PI Tuning of Coupled Four Tank System by model Reference Adaptive Control using MIT Rule

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ABSTRACT: *Application of proportional and integral (PI) based model reference adaptive control (MRAC) using MIT rule on a nonlinear model of cross coupled four-tank liquid level control system through simulation has been investigated. The cross coupled four -tank liquid level control system is regarded as the relevant plant to emulate the process control in petroleum and chemical industries. The processing plants in these industries largely involve in controlling the liquid level and the flow rate from one reservoir to another in the presence of nonlinearity and disturbance. This requires the use of adaptive techniques such as MRAC in the process control system. Cross coupled-tank which resembles the model of the chemical or mixing process plant is used to evaluate the performance of PI based MRAC under various conditions. A simulation is carried out using MATLAB® and Simulink® to control the modeled nonlinear cross coupled four-tank using the adaptive control algorithm. It is also utilized to show that the controller can produce the appropriate control signals to the coupled-tank system to control the liquid level in the presence of plant nonlinearity, and disturbance. Considerable matching between reference model response and tank level response is obtained.*

KEYWORDS: PI Controller, MRAC, MIT rule and Coupled Tanks System.

1. INTRODUCTION

Process in chemical industries largely involves controlling the liquid level and the flow rate from one reservoir to another in the presence of nonlinearity and disturbance. This requires the use of adaptive techniques such as MRAC in the process control system as fixed controllers are often not capable of controlling a process whose system parameters are varying and disturbances acting upon the system during the operation. The simulated cross coupled nonlinear plant model is based on the principle of mass balance, i.e. the rate of change of liquid volume in each tank equals to the net flow of liquid into the tank. This nonlinear plant model is used instead of linearised perturbation model of coupled tank as to evaluate the controller performance in the presence of nonlinearity. The model reference adaptive control (MRAC) may be regarded as an adaptive scheme in which the desired performance is expressed in terms of a reference model, which gives the desired response to a command signal. Alternatively, if it is specified (as a map) as the response of a reference model to an input from a permissible class, then the control scheme is referred to as model reference adaptive control (MRAC) [8]. MIT rule is used for designing PI based MRAC controller

The design and analysis of PI tuning technique using MRAC and MIT rule will provide online automatic tuning of PI parameters. Adaptation algorithm tunes the PI parameters until the performance is satisfactory and then the system continues with updated parameters.

2. DYNAMICS OF CROSS COUPLED FOUR TANKS SYSTEM

The mathematical model of the coupled-tank liquid level control system in the simulation is formulated based on a real laboratory-scale coupled-tank system, developed by Feedback Instruments Ltd.

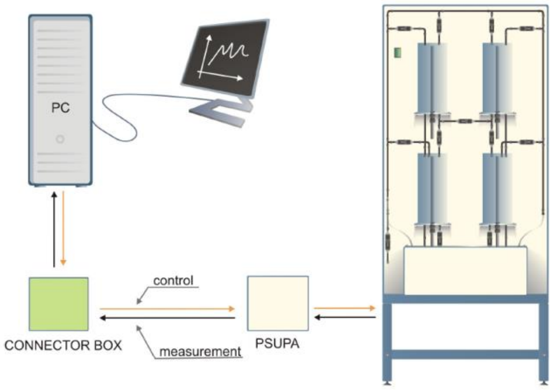

Fig.1: Schematic representation of experimental set-up showing the hardware [2]

Fig.1 illustrates the basic schematic representation of a coupled tank system. It consists of four equal-sized translucent cylindrical tanks and each tank is fitted with an outlet pipe in order to transmit the water to reservoir. In this process, a bottom tank (fifth tank) is used for water storage purposes i.e. as a reservoir. A level sensor is attached at the base of each tank in order to measure

