$.

(d) $G_E \tilde{\subseteq} H_E) \Rightarrow smcl_p(G_E) \tilde{\subseteq} smcl_p(H_E)$.

(e) $smcl_p(G_E) \tilde{\cup} smcl_p(H_E) \tilde{\subseteq} smcl_p(G_E \tilde{\cup} H_E)$.

(f) $smcl_p[smcl_p(G_E)] = smcl_p(G_E)$ i.e, $smcl_p(G_E)$ is a p-closed soft multi set.

Proof. Straight forward.

Theorem 4.11. Let (U,τ_1,τ_2,E) be a soft multi bitopological space and let $G_E \in SMS(U_E)$. Then,

$e_{(F,A)} \in smcl_p(G_E) \Leftrightarrow G_E^{e(F,A)} \tilde{\cap} G_E \neq \tilde{\phi}_E$, $\forall G_E^{e(F,A)} \in \tau_{12}(e_{(F,A)})$, where, $G_E^{e(F,A)}$ is any p-open soft multi set contains $e_{(F,A)}$ and $\tau_{12}(e_{(F,A)})$ is the family of all p-open soft sets contains $e_{(F,A)}$.

Proof. Let $e_{(F,A)} \in smcl_p(G_E)$ and assume that there exists $G_E^{e(F,A)} \in \tau_{12}(e_{(F,A)})$ such that

$G_E^{e(F,A)} \tilde{\cap} G_E = \tilde{\phi}_E$. Then, $G_E \tilde{\subseteq} G_E^{e(F,A)} \Rightarrow smcl_p(G_E) \tilde{\subseteq} smcl_p(G_E^{e(F,A)}) = (G_E^{e(F,A)})^c$ which implies that $smcl_p(G_E^{e(F,A)}) \tilde{\cap} G_E = \tilde{\phi}_E$, which is a contradiction.

Conversely, assume that $e_{(F,A)} \notin smcl_p(G_E)$, then $e_{(F,A)} \in [smcl_p(G_E)]^c$. Therefore, $[smcl_p(G_E)]^c \in \tau_{12}(e_{(F,A)})$.So, by hypothesis, $[smcl_p(G_E)]^c \tilde{\cap} G_E \neq \tilde{\phi}_E$, a contradiction.

Theorem 4.12. Let (U,τ_1,τ_2,E) be a soft multi bitopological space. *A soft multi set* F_E *over U is a p-closed soft multi set if and only if* $F_E = smcl_{\tau_1}(F_E) \tilde{\cap} smcl_{\tau_2}(F_E)$.

Proof. Let F_E be a p-closed soft multi set and let $e_{(F,A)} \notin F_E$. Then, $e_{(F,A)} \notin smcl_p(F_E)$[for $smcl_p(F_E) = F_E$]. Therefore, by Theorem 4.11, there exists $G_E^{e(F,A)} \in \tau_{12}(e_{(F,A)})$ such that $G_E^{e(F,A)} \tilde{\cap} G_E = \tilde{\phi}_E$. Since $G_E^{e(F,A)} \in \tau_{12}(e_{(F,A)})$, then there exists $G_E^1 \in \tau_1$ and $G_E^2 \in \tau_2$ such that $G_E^{e(F,A)} = G_E^1 \tilde{\cup} G_E^2$. Therefore, $[G_E^1 \tilde{\cup} G_E^2] \tilde{\cap} F_E = \tilde{\phi}_E$, it follows that $G_E^1 \tilde{\cap} F_E = \tilde{\phi}_E$ and $G_E^2 \tilde{\cap} F_E = \tilde{\phi}_E$. Since, $e_{(F,A)} \in G_E^{e(F,A)}$, then $e_{(F,A)} \in G_E^1$ or $e_{(F,A)} \in G_E^2$ implies $e_{(F,A)} \notin smcl_{\tau_1}(F_E)$ *or* $e_{(F,A)} \notin smcl_{\tau_2}(F_E)$.Therefore, $e_{(F,A)} \notin smcl_{\tau_1}(F_E) \tilde{\cap} smcl_{\tau_2}(F_E)$. Hence, $smcl_{\tau_1}(F_E) \tilde{\cap} smcl_{\tau_2}(F_E) \tilde{\subseteq} F_E$. On the other hand, we have, $F_E \tilde{\subseteq} smcl_{\tau_1}(F_E) \tilde{\cap} smcl_{\tau_2}(F_E)$ Hence, $F_E = smcl_{\tau_1}(F_E) \tilde{\cap} smcl_{\tau_2}(F_E)$.

