Economic-Production-Inventory Model for a Single-Vendor and Multiple-Buyers with Time-Varying Demand

Shalini Rathore
Department of Mathematics
Amity University, Mumbai
drshalinirathore82@gmail.com, ssingh@mum.amity.edu

ABSTRACT

In this paper, we consider a deteriorating production-inventory system having a single-vendor and multiple-buyers. The time-varying demand rate is considered. It is assumed that the items deteriorate at a constant rate and production rate is finite. Shortages are not allowed. Our aim in the present investigation is to determine the total joint cost for both vendors and buyers over a finite planning horizon. Numerical examples are presented to validate the analytical results of the proposed model.

Keywords: *Production-inventory system, Single-vendor, Multi-buyer, Deterioration, Time-varying demand, Shortages, Cost analysis.*

1. INTRODUCTION

The production system is specified as arrangements of people, resources, energy, machines and technologies, whereby agreed upon forms of work are accomplished. Thus, any working arrangement can be considered as a productive system, including, for example, private entrepreneurs, charities, multinational corporations and public utilities, etc.. The product of these organizations can be a manufactured item to be sold in the markets or a particular service delivered through an agency. The productive process, accordingly, is then taken to be the transfer function, which generates the flow of required outputs from the flow of available inputs.

Organizations should evidently seek high productivity. Their respective measures of productivity reflect their objectives and the attained valued on these measures indicate how efficiently they are performing. Low productivity compared with other businesses in an industry will lead to a company having relatively higher costs and ultimately losing out in the market place. Decreasing productivity across a whole industry will generate price-inflation, loss of demand and ultimately a recession.

Xu. et al. (2017) proposed an inventory system with periodic review base stock and partial backlogging. Guchhait et al. (2015) developed a model for inventory system with time dependent deteriorating items to determine the profit maximization. Sicilia et. al. (2014) analyzed shortages in inventory model where demand is constant and varies with time and follow power pattern. Agrawal et al. (2013) also considered an inventory system with two warehouses where demand rate is ramp type and deterioration rate is constant. Min et al. (2012) developed inventory model in which items are deteriorating exponentially and shortages are allowed. Again Madhavilata et al. (2011) introduced two levels of storage for inventory of single item in their research work. Rong et al. (2008) presented an EOQ model with two-warehouses for the perishable goods with fuzzy lead time and partially/ fully backlogged shortage. Lebacque et al. (2007) suggested the methods which are aimed at maintaining the production rate of each type of part as smooth as possible and therefore holding small inventory and shortage costs. Burke et al. (2007) analyzed single period, single product sourcing decisions under demand uncertainty. Their approach includes the product prices, supplier costs, supplier capacities, historical supplier reliabilities and firm specific inventory costs. Bretthauer et al. (2006) presented a model and solution methodology for production and inventory management problems that involve multiple resource constraints. Their model formulation was general, allowing organizations to handle a variety of multi-item decisions such as determining order quantities, production batch sizes, number of production runs, or cycle times. They presented efficient algorithms for solving both continuous and integer variable versions of the resource constrained production and inventory management model. Chan et al. (2006) addressed the supplier-scheduling problem by considering the deliveries scheduling issue, once the optimal replenishment cycles are determined. They considered four large integer programming problems according to four different objectives in cost and resource minimization and solved them by converting them into network flow problems.

Chen and Chen (2005) considered a single product that is subject to continuous decay, facing a price-dependent and time-varying demand, and time-varying deteriorating rate, with the objective of maximizing the profit stream over multi-period planning horizon. Lin et al. (2005) considered the economic lot-scheduling problem (ELSP) for a production-inventory system where items produced are subject to continuous deterioration. Wang and Sheu (2003) studied the effects of general time to shift distributions, two types of process inspection errors and general repair policy on the optimal production/inspection/maintenance policy.

Wang and Sheu (2001) considered the relationship between production, inventory and inspection in a deteriorating production system, which may transit from the "in-control" state to the "out-of-control" state after a period of operation. Wu and Wee (2001) examined the buyer-seller joint cost for deteriorating items with multiple-lot-size deliveries. Wang and Sheu (2000) studied the economical manufacturing quality (EMQ) problem in the presence of an imperfect process. When the production process goes out of control, the production process produces some proportion of defective items. Yang and Wee (2000) developed an economic ordering policy of deteriorated items for vendors and buyers. Most of the previous works on classical inventory models are based on the assumptions that the value (or utility) of inventory remains constant over time. A special case of inventory model is studied by Hwang (1999) for both ameliorating when the items stay at breeding yard (fish culture facility or farming yard) and deteriorating when the items stay in the distribution systems.

Wee and Law (1999) applied the discounted cash-flow approach to a deterministic inventory model of an item that deteriorates over time at a varying rate. An economic production scheduling policy which allows for variations in production quantity, scheduling time and shortages is developed by Wee and Wang (1999). Andijani and Dajani (1998) considered an inventory-production system where items deteriorate at a constant rate. The inventory problem is first modeled as a linear optimal control problem. Then linear quadratic regulator (LQR) technique is applied to the control problem in order to determine the optimal production policy. Generally, inventory control policies for deteriorating items are very sensitive to different marketing policies especially in chemical, food and pharmaceutical industries. Realizing the importance of such inventory policies in practice, an integrated production-inventory-marketing model was developed by Goyal and Gunasekaran (1995) for determining the economic production quantity (EPQ) and economic order quantity (EOQ) for raw materials in a multi-stage production system.