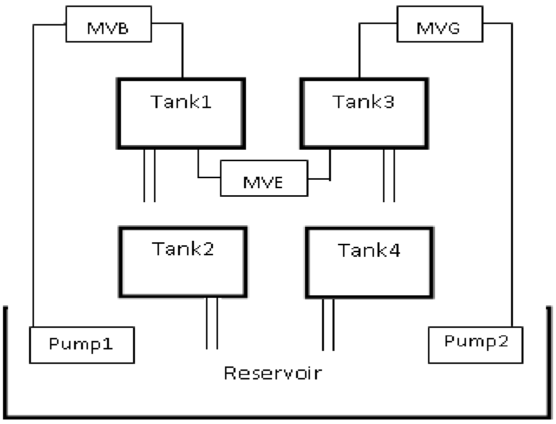

Fig.2: Representation of cross coupled tanks model

the water level of the corresponding tank [2]. The output of the level sensor is converted to 0- 5 volt DC by the help of a signal conditioning circuit. There are two pumps installed in the reservoir in order to drive the water from bottom to the top of the tank.

The objective of this problem was to control the water level in tanks. The phenomenological model of cross coupled four tanks is shown in Fig. 2. The coupled tanks system is a nonlinear plant. In Fig. 2, h_1, h_2, h_3 and h_4 are the water levels in tank1, tank2, tank 3 and tank 4 respectively. $u_1(t)$ and $u_2(t)$ is defined as a control signal in voltage term. Then, liquid levels can be represented by the following equations;

$$\frac{dh_1}{dt} = \frac{a_1}{A}\sqrt{2gh_1(t)} + \eta u_1(t) - \frac{a_{13}}{A}\sqrt{2g(h_1(t) - h_2(t)}$$

$$\frac{dh_2}{dt} = \frac{a_1}{A}\sqrt{2gh_1(t)} - \frac{a_2}{A}\sqrt{2gh_2(t)}$$

$$\frac{dh_3}{dt} = -\frac{a_3}{A}\sqrt{2gh_3(t)} + \eta u_2(t) + \frac{a_{13}}{A}\sqrt{2g(h_1(t) - h_3(t)} \quad \frac{dh_4}{dt} = \frac{a_3}{A}\sqrt{2gh_3(t)} - \frac{a_4}{A}\sqrt{2gh_4(t)}$$

(1)

where, a_1 = tank1 outlet area, a_2 = tank2 outlet area, a_3 = tank3 outlet area, a_4 = tank4 outlet area, a_{13} = cross flow area of pipe between tank1 and tank3 η = constant relating the control voltage with the water flow from the pump, g = gravitational constant, A = cross-sectional area of the tanks.

After linearization and Laplace transformation of equations (1), considering the operating points h_{10}, h_{20}, h_{40} and h_{30} as below

$$\Delta H_1(s) = \frac{\eta}{s+(k_1+k_{13})}\Delta U_1(s) + \frac{k_{13}}{s+(k_1+k_{13})}\Delta H_3(s) \quad \Delta H_2(s) = \frac{k_1}{s+(s+k_2)}\Delta H_1(s)$$

$$\Delta H_3(s) = \frac{\eta}{s+(k_3+k_{13})}\Delta U_2(s) + \frac{k_{13}}{s+(k_3+k_{13})}\Delta H_1(s)$$

$$\Delta H_4(s) = \frac{k_1}{s+(s+k_4)}\Delta H_3(s) \qquad (2)$$

$$k_1 = \frac{ga_1}{A\sqrt{2gh_{10}})}$$

$$k_2 = \frac{ga_2}{A\sqrt{2gh_{20}})}$$

$$k_3 = \frac{ga_3}{A\sqrt{2gh_{30}})}$$

$$k_4 = \frac{ga_4}{A\sqrt{2gh_{40}})}$$

$$k_{13} = \frac{ga_{13}}{A\sqrt{2g(h_{10}} - h_{30})}$$

The above equations can be represented as cross coupled transfer function matrix relating the inputs pump voltages and the outputs water level in tanks.