Conversely, assume that $F_E = smcl_{\tau_1}(F_E) \tilde{\cap} smcl_{\tau_2}(F_E)$. Since, $smcl_{\tau_1}(F_E)$ is a closed soft multi set in (U,τ_1,E) and $smcl_{\tau_2}(F_E)$ is a closed soft multi set in (U,τ_2,E), then, by Definition 4.2, $smcl_{\tau_1}(F_E) \tilde{\cap} smcl_{\tau_2}(F_E)$ is a p-closed soft multi set in (U,τ_1,τ_2,E), therefore F_E is a p-closed soft set.

Corollary 4.13. *Let* (U,τ_1,τ_2,E) *be a soft multi bitopological space. Then,* $smcl_p(G_E) = smcl_{\tau_1}(G_E) \tilde{\cap} smcl_{\tau_2}(G_E)$, $\forall G_E \in SMS(\tilde{U}_E)$.

Definition 4.14. Let (U,τ_1,τ_2,E) be a soft multi bitopological space and let $F_E \in SMS(\tilde{U}_E)$. The pairwise soft multi interior of G_E, denoted by $smint_p(F_E)$, is the union of all p-closed soft multi subsets of (F_E), i.e., $smint_p(F_E) = \tilde{\cup}\{H_E \in \tau_{12}: H_E \tilde{\subseteq} F_E\}$.

Clearly, $smint_p(F_E)$ is the largest p-open soft multi set containing F_E.

Theorem 4.15. Let (U,τ_1,τ_2,E) be a soft multi bitopological space and let G_E, $H_E \in SMS(\tilde{U}_E)$. Then, **(a)** $smint_p(\tilde{\phi}_E) = \tilde{\phi}_E$ and $smint_p(\tilde{U}_E) = \tilde{U}_E$.

(b) if $smint_p(G_E) \tilde{\subseteq} G_E$

(c) G_E is a p-open soft multi set if and only if $smint_p(G_E) = G_E$.

(d) $G_E \tilde{\subseteq} H_E) \Rightarrow smint_p(G_E) \tilde{\subseteq} smint_p(H_E)$.

(e) $smint_p(G_E \tilde{\cap} H_E) \tilde{\subseteq} smint_p(G_E) \tilde{\cap} smint_p(H_E)$.

(f) $smint_p[smint_p(G_E)] = smint_p(G_E)$ i.e, $smint_p(G_E)$ is a p-open soft multi set.

Proof. Straight forward.

Theorem 4.16. Let (U,τ_1,τ_2,E) be a soft multi bitopological space and $G_E \in SMS(U_E)$.Then, $e_{(F,A)} \in smint_p(G_E)$ iff $\exists$ $G_E^{e(F,A)} \in \tau_{12}(e_{(F,A)})$, such that $G_E^{e(F,A)} \tilde{\subseteq} G_E$.

Proof. Straightforward.

Theorem 4.17. Let (U,τ_1,τ_2,E) be a soft multi bitopological space. A soft set G_E over U is a p-open soft multi set if and only if $G_E = smcl_{\tau 1}(G_E) \tilde{\cup} smcl_{\tau 2}(G_E)$.

Proof. Let G_E be a p-open soft multi set. Since, $smcl_{\tau i}(G_E) \tilde{\supseteq} G_E$, i = 1, 2, then $smcl_{\tau 1}(G_E) \tilde{\cup} smcl_{\tau 2}(G_E) \tilde{\supseteq} G_E$. Now, let $e_{(F,A)} \in G_E$. Then, by Lemma 4.8, there exists $G_{E1}^{e(F,A)} \in \tau_1$ such that $G_{E1}^{e(F,A)} \tilde{\subseteq} G_E$ or there exists $G_{E2}^{e(F,A)} \in \tau_2$ such that $G_{E2}^{e(F,A)} \tilde{\subseteq} G_E$. Therefore, $e_{(F,A)} \in smcl_{\tau 1}(G_E)$ or $e_{(F,A)} \in smcl_{\tau 2}(G_E)$. Hence, $e_{(F,A)} \in smcl_{\tau 1}(G_E) \tilde{\cup} smcl_{\tau 2}(G_E)$. Consequently, $G_E = smcl_{\tau 1}(G_E) \tilde{\cup} smcl_{\tau 2}(G_E)$.