Wee (1993) formulated an economic production plan for deteriorating items with partial back-ordering. Yang and Wee (2002) developed a production-inventory system model of a deteriorating item, taking into account the view of both the vendor and the multi-buyers. Heng et al. (1991) assumed an order-level lot size inventory model for deteriorating items with finite replenishment rate.

In the present investigation, we analyze a production inventory system with, deterioraring items. The time varying demands are considered. The rest of the paper is organized as follows. In section 2, we describe inventory model by defining requisite assumptions and notations. The mathematical analysis is presented in section 3. In section 4, we derive some performance indices such as maximum inventory level, deteriorated cost, holding cost, etc.. In section 5, cost minimization problem is considered. In section 6, some special cases are deduced by setting appropriate parameters. In section 7, a graph is depicted. Finally in section 8, we summarize our findings, and discuss how our model can be further modified.

2. MODEL DISCRIPTION

Consider a single-vendor and multi-buyers inventory-production system with time-varying demand. The inventory system involves only one type of item, which deteriorates at a constant rate. Deterioration of the units is considered only after these items have been received into the inventory. No replacement or repair of deteriorated items is made during a given cycle. Shortages are not allowed and production rate is finite.

Production rate is greater than the buyers' demand. Here we establish a stationary policy where the buyers order the same lot size. Holding cost is relevant only with good units.

The following notations are used for modeling purpose:

N	Number of buyers
θ	Deterioration rate
d_i	Demand rate per year for buyer i, $d_i = \alpha_i I(t) + \beta_i$, i=1,2,…,N
α_i	Parameter corresponding to the demand rate.
β_i	Initial demand rate.
P	Production rate per year
T	Time length of each cycle, $T=T_1+T_2$
T_1	Length of production time in each production time in each production cycle T
T_2	Length of non-production time in each production time in each production cycle T
I(t)	Inventory level at any time t, t ≥ 0
$I_{v1}(t_1)$	Inventory level for vendor when t_1 is between 0 and T_1
$I_{v2}(t_2)$	Inventory level for vendor when t_2 is between 0 and T_2
$I_{bi}(t)$	Inventory level for buyer i when t is between 0 and T/n_i.
n_i	Delivery times per period T for buyer i.
I_{mv}	Maximum inventory level of vendor
I_{mi}	Maximum inventory level of buyer i.
C_{pv}	Unit production cost for vendor
C_{pb}	Unit price for buyer
C_{sv}	The setup cost of each production cycle for vendor
C_{sb}	The setup or ordering cost per order for buyer
C_{hv} (C_{hb})	The holding cost per year for vendor (buyer)
VC	The cost of vendor per unit time
BC	The cost of all buyers per unit time
TC	The integrated cost of vendor and all buyers per unit time

3. MATHEMATICS MODELLING AND ANALYSIS

The differential equations governing the inventory model are given by

$$\frac{dI_{v1}(t_1)}{dt_1} + \theta I_{v1}(t_1) = P - \sum_{i=1}^{N}\left(\alpha_i I_{v1}(t_1) + \beta_i\right), \; 0 \leq t_1 \leq T_1 \qquad \ldots(1)$$

$$\frac{dI_{v2}(t_2)}{dt_2} + \theta I_{v2}(t_2) = -\sum_{i=1}^{N}\left(\alpha_i I_{v2}(t_2) + \beta_i\right), \quad 0 \leq t_2 \leq T_2 \qquad \ldots(2)$$

and

$$\frac{dI_{bi}(t)}{dt}+\theta I_{bi}(t)=-(\alpha_i I_{bi}(t)+\beta_i), \qquad 0\le t\le \frac{T}{n_i} \qquad \ldots(3)$$

The boundary conditions, are

$I_{v1}(t_1)=0$ at $t_1=0$

$I_{v2}(T_2)=0$ at $t_2=T_2$

and

$I_{bi}\left(\frac{T}{n_i}\right)=0$ at $t=\frac{T}{n_i}$

After adjusting for the constant of integration, equations (1)-(3) are clearly equivalent to the following equations

$$I_{v1}=\left(\frac{P-\sum_{i=1}^{N}\beta_i}{z_1}\right)\left(1-e^{-z_1t_1}\right) \qquad \ldots(4)$$

$$I_{v2}(t_2)=\frac{\sum_{i=1}^{N}\beta_i}{z_1}\left(e^{z_1(T_2-t_2)}-1\right) \qquad \ldots(5)$$

where

$z_1=-\left(\theta+\sum\alpha_i\right)$

and

$$I_{bi}(t)=\frac{\beta_i}{z}\left(e^{z\left(\frac{T}{n_i}-t\right)}-1\right) \qquad \ldots(6)$$

where $z=-(\theta+\alpha_i)$

4. PERFORMANCE INDICES

The maximum inventory level of vendor obtained by the boundary condition, $I_{mv}=I_{v2}(0)$ is

$$I_{mv}=\frac{\sum_{i=1}^{N}\beta_i}{z_1}\left(e^{z_1T_2}-1\right) \qquad \ldots(7)$$