$$\begin{matrix}\Delta H_1(s) \\ \Delta H_3(s)\end{matrix} = \begin{matrix}G_{11} & G_{12} \\ G_{21} & G_{22}\end{matrix} \begin{matrix}\Delta U_1(s) \\ \Delta U_2(s)\end{matrix}$$

$$G_{11} = \frac{\eta[s+(k_3+k_{13})]}{[s+(k_1+k_{13})].[s+(k_3+k_{13})] - k_{13}{}^2}$$

$$G_{12} = \frac{\eta k_{13}}{[s+(k_1+k_{13})].[s+(k_3+k_{13})] - k_{13}{}^2}$$

$$G_{21} = \frac{\eta k_{13}}{[s+(k_1+k_{13})].[s+(k_3+k_{13})] - k_{13}{}^2}$$

$$G_{22} = \frac{\eta[s+(k_1+k_{13})]}{[s+(k_1+k_{13})].[s+(k_3+k_{13})] - k_{13}{}^2} \qquad (3)$$

Table 1 shows the parameters of the coupled-tank system used in the plant modeling and simulation.

Table1. Parameters of the coupled tank system

Physical parameters	**Value**
Cross section area of tank (*A*)	0.01389 m^2
Outlet area of tank 1 (a_1)	$50.265\text{x}10^{-6}$ m^2
Outlet area of tank 2 (a_2)	$50.265\text{x}10^{-6}$ m^2
Gravitational constant (*g*)	9.81 m/s^2
Constant relating the control voltage with water flow from the pump (η)	0.0022

3. MODEL REFERENCE ADAPTIVE CONTROL (MRAC) USING MIT RULE.

The MRAC is a reference model based adaptive control system. The adjustment mechanism is designed using MIT rule. The block diagram of MRAC is shown in Fig. 3

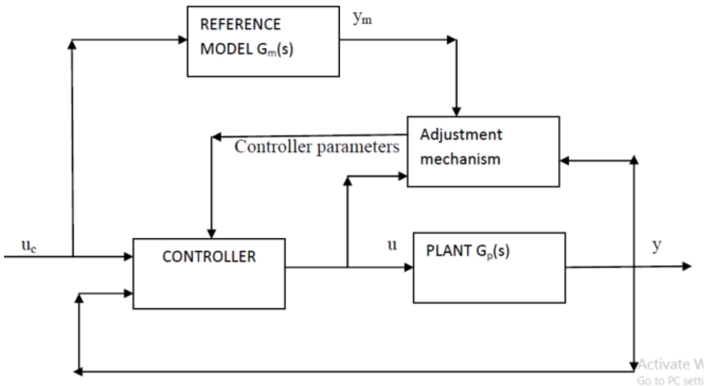

Fig. 3: Block diagram of MRAC [1]

In this work, MIT rule is used for adjustment mechanism. The MRAC control strategy is obtained using gradient approach of MIT rule [3]. According to gradient approach, a cost function is decided in terms of tracking error as shown below. The tracking error is defined as error between output of reference model and output of plant. The MIT rule performs the algorithms as follows.

Tracking error, $e = y - y_m$

From cost function $j(\theta) = ½\, e^2(\theta)$

MIT Rule says that the time rate of change of θ is proportional to negative gradient of *J*. That is

$$\frac{\partial\theta}{\partial t} = -\gamma\frac{\partial j}{\partial\theta} = -\gamma e\frac{\partial e}{\partial\theta}$$

where, *e* denotes the model error and θ is the controller parameter vector. The components of $\frac{\partial e}{\partial\theta}$ are the sensitivity derivatives of the error with respect to θ. The parameter γ is known as the adaptation gain. The MIT rule is a gradient scheme that aims to minimize the squared model cost function.

4. PI TUNING ALGORITHM USING MRAC FOR CROSS COUPLED FOUR TANKS PROCESS

The proposed algorithm is an adaptive control law in which the value of proportional gain, derivative gain and integral gain are automatically updated in such a way that plant follows the reference model.