Conversely, since $smcl_{\tau 1}(G_E)$ is an open soft multi set in (U,τ_1,E) and $smcl_{\tau 2}(G_E)$ is an open soft multi set in (U,τ_2,E), then, by Definition 4.1, $smcl_{\tau 1}(G_E) \tilde{\cup} smcl_{\tau 2}(G_E)$ is a p-open soft multi set in (U,τ_1,τ_2,E). Hence, G_E is a p-open soft multi set.

Corollary 4.18. Let (U,τ_1,τ_2,E) be a soft multi bitopological space. Then, $smcl_p(G_E) = smcl_{\tau 1}(G_E) \tilde{\cup} smcl_{\tau 2}(G_E)$.

Remark 4.19. Let (U,τ_1,τ_2,E) be a soft multi bitopological space. Then,

(a) $smcl_p(G_E) \tilde{\cup} smcl_p(H_E) \neq smcl_p(G_E \tilde{\cup} H_E)$, in general.

(b) $smcl_p(G_E) \tilde{\cap} smcl_p(H_E) \neq smcl_p(G_E \tilde{\cap} H_E)$, in general.

This can be seen from the following example:

Example 4.20. Let (U,τ_1,τ_2,E) be a soft multi bitopological space in Example 4.6 and let

$G_E = \{((e_{U1,1}, e_{U2,1}), (\{a_1\},\{b_1,b_2\})), ((e_{U1,2}, e_{U2,2}), (\{a_2,a_3\}, \{b_1\}))\}$

$H_E = \{((e_{U1,1}, e_{U2,1}), (\{a_3\},\{b_2,b_3\})), ((e_{U1,2}, e_{U2,2}), (\{a_1\},\{b_1,b_3\}))\}$.

Then, **(a)** Here, $smcl_p(G_E) = F_E^{2c}$, $smcl_p(H_E) = G_E^{2c}$.

Now, $smcl_p(G_E) \tilde{\cup} smcl_p(H_E) = \{((e_{U1,1}, e_{U2,1}), (U,U)), ((e_{U1,2}, e_{U2,2}), (U,\{b_1,b_3\}))\}$

$G_E \tilde{\cup} H_E = \{((e_{U1,1}, e_{U2,1}), (\{a_1,a_3\},,U)), ((e_{U1,2}, e_{U2,2}), (U,\{b_1,b_3\}))\}$, $smcl_p(G_E \tilde{\cup} H_E) = \tilde{U}_E$.

Thus, $smcl_p(G_E) \tilde{\cup} smcl_p(H_E) \tilde{\subseteq} smcl_p(G_E \tilde{\cup} H_E)$ but $smcl_p(G_E) \tilde{\cup} smcl_p(H_E) \neq smcl_p(G_E \tilde{\cup} H_E)$.

(b) *Here,* $smcl_p(G_E)\,\tilde{\cap}\,smcl_p(H_E)=\{((e_{U1,1}, e_{U2,1}), (\{a_2\},\{b_2\})), ((e_{U1,2}, e_{U2,2}), (\{a_3\}, \{b_1,b_3\}))\}$,

$G_E\,\tilde{\cap}\,H_E=\{((e_{U1,1}, e_{U2,1}), (\phi, \{b_2\})), ((e_{U1,2}, e_{U2,2}), (\phi,\{b_1\}))\}$

Now, $smcl_p(G_E\,\tilde{\cap}\,H_E)=\{((e_{U1,1}, e_{U2,1}), (\{a_2\},\{b_2\})), ((e_{U1,2}, e_{U2,2}), (\phi, \{b_1,b_3\}))\}$.

Thus, $smcl_p(G_E\,\tilde{\cap}\,H_E) \neq smcl_p(G_E)\,\tilde{\cap}\,smcl_p(H_E)$.

Remark 4.21. Let (U,τ_1,τ_2,E) be a soft multi bitopological space. Then,

(a) $smint_p(P_E\,\tilde{\cap}\,C_E) \neq smint_p(P_E)\,\tilde{\cap}\,smint_p(C_E)$, in general.

(b) $smint_p(B_E)\,\tilde{\cup}\,smint_p(C_E) \neq smint_p(B_E\,\tilde{\cup}\,C_E)$, in general.