The maximum inventory level of buyer 'i' obtained by the boundary condition

$I_{mi}=I_{bi}(0)$ is

$$I_{mi}=\frac{\beta i}{Z}\left(e^{zT/ni}-1\right) \qquad \ldots(8)$$

Now by the boundary condition $I_{v1}(T_1)=I_{v2}(0)$, we can derive the following equation.

$$T_1 \approx \frac{\sum_{i=1}^{N}\beta_i}{P-\sum_{i=1}^{N}\beta_i} T_2\left(1+\frac{z_1 T_2}{2}\right) \quad \ldots(9)$$

We know that

$$T = T_1 + T_2 \quad \ldots(10)$$

which gives,

$$T = \frac{T_2}{P-\sum_{i=1}^{N}\beta_i}\left(P+\frac{zT_2}{2}\sum_{i=1}^{N}\beta_i\right) \quad \ldots(11)$$

The yearly holding costs for all buyers and vendors are

$$HC_b = \frac{C_{pb}C_{hb}}{T}\sum_{i=1}^{N} n_i \int_0^{T/n_i} I_{bi}(t)dt \quad \ldots(12)$$

$$= \frac{C_{pb}C_{hb}}{T}\sum_{i=1}^{N} n_i \frac{\sum_{i=1}^{N}\beta_i}{z}\left(\frac{e^{\frac{zT}{n_i}}-1}{z}-\frac{T}{n_i}\right) \quad \ldots(13)$$

and

$$HC_v = \frac{C_{pv}C_{hv}}{T}\left[\int_0^{T_1} I_{v1}(t_1)dt_1 + \int_0^{T_2} I_{v2}(t_2)dt_2 - \sum_{i=1}^{N} n_i \int_0^{T/n_i} I_{bi}(t)dt\right] \quad \ldots(14)$$

$$= \frac{C_{pv}C_{hv}}{T}\left[\left(\frac{P-\sum_{i=1}^{N}\beta_i}{z_1}\right)\left(T_1+\frac{e^{-z_1T_1}-1}{z_1}\right)+\frac{\sum_{i=1}^{N}\beta_i}{z_1}\left(\frac{1-e^{z_1T_2}}{z_1}-T_2\right)-\sum_{i=1}^{N} n_i\frac{\beta_i}{z}\left(\frac{e^{\frac{zT}{n_i}}-1}{z}-\frac{T}{n_i}\right)\right] \quad \ldots(15)$$

The deteriorated cost per annum for all buyers and vendor are

$$DC_b = \sum_{i=1}^{N}\left(I_{mi}-\frac{Td_i}{n_i}\right)\frac{n_iC_{pb}}{T} \quad \ldots(16)$$

$$= \sum_{i=1}^{N}\left(\frac{\beta_i}{z}\left(e^{zT/n_i}-1\right)-\frac{Td_i}{n_i}\right)\frac{n_iC_{pb}}{T} \quad \ldots(17)$$

$$DC_v = \frac{C_{pv}}{T}\left(PT_1 - \sum_{i=1}^{N} n_i I_{mi} \right) \qquad \text{...(18)}$$

$$= \frac{C_{pv}}{T}\left(PT_1 - \sum_{i=1}^{N} n_i \frac{\beta_i}{z}\left(e^{zT/n_i} - 1 \right) \right) \qquad \text{...(19)}$$

The annually setup costs for all buyers and vendor are given by

$$SC_b = \sum_{i=1}^{N} \frac{n_i C_{sb}}{T} \qquad \text{...(20)}$$

$$SC_v = \frac{C_{sv}}{T} \qquad \text{...(21)}$$

The buyers' cost and the vendor's cost can be obtained using

$$BC = HC_b + DC_b + SC_b \qquad \text{...(22)}$$

and

$$VC = HC_v + DC_v + SC_v \qquad \text{...(23)}$$

Hence the total costs incurred is expressed by

$$TC = BC + TC \qquad \text{...(24)}$$

5. COST MINIMIZATION

For cost analysis purpose, we set i=1. The objective of our study in the present section is to obtain the minimum TC, for this purpose we determine the derivative of TC from equation (24) with respect to T_2 and set it to zero, so that

$$T = n_1 \sqrt{\frac{2\left(C_{sb} + C_{sv}/n_1\right)}{\beta_1\left\{ C_{pb}\left(C_{hb} + z\right) + C_{pv}\left[n_1\left(C_{hv} + z\right)\left(1 - \beta_1/P\right) - C_{hv} \right] \right\}}} \qquad \text{...(25)}$$

$$= n_1 R \text{ (say)}$$

where

$$R = \sqrt{\frac{2\left(C_{sb} + C_{sv}/n_1\right)}{\beta_1\left\{ C_{pb}\left(C_{hb} + z\right) + C_{pv}\left[n_1\left(C_{hv} + z\right)\left(1 - \beta_1/P\right) - C_{hv} \right] \right\}}}$$