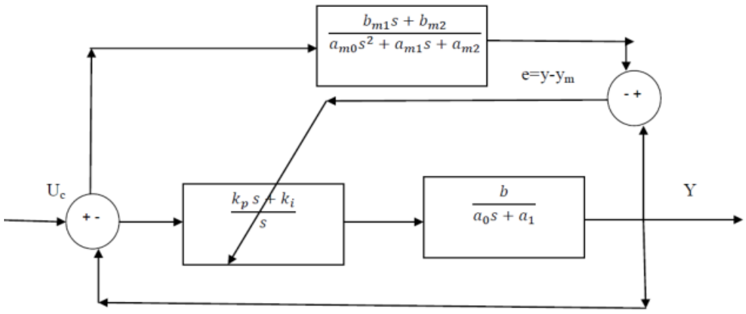

Fig. 4: Block diagram of MRAC based PI

The closed loop transfer function obtained from the block diagram is given by

$$\frac{Y(s)}{U_c(s)} = \frac{b(k_p s + k_i)}{a_0 s^2 + (a + bk_p)s + bk_i} \tag{4}$$

Since the closed loop transfer function is of second order the appropriate reference model is chosen as

$$\frac{Y_m(s)}{U_c(s)} = \frac{b_{m1}s + b_{m2}}{a_{m0}s^2 + a_{m1}s + a_{m2}} \tag{5}$$

Now, the MIT rule of MRAC can be applied to the coupled tanks to obtain the controller parameters. The controller parameter vectors are $u=[k_p , k_i]$. Using MIT rule the rate of change of controller parameter with respect to time is found as:

$$\frac{dk_p}{dt} = -\gamma_p \frac{\partial j}{\partial k_p} = -\gamma_p \frac{\partial j}{\partial e}.\frac{\partial e}{\partial y}.\frac{\partial y}{\partial k_p} \tag{6}$$

$$\frac{dk_i}{dt} = -\gamma_i \frac{\partial j}{\partial k_i} = -\gamma_i \frac{\partial j}{\partial e}.\frac{\partial e}{\partial y}.\frac{\partial y}{\partial k_i} \tag{7}$$

But, $e = y - y_m$, So, $\frac{\partial e}{\partial y} = 1$ (8)

and, $\frac{\partial j}{\partial e} = e$ (9)

By substituting equation (6) and (7) in the equations (8) and (9) it becomes

$$\frac{dk_p}{dt} = -\gamma_p e \frac{\partial y}{\partial k_p} \tag{10}$$

$$\frac{dk_i}{dt} = -\gamma_i e \frac{\partial y}{\partial k_i} \tag{11}$$

By differentiating equation 4 with respect to k_p and k_i

$$\frac{\partial y}{\partial k_p} = \frac{bs}{a_0 s^2 + (a + bk_p)s + bk_i}(u_c - y) \tag{12}$$

$$\frac{\partial y}{\partial k_i} = \frac{b}{a_0 s^2 + (a + bk_p)s + bk_i}(u_c - y) \tag{13}$$

Substituting equations (12) and (13) in equations (10) and (11) it becomes

$$\frac{dk_p}{dt} = -\gamma_p e \frac{bs}{a_0 s^2 + (a + bk_p)s + bk_i}(u_c - y)$$

$$\frac{dk_i}{dt} = -\gamma_i e \frac{b}{a_0 s^2 + (a + bk_p)s + bk_i}(u_c - y)$$

Using the above equations MRAC based PI controller is implemented in Simulink programme of MATLAB

5. EXPERIMENT & RESULTS

Referring to the parameters and the operating points of process in table 1 and table 2, this process can be placed into the equation (3). It will be obtained the plant transfer function as in equation (14).

$$\begin{matrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{matrix} =$$

$$\begin{matrix} \frac{0.0022\ s + 1.0098 \times 10^{-4}}{s^2 + 0.0872\ s + 1.254 \times 10^{-3}} & \frac{5.566 \times 10^{-5}}{s^2 + 0.0872\ s + 1.254 \times 10^{-3}} \\ \frac{5.566 \times 10^{-5}}{s^2 + 0.0872\ s + 1.254 \times 10^{-3}} & \frac{0.0022\ s + 9.086 \times 10^{-5}}{s^2 + 0.0872\ s + 1.254 \times 10^{-3}} \end{matrix} \quad (14)$$