This can be seen from the following example:

Example 4.22. Let (U,τ_1,τ_2,E) be a soft multi bitopological space in Example 4.6 and let

$B_E=\{((e_{U1,1}, e_{U2,1}), (\{a_1,a_2,a_3\},\{b_2,b_3\})), ((e_{U1,2}, e_{U2,2}), (\{a_1,a_2,a_3\}, \{b_2,b_3\}))\}$

$C_E=\{((e_{U1,1}, e_{U2,1}), (\{a_2,a_3\},\{b_1,b_3\})), ((e_{U1,2}, e_{U2,2}), (\{a_1,a_2\}, \{b_1,b_2\}))\}$

$P_E=\{((e_{U1,1}, e_{U2,1}), (\{a_1,a_2,a_3\},\{b_1,b_3\})), ((e_{U1,2}, e_{U2,2}), (\{a_2,a_3\}, \{b_1,b_2\}))\}$

Then, **(a)** Here, $smint_p(C_E)= F_E^2$, $smint_p(P_E)= H_E^3$,

$smint_p(C_E)\,\tilde{\cap}\,smint_p(P_E)=\{((e_{U1,1}, e_{U2,1}), (\{a_3\},\{b_3\})), ((e_{U1,2}, e_{U2,2}), (\phi,\{b_2\}))\}$.

$C_E\,\tilde{\cap}\,P_E=\{((e_{U1,1}, e_{U2,1}), (\{a_2,a_3\},\{b_1,b_3\})), ((e_{U1,2}, e_{U2,2}), (\{a_2\},\{b_1,b_2\}))\}$,

$smint_p(C_E\,\tilde{\cap}\,P_E)=\tilde{\phi}_E$. Thus, $smint_p(C_E\,\tilde{\cap}\,P_E)\,\tilde{\subseteq}\,smint_p(C_E)\,\tilde{\cap}\,smint_p(P_E)$ *but* $smint_p(C_E\,\tilde{\cap}\,P_E) \neq smint_p(C_E)\,\tilde{\cap}\,smint_p(P_E)$.

(b) Here, $smint_p(B_E)= F_E^3$, $smint_p(C_E)= F_E^2$,

$smint_p(B_E)\,\tilde{\cup}\,smint_p(C_E)= F_E^3$. $(B_E\,\tilde{\cup}\,C_E)=\tilde{U}_E$, $smint_p(G_E\,\tilde{\cup}\,H_E)=\tilde{U}_E$.

Thus, $smint_p(B_E)\,\tilde{\cup}\,smint_p(C_E) \neq smint_p(B_E\,\tilde{\cup}\,C_E)$.

5. OPEN PROBLEM:

5. OPEN PROBLEM: In theorem, 4.10 and 4.15, it was found that $smcl_p(G_E)\,\tilde{\cup}\,smcl_p(H_E)\,\tilde{\subseteq}\,smcl_p(G_E\,\tilde{\cup}\,H_E)$ and $smint_p(G_E\,\tilde{\cap}\,H_E)\,\tilde{\subseteq}\,smint_p(G_E)\,\tilde{\cap}\,smint_p(H_E)$. But in remark 4.19 and 4.21, we find that in general, $smcl_p(G_E)\,\tilde{\cap}\,smcl_p(H_E) \neq smcl_p(G_E\,\tilde{\cap}\,H_E)$ *and* $smint_p(B_E)\,\tilde{\cup}\,smint_p(C_E) \neq smint_p(B_E\,\tilde{\cup}\,C_E)$. Naturally question arises is it generally true that $smcl_p(G_E\,\tilde{\cap}\,H_E)\,\tilde{\subseteq}\,smcl_p(G_E)\,\tilde{\cap}\,smcl_p(H_E)$ and $smint_p(B_E)\,\tilde{\cup}\,smint_p(C_E)\,\tilde{\subseteq}\,smint_p(B_E\,\tilde{\cup}\,C_E)$??

6. CONCLUSION

The soft multi set theory proposed by D.Tokat and I.Osmanoglu offers a general mathematical tool for dealing with uncertain or vague objects. It is shown that soft sets are special type of information system known as multi valued information system. In this work, the notions of soft multi bitopological spaces are introduced which are defined over an initial universe with a fixed set of parameters and also generalize the notions of soft topological spaces such as open soft sets, closed soft sets, soft interior, soft closure in a soft multi bitopological spaces. Also the notions of pairwise open (closed) soft multi sets, pairwise soft multi interior (multi closure) operators are introduced in a soft multi bitopological space (U,τ_1,τ_2,E). The properties of these notions and some important results related to it are obtained. The following definitions which are prerequisites for present study are considered unless otherwise stated. I hope that the findings in this paper will motivate researcher and promote the further study on soft topological spaces to carry out a general framework for their applications practical life.

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