Now the maximum inventory level of buyer for i=1 is given by

$$I_{m1} = \sqrt{\frac{2\beta_1\left(C_{sb} + C_{sv}/n_1\right)}{\left\{ C_{pb}\left(C_{hb} + z\right) + C_{pv}\left[n_1\left(C_{hv} + z\right)\left(1 - \beta_1/P\right) - C_{hv} \right] \right\}}} \qquad \text{...(26)}$$

If we do not consider the deterioration rate and the production rate is infinite, then above results become

$$T = n_1 \sqrt{\frac{2\left(C_{sb} + C_{sv}/n_1\right)}{\beta_1\left\{ C_{pb}\left(C_{hb} - \alpha_1\right) + C_{pv}\left[n_1\left(C_{hv} - \alpha_1\right) - C_{hv} \right] \right\}}} \qquad \text{...(27)}$$

and

$$I_{m1} = \sqrt{\frac{2\beta_1\left(C_{sb} + C_{sv}/n_1\right)}{\left\{C_{pb}\left(C_{hb} - \alpha_1\right) + C_{pv}\left[n_1\left(C_{hv} - \alpha_1\right) - C_{hv}\right]\right\}}} \qquad \ldots(28)$$

6. SPECIAL CASES

Here we consider some special cases for production inventory model by setting appropriate parameter values.

Case I: In this case, the deterioration rate is not considered, i.e. θ = 0, so that the maximum inventory level of vendor becomes

$$I_{mv} = \frac{\sum_{i=1}^{N}\beta_i}{\sum_{i=1}^{N}\alpha_i}\left(1 - e^{-\sum_{i=1}^{N}\alpha_i T_2}\right) \qquad \ldots(29)$$

For this, the maximum inventory level of buyer 'i' becomes

$$I_{mi} = \frac{\beta i}{\alpha_i}\left(1 - e^{-\alpha_i T / ni}\right) \qquad \ldots(30)$$

Also, the yearly holding costs for all buyers and vendors respectively reduce to

$$HC_b = \frac{C_{pb}C_{hb}}{T}\sum_{i=1}^{N} n_i \frac{\sum_{i=1}^{N}\beta_i}{\alpha_i}\left(\frac{e^{\frac{-\alpha_i T}{n_i}} - 1}{\alpha_i} + \frac{T}{n_i}\right) \qquad \ldots(31)$$

and

$$HC_v = \frac{C_{pv}C_{hv}}{T}\left[\left(\frac{P - \sum_{i=1}^{N}\beta_i}{-\sum_{i=1}^{N}\alpha_i}\right)\left(T_1 + \frac{1 - e^{\sum_{i=1}^{N}\alpha_i T_1}}{\sum_{i=1}^{N}\alpha_i}\right) + \frac{\sum_{i=1}^{N}\beta_i}{\sum_{i=1}^{N}\alpha_i}\left(\frac{1 - e^{-\sum_{i=1}^{N}\alpha_i T_2}}{\sum_{i=1}^{N}\alpha_i} + T_2\right)\right.$$

$$\left. - \sum_{i=1}^{N} n_i \frac{\beta_i}{\alpha_i}\left(\frac{e^{-\frac{\alpha_i T}{n_i}} - 1}{\alpha_i} + \frac{T}{n_i}\right)\right] \qquad \ldots(32)$$

Case II: In this case the parameter α, corresponding to the demand rate is ignored, i.e. α=0, so that we come across the model having constant demand rate $d_i=\beta_i$, which was studied by Yang and Wee (2002). Various performance indices become as follows:

The maximum inventory level of vendor and buyer ‘i’ are

$$I_{mv} = \frac{\sum_{i=1}^{N} d_i}{\theta}\left(1 - e^{-\theta T_2}\right) \quad \ldots(33)$$

and

$$I_{mi} = \frac{d_i}{\theta}\left(1 - e^{-\theta_i T / ni}\right) \quad \ldots(34)$$

The yearly holding costs for all buyers and vendors reduce to

$$HC_b = \frac{C_{pb} C_{hb}}{T} \sum_{i=1}^{N} n_i \frac{\sum_{i=1}^{N} d_i}{\theta}\left(\frac{e^{\frac{-\theta T}{n_i}} - 1}{\theta} + \frac{T}{n_i}\right) \quad \ldots(35)$$

and

$$HC_v = \frac{C_{pv} C_{hv}}{T}\left[\left(\frac{P - \sum_{i=1}^{N} d_i}{-\theta}\right)\left(T_1 + \frac{1 - e^{\theta T_1}}{\theta}\right) + \frac{\sum_{i=1}^{N} d_i}{\theta}\left(\frac{1 - e^{-\theta T_2}}{\theta} + T_2\right)\right.$$

$$\left. - \sum_{i=1}^{N} n_i \frac{d_i}{\theta}\left(\frac{e^{-\frac{\theta T}{n_i}} - 1}{\theta} + \frac{T}{n_i}\right)\right] \quad \ldots(36)$$

The deteriorated costs per annum for all buyers and vendor are

$$DC_b = \sum_{i=1}^{N}\left(\frac{d_i}{\theta}\left(1 - e^{-\theta T / n_i}\right) - \frac{T d_i}{n_i}\right)\frac{n_i C_{pb}}{T} \quad \ldots(37)$$

$$DC_v = \frac{C_{pv}}{T}\left(P T_1 + \sum_{i=1}^{N} n_i \frac{d_i}{\theta}\left(e^{\theta T / n_i} - 1\right)\right) \quad \ldots(38)$$

7. NUMERICAL ILLUSTRATIONS

The explicit analytical results obtained have been computed by to developing computer program in software MATLAB in order to analyze the effect of deterioration rate on maximum inventory level and time length (T).