From (14), decoupling controllers are designed $D_{12}(s)$ and $D_{21}(s)$ for decoupling the interaction in couple-tank process as equation (15), (16).

$$D_{12}(s) = -\frac{G_{12}(s)}{G_{11}(s)}$$

$$D_{12}(s) = -\frac{k_{13}}{s + (k_3 + k_{13})} = -\frac{0.0253}{s + 0.0459} \quad (15)$$

$$D_{21}(s) = -\frac{G_{21}(s)}{G_{22}(s)}$$

$$D_{21}(s) = -\frac{k_{13}}{s + (k_1 + k_{13})} = -\frac{0.0253}{s + 0.0413} \quad (16)$$

After applying decoupling equations (15) and (16) to the cross coupled matrix transfer function equation (14), the cross coupled matrix is diagonalised and first order plant transfer function model is obtained. The decoupled and diagonalised plant transfer function matrix is given by

$$\begin{matrix} \Delta H_1(s) \\ \Delta H_3(s) \end{matrix} = \begin{matrix} G_{p11} & 0 \\ 0 & G_{p22} \end{matrix} * \begin{matrix} \Delta U_1(s) \\ \Delta U_2(s) \end{matrix}$$

where,

$$G_{p11}(s) = G_{11}(s) - \frac{G_{21}(s)}{G_{22}(s)} G_{12}(s)$$

$$G_{p11}(s) = \frac{\eta}{s + (k_1 + k_{13})} = \frac{0.0022}{s + 0.0413} \text{, for tank 1}$$

$$G_{p22}(s) = G_{22}(s) - \frac{G_{12}(s)}{G_{11}(s)} G_{21}(s)$$

$$G_{p22}(s) = \frac{\eta}{s + (k_3 + k_{13})} = \frac{0.0022}{s + 0.0459} \text{, for tank 3.}$$

Thus the plant transfer function for tank 1 and tank3 are obtained.

Simulation of tank 1 control loop gave

$$G_p(s) = \frac{\eta}{s+(k_1+k_{13})} = \frac{0.0022}{s+0.0413}$$

From the specified properties of the desired control system overshoot percentage, $P.O.$ = 5.02 % and settling time, t_s= 95.3sec , reference model for tank 1 is

$$G_m(s) = \frac{y_m(s)}{u_c(s)} = \frac{0.04577s+0.002823}{s^2+0.08176s+0.002823}$$

Simulation of tank 3 control loop gave

$$G_p(s) = \frac{\eta}{s+(k_3+k_{13})} = \frac{0.0022}{s+0.0459}$$

From the specified properties of the desired control system $P.O.$ = 5.38 % and t_s= 216 sec reference model for tank 3 is

$$G_m(s) = \frac{y_m(s)}{u_c(s)} = \frac{0.01977s+0.0005588}{s^2+0.0357s+0.0005588}$$

The input for simulation test given to tank 1 and tank 3 with the control system (G_{p11},G_{p22}) are as per the details in table 3.

Table 2. Operating point for simulation

Tank	**Time = 0 second**	**Time = 500 second**
Tank 1	5 cm	10 cm
Tank 3	5 cm	10 cm

Determining the value of PI controller parameters k_p and k_i by MRAC algorithm, it was found that, the value of adaptation gain γ as following

Table 3. Adaptation gains

Tank	γ_p	γ_i
Tank 1	0.0103	0.0000103
Tank 3	0.02119	0.00002119

5.1 SIMULATION RESPONSES

After simulations done for various adaptation gains of MRAC based PI controller, the mean square error between reference model response and response for level of tank 1 and tank3 is obtained and it is found that the mean square error is minimum in tank 1 for adaptation gains γ_p=0.02066, γ_i=0.00002066 and mean square error is minimum in tank 3 for adaptation gain γ_p=0.04238, γ_i=0.00004238. So, the validation is carried out for the gains mentioned above.