Table 1 shows the effects of (C_{hb}, C_{hv}), β, θ, and n on time length (T) and maximum inventory level (I_{m1}). We observe that the maximum inventory level (I_{m1}) and time length (T) decrease as holding costs of buyer and vendor increase. There is an increase in I_{m1} and T, as we increase the value of 'n' but if we increase the value of β, T decreases and I_{m1} increases. Time length (T) and maximum inventory level (I_{m1}) decrease slowly by increasing the value of θ.

From table 2 we see the effects of (C_{pb}, C_{pv}), β, θ, and n on time length (T) and maximum inventory level (I_{m1}). We observe that the maximum inventory level (I_{m1}) and time length (T) decrease as production cost of buyer and vendor, β andθ, increase. It is also noted that I_{m1} increases with β but T decreases with the increase inβ.

Table 3 exhibits the effect of parameters (C_{sb}, C_{sv}, β,θ, n) on time length (T) and maximum inventory level (I_{m1}). We see that maximum inventory level and time length vary with the increase in setup costs of buyer and vendor. Time length (T) and maximum inventory level (I_{m1}) decrease slowly with the increased in of θ. It is noted that I_{m1} increases with β but T decreases with the increase in β. We also observe that T increases with n but I_{m1} decreases with the increase in n.

Figures 1(a) – 1(c) depict the variation of the maximum inventory level I_{m1} for single buyer (i.e. i=1) with the deterioration rate (θ).

We fix parameters as follows: Setup cost of buyer and vendor is $2000 and $5000 respectively, production cost of buyer and vendor as $12 and $10 respectively, the yearly percentage of holding cost of buyer and vendor are taken as 0.17 and 0.15, respectively. The capacity of production is chosen as 20,00000 units per year.

It is noticed from figure 1(a) that maximum inventory level increases with θ for α=1, first slowly up to θ=13 and then after it increases sharply. We also observe that as we increase the value of α (say α=2, α=3), it remain linear and constant for increasing value of θ. Figures 1(b) and 1(c) display the increasing trend in maximum inventory level with θ. The increase is gradual up to θ=13 and it is quite sharp then after in both the figures. As we increase the value of β, the level of maximum inventory level decreases. The same pattern is noted for increasing value of n.

Figures 2(a) – 2(c) visualize the effect of deterioration rate (θ) on time length (T). We observe that the time length (T) decreases slowly with the increase in θ. It can be seen from figures 2(a) and 2(b), that as we increase the values of α and β, time length (T) reduces. In figure 2(c), time length (T) increases as n increases.

8. CONCLUSION

This study presents a production inventory model for deteriorating items with a time varying demand rate for single vendor and multiple buyers. In our investigation items are assumed to deteriorate at a constant rate and production rate is infinite. Total cost is find as the sum of vendors cost and buyers cost and numerical analysis is incorporate with some fix parameters to show the effect of many factor like deterioration, holding cost (both for vendor and buyer), demand parameters, etc.. A future study will incorporate more realistic assumptions in the proposed model, such as variable deterioration rate, stochastic nature of demand and production rate, which depends on both on-hand inventory and demand.

REFRENCES

1. Chen, J.M. and Chen, L.T. (2005): Pricing and production lot-size/scheduling with finite capacity for a deteriorating item over a finite horizon. Comput. Oper. Res., Vol. 32, No. 11, pp. 2801-2819.
2. Lin, G.C., Kroll, E.K. and Lin, C.J. (2005): Determining a common production cycle time for an economic lot scheduling problem with deteriorating items, Euro. J. Oper. Res..
3. Wang, C.H. and Sheu, S.H. (2003): Determining the optimal production–maintenance policy with inspection errors: using a Markov chain, Comput. Oper. Res., Vol. 30, No. 1, pp. 1-17.
4. Yang, P.C. and Wee, H.M. (2002): A single-vendor and multiple-buyers production-inventory policy for a deteriorating item, Euro. J. Oper. Res., Vol. 143, pp. 570-581.
5. Wang, C.H. and Sheu, S.H. (2001): Simultaneous determination of the optimal production–inventory and product inspection policies for a deteriorating production system, Comput. Oper. Res., Vol. 28, No. 11, pp. 1093-1110.
6. Wu, M.Y. and Wee, H.M. (2001): Buyer-seller joint cost for deteriorating items with multiple-lot-size deliveries, Jour. Chinese Inst. Indust. Eng., Vol. 18, No. 1, pp. 109-119.
7. Wang, C.H. and Sheu, S.H. (2000): Fast approach to the optimal production/PM policy, Comput. Math. Appl., Vol. 40, No. 10-11, pp. 1297-1314.
8. Yang, P.C. and Wee, H.M. (2000): Economic ordering policy of deteriorated items for vendors and buyer: An integrated approach, Prod. Plan. Control, Vol. 11, No. 5, pp. 474-480.
9. Hwang, H.S. (1999): Inventory models for both deteriorating and ameliorating items. Comput. Indust. Eng., Vol. 37, No. 1-2, pp. 257-260.
10. Wee, H.M. and Law, S.T. (1999): Economic production lot size for deteriorating items taking account of the time-value of money, Comput. Oper. Res., Vol. 26, No. 6, pp. 545-558.
11. Wee, H.M. and Wang, W.T. (1999): A variable production scheduling policy for deteriorating items with time-varying demand, Comput. Oper. Res., Vol. 26, No. 3, pp. 237-254.
12. Andijani, A. and Dajani, M.A. (1998): Analysis of deteriorating inventory/production systems using a linear quadratic regulator, Euro. J. Oper. Res., Vol. 106, No.1, pp. 82-89.
13. Goyal, S. K. and Gunasekaran, A. (1995): An integrated production-inventory-marketing model for deteriorating items, Comput. Indust. Eng., Vol. 28, No. 4, pp. 755-762.
14. Wee, H.M. (1993): Economic production lot size model for deteriorating items with partial back-ordering, Comput. Indust. Eng., Vol. 24, No. 3, pp. 449-458.
15. Heng, K.J., Labban, J. and Linn, R.L. (1991): An order-level lot size inventory model for deteriorating items with finite replenishment rate, Comput. Indust. Eng., Vol. 20, pp. 187-197.
16. Bretthauer, K. M., Shetty, B., Syam, S. and Vourka, R. J. (2006): Production an inventory management under multiple resource constraints, Mathematical and computer modeling, Vol. 44, issue 1-2, pp 85-95 .
17. Burke, G. J., Carrillo, J. E. and Vakharia, A. J. (2007): Single versus multiple supplier sourcing strategies, Euro. J. Oper. Res., Vol. 182, No. 1, pp. 95-112.
18. Chan, C. K., L. Y., Ng, C.T., Cheung, B.K. and Langevin, A. (2006): Scheduling of multi – buyer joint replenishments', International Journal of Production Economics, Vol. 102, Issue 1, pp.132-142.
19. Lebacque, V., Jost, V. and Brauner, N. (2007): Simultaneous optimization of classical objectives in JIT scheduling, Euro. J. Oper. Res., Vol. 182, No. 1, pp. 29-39.
20. Rong, M., Mahapatra, N.K. and Maiti, M. (2008): A two-warehouse inventory model for a deteriorating item with partially/ fully backlogged shortage and fuzzy lead time, Eurpean Journal of Operational Research, vol. 189, pp. 59-75
21. Madhavilata, M., Rao, K.S. and Ravindranath, V. (2011): An order level inventory model under L2-system with exponentially increasing demand, International Journal of Engineering Research and Applications, vol. 1(4), pp. 1744-1750.
22. Min, J., Zhou, Y.W., Liu, G.Q. and Wang, S.D. (2012): An EPQ model for deteriorating items with inventory level dependent demand and permissible delay in payments, International Journal of Systems Sciences, vol. 43, pp. 1039-1053.

23. Agrawal, S., Banerjee, S. and Papachristos, S. (2013): Inventory model with deteriorating items, ramp-type demand and partially backlogged shortages for a two warehouse system, Applied Mathematical Modelling, vol. 37, pp. 8912–8929.
24. Sicilia, J., González-De-la-Rosa, M., Febles-Acosta, J. and Alcaide-López-de-Pablo, D. (2014): An inventory model for deteriorating items with shortages and time-varying demand, International Journal of Production Economics, vol. 155, pp. 155–162.
25. Guchhait, P., Maiti, M. K. and Maiti, M. (2015): An EOQ model of deteriorating item in imprecise environment with dynamic deterioration and credit linked demand, Applied Mathematical Modelling, vol. 39, pp. 6553–6567.
26. Xu, Y., Bisi, A. and Dada, M. (2017): A finite-horizon inventory system with partial backorders and inventory holdback, Operations Research Letters, vol. 45, pp. 315–322,.

.

			T			I_{m1}		
(C_{hb}, C_{hv})	n	β	θ=0.1	θ=0.5	θ=0.9	θ=0.1	θ=0.5	θ=0.9
(3, 2)	1	1.5	13.95	13.04	12.28	20.93	19.56	18.42
		2	12.16	11.36	10.71	24.33	22.73	21.41
		2.5	10.95	10.23	9.64	27.38	25.58	24.09
	2	1.5	18.40	17.23	16.27	13.80	12.93	12.20
		2	16.08	15.06	14.21	16.08	15.06	14.21
		2.5	14.51	13.59	12.82	18.14	16.98	16.03
	3	1.5	21.66	20.31	19.18	10.83	10.15	9.59
		2	18.94	17.76	16.78	12.63	11.84	11.18
		2.5	17.12	16.05	15.16	14.26	13.37	12.63
(4, 3)	1	1.5	12.76	12.05	11.44	19.14	18.07	17.17
		2	11.13	10.51	9.98	22.26	21.02	19.96
		2.5	10.03	9.47	8.99	25.07	23.66	22.47
	2	1.5	16.54	15.68	14.94	12.41	11.76	11.21
		2	14.46	13.71	13.06	14.46	13.71	13.06
		2.5	13.06	12.38	11.79	16.33	15.47	14.74
	3	1.5	19.31	18.33	17.49	9.65	9.17	8.75
		2	16.90	16.04	15.31	11.27	10.70	10.20
		2.5	15.28	14.50	13.84	12.73	12.09	11.53
(5, 4)	1	1.5	11.83	11.25	10.76	17.74	16.88	16.14
		2	10.32	9.82	9.38	20.64	19.64	18.77
		2.5	9.30	8.85	8.46	23.26	22.12	21.14
	2	1.5	15.16	14.48	13.89	11.37	10.86	10.42
		2	13.26	12.67	12.15	13.26	12.67	12.15
		2.5	11.98	11.44	10.97	14.97	14.30	13.72
	3	1.5	17.59	16.84	16.18	8.79	8.42	8.09
		2	15.40	14.74	14.17	10.27	9.83	9.44
		2.5	13.93	13.33	12.81	11.61	11.11	10.67