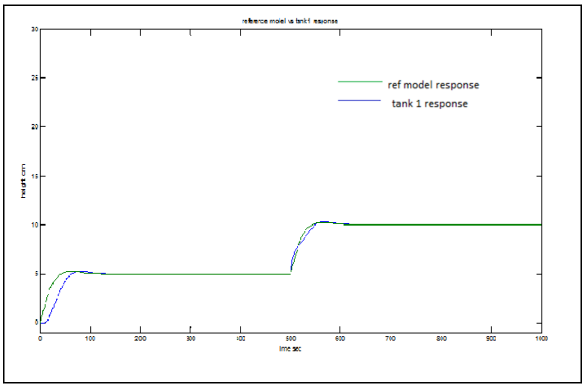

Fig.5: Reference model response versus tank 1 response γ_p=0.02066, γ_i=0.00002066

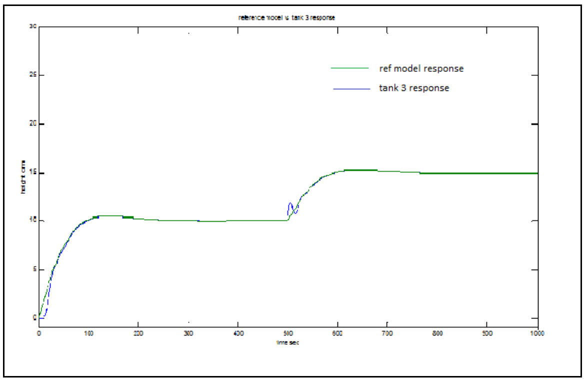

Fig.6 Reference model vs tank 3 response γ_p=0.04238, γ_i=0.00004238

Table 4 shows the mean square error corresponding to various gains for tank 1 and tank 3.

Table 4 Mean square error for various gains for tank1 and tank3

Adaptation gains	**Tank 1 (mean square error)**
γ_p=0.0103, γ_i=0.0000103	4875.8
γ_p=0.00515, γ_i=0.00000515	6944.1
γ_p=0.02066, γ_i=0.00002066	3305.8
Adaptation gains	**Tank 3 (mean square error)**
γ_p=0.02119, γ_i=0.00002119	1267.7
γ_p=0.010595,γ_i=0.00001059	2134.6
γ_p=0.04238, γ_i=0.00004238	751.4

5.2. VALIDATION

Usually, the Real-Time Workshop (RTW) is an extension of Simulink which has rapid prototyping ability for real-time software applications. It has the following features;

1) Automatic code generation tailored for a variety of target platforms
2) A rapid and direct path from system design to implementation

3) Seamless integration with MATLAB and Simulink
4) A simple graphical user interface
5) An open architecture and extensible make process

The toolbox has an automatic code for building up the process for real time process.

The steps adopted for real-time build process are as follows;

1) Real-Time Workshop analysis the block diagram and compiles it into an intermediate hierarchical depiction of the form **model.rtw**.
2) Target language compiler (TLC) reads the **model.rtw** and converts it into C code which is placed in the build directory within the MATLAB working directory.
3) Further, the TLC constructs a make file from an appropriate target make file template and places in the build directory.
4) Then the system reads the make file to compile the source code and links object files and libraries and generates an executable file i.e. **model.exe**.

5.2.1 REAL TIME IMPLEMENTATION RESPONSES

Real time responses for reference model and desired level against tank1 and tank3 are as shown in figures below;