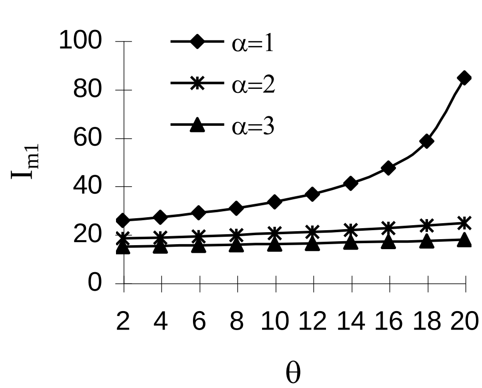
Table 1: Effect of holding cost (C_{hb}, C_{hv}), demand parameter (β) and, number of cycle (n) on time length (T) and maximum inventory level (IM_1).

			T			I_{m1}		
(C_{pb}, C_{pv})	n	β	θ=0.1	θ=0.5	θ=0.9	θ=0.1	θ=0.5	θ=0.9
(10, 8)	1	1.5	21.21	18.35	16.41	31.81	27.53	24.61
		2	18.48	15.99	14.29	36.96	31.98	28.59
		2.5	16.63	14.39	12.86	41.57	35.97	32.16
	2	1.5	28.28	24.52	21.94	21.21	18.39	16.46
		2	24.70	21.41	19.16	24.70	21.41	19.16
		2.5	22.28	19.31	17.28	27.85	24.14	21.61
	3	1.5	33.48	29.05	26.02	16.74	14.53	13.01
		2	29.28	25.40	22.75	19.52	16.94	15.16
		2.5	26.44	22.94	20.54	22.04	19.12	17.12
(11, 9)	1	1.5	20.13	17.42	15.57	30.20	26.13	23.36
		2	17.54	15.18	13.57	35.09	30.36	27.14
		2.5	15.79	13.66	12.21	39.47	34.15	30.52
	2	1.5	26.79	23.23	20.79	20.09	17.42	15.59
		2	23.40	20.28	18.15	23.40	20.28	18.15
		2.5	21.11	18.30	16.37	26.39	22.87	20.47
	3	1.5	31.68	27.49	24.62	15.84	13.74	12.31
		2	27.70	24.04	21.52	18.47	16.02	14.35
		2.5	25.03	21.71	19.44	20.86	18.09	16.20
(14, 12)	1	1.5	17.68	15.30	13.67	26.52	22.94	20.51
		2	15.41	13.33	11.91	30.82	26.66	23.83
		2.5	13.87	12.00	10.72	34.68	29.99	26.81
	2	1.5	23.43	20.31	18.17	17.57	15.23	13.63
		2	20.46	17.74	15.87	20.46	17.74	15.87
		2.5	18.47	16.00	14.32	23.08	20.00	17.90
	3	1.5	27.64	23.98	21.47	13.82	11.99	10.74
		2	24.17	20.97	18.78	16.11	13.98	12.52
		2.5	21.84	18.95	16.96	18.20	15.79	14.14

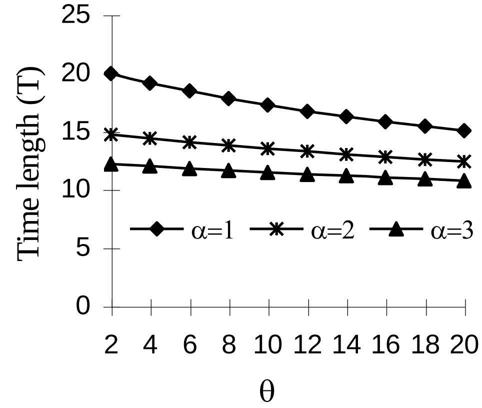
Table 2: Effect of production cost (C_{pb}, C_{pv}), demand parameter (β) and, number of cycle (n) on time length (T) and maximum inventory level (IM_1).

			T			I_{m1}		
(C_{sb}, C_{sv})	n	β	θ=0.1	θ=0.5	θ=0.9	θ=0.1	θ=0.5	θ=0.9
(2, 5) In thousand	1	1.5	17.68	15.30	13.67	26.52	22.94	20.51
		2	15.41	13.33	11.91	30.82	26.66	23.83
		2.5	13.87	12.00	10.72	34.68	29.99	26.81
	2	1.5	23.43	20.31	18.17	17.57	15.23	13.63
		2	20.46	17.74	15.87	20.46	17.74	15.87
		2.5	18.47	16.00	14.32	23.08	20.00	17.90
	3	1.5	27.64	23.98	21.47	13.82	11.99	10.74
		2	24.17	20.97	18.78	16.11	13.98	12.52
		2.5	21.84	18.95	16.96	18.20	15.79	14.14
(3.5, 4.5) In thousand	1	1.5	18.90	16.35	14.62	28.35	24.53	21.92
		2	16.47	14.25	12.74	32.95	28.50	25.47
		2.5	14.83	12.83	11.46	37.07	32.06	28.66
	2	1.5	26.48	22.95	20.54	19.86	17.21	15.40
		2	23.13	20.05	17.94	23.13	20.05	17.94
		2.5	20.87	18.09	16.19	26.09	22.61	20.23
	3	1.5	32.27	28.00	25.07	16.14	14.00	12.54
		2	28.23	24.49	21.93	18.82	16.33	14.62
		2.5	25.50	22.12	19.81	21.25	18.44	16.51
(6, 8) In thousand	1	1.5	25.00	21.63	19.33	37.51	32.45	29.00
		2	21.79	18.85	16.85	43.58	37.70	33.70
		2.5	19.62	16.97	15.16	49.04	42.42	37.91
	2	1.5	34.92	30.27	27.09	26.19	22.70	20.32
		2	30.51	26.44	23.66	30.51	26.44	23.66
		2.5	27.53	23.86	21.35	34.41	29.82	26.68
	3	1.5	42.49	36.87	33.01	21.24	18.43	16.50
		2	37.16	32.24	28.87	24.77	21.49	19.25
		2.5	33.57	29.13	26.08	27.98	24.27	21.73

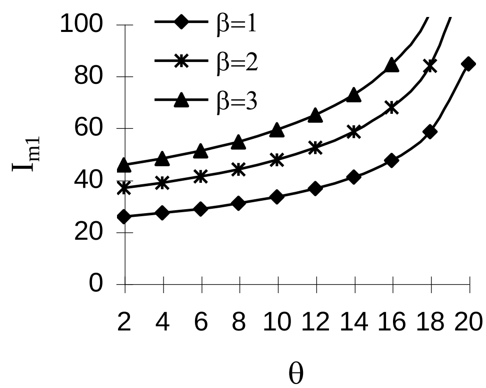
Table 3: Effect of setup cost (C_{sb}, C_{sv}), demand parameter (β) and, number of cycle (n) on time length (T) and maximum inventory level (IM_1).



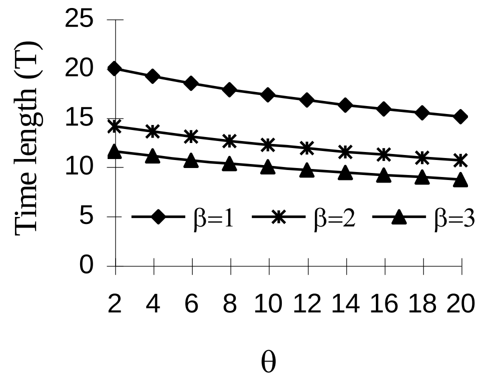
1(a)



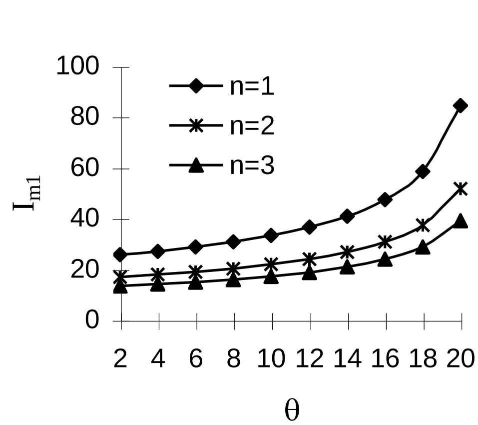
2(a)



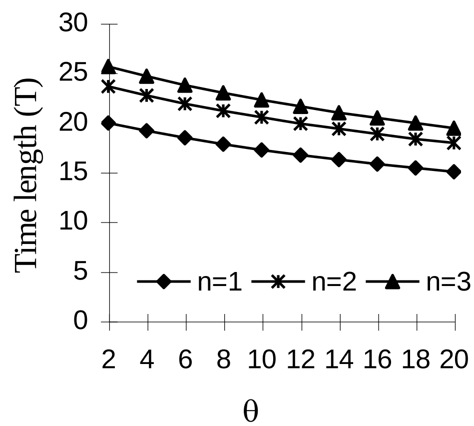
1(b)



2(b)



1(c)



2(c)

Fig. 1: Maximum inventory level (I_{m1}) vs. deterioration rate (θ) for different parameter (a) α, (b) β and (c) n

Fig. 2: Time length (T) vs. deterioration rate (θ) for different parameter (a) α, (b) β and (c) n