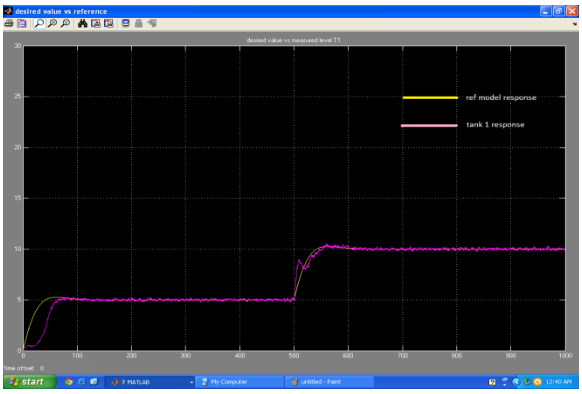

Fig. 7 Real time response Reference model vs tank 1 γ_p=0.02066, γ_i=0.00002066

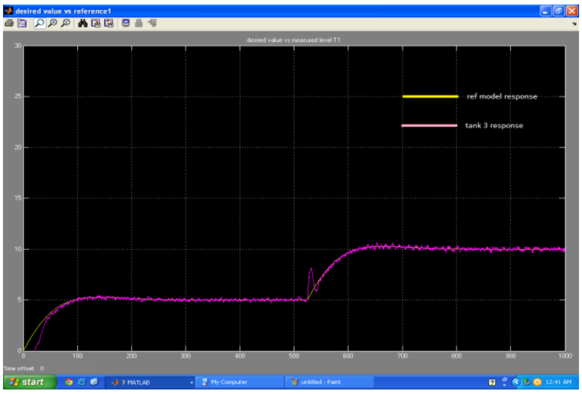

Fig. 8 Real time response Reference model vs tank 3 γ_p=0.04238, γ_i=0.00004238

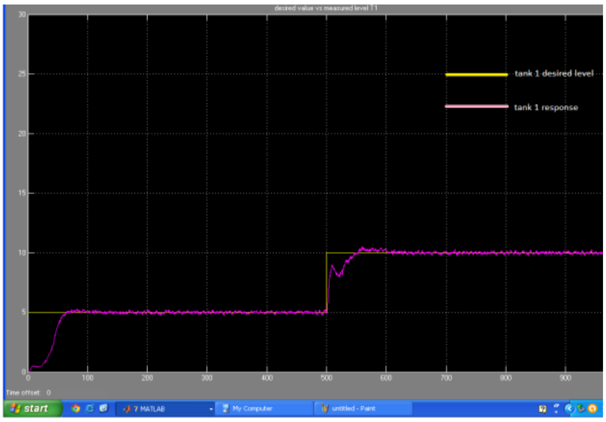

Fig 7.5 Real time response Desired level vs tank 1 γ_p=0.02066, γ_i=0.00002066

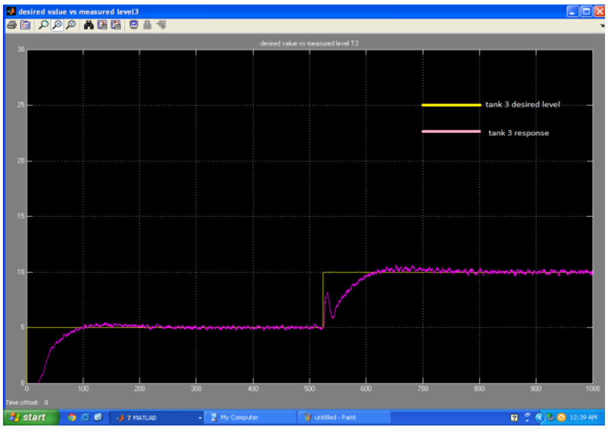

Fig 7.6 Real time response Desired level vs tank 3 γ_p=0.04238, γ_i=0.00004238

From simulation results, it is observed that proposed controller gives robust performance. From the above results, it can be seen that during first step, adaptation mechanism tunes the PI parameters to track the output of reference model. Results show that updated PI controller is enabling to track the reference model quite smoothly

7. CONCLUSION

The design of PI controller using MRAC techniques for couple-tanks process can adjust controller parameters in response to changes in plant and disturbance and with specified properties of the desired control system. It is shown by the experiment results that MRAC technique solves the dynamic problem of the coupled-tanks process and it is convenient for controller design under the requirement of the system.

In order to get robust response in face of model disturbance and also parametric uncertainties a sliding mode control should be designed and implemented in real-time. But usually the sliding mode controller suffers from the chattering problem. Hence due to that, an adaptive fuzzy sliding mode control has to be developed for the improvements of chattering where a fuzzy variable is considered while designing the sliding surface. It can be further improved by utilizing higher order sliding mode (HOSM) and the super twisting algorithm.

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