Application of the affine theorem to an orthotropic rectangular reinforced concrete slab continuous over one long side and simply supported on other three sides having an interior corner opening

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ABSTRACT

An attempt has been made to apply the affinity theorem to determine collapse load of two-way orthotropic slab with interior corner opening with One Long Side Continuous and other three sides Simply Supported Slab (OLC). Keeping in view the basic principles of yield line theory, all possible admissible yield line patterns are considered for the given configuration of the slab subjected to uniformly distributed load (u.d.l.). A computer program has been developed to solve the virtual work equations derived in this paper. Illustration of above methodology has been brought out with numerical examples. The analysis is carried out with aspect ratio of opening quite different to that the slab.

Key Words: aspect ratio, interior corner opening, configuration, affinity theorem, orthotropic slab, uniformly distributed load, ultimate load, ultimate moment and virtual work equations.

Introduction

Openings in slabs are usually required for plumbing, fire protection pipes, heat and ventilation ducts and air conditioning. Larger openings that could amount to the elimination of a large area within a slab panel are sometimes required for stairs and elevators shafts. For newly constructed slabs, the locations and sizes of the required openings are usually predetermined in the early stages of design and are accommodated accordingly. Such two way slabs subjected to uniformly distributed load and supported on various edge conditions are being analyzed by using yield-line method as suggested by Johansen, K.W[1]. Many researchers (Goli,H.B. et al[2],Islam,S. et al[3], Zaslavsky.Aron[4], Siva Rama Prasad et al[5], Sudhakar,K.J. et al[6], Veerendra Kumar et al[7]) adopted the yield-line analysis and virtual work method in deriving the virtual work equations of the rectangular reinforced concrete solid slabs subjected to uniformly distributed load and supported on various edge conditions.

Methodology

The method of determining collapse loads based on principle of virtual work has proved to be a powerful tool for a structural engineer, despite it gives an upper bound value. The work equations are formed by equating the energy absorbed by yield lines and the work done by the external load of the orthogonal rectangular slab with interior corneropenings where a small virtual displacement is given to the slab. The same principle was also used by Islam,S. et al[4]in their paper. In other words, the work equation is given by

$$\iint W_{ult}\delta(x,y)dxdy = \sum\left(m_{ult,x}\theta_x y_0 + m_{ult,y}\theta_y x_0\right) \text{-------------------(1)}$$

where W_{ult} is the ultimate load per unit area of slab, $\delta(x,y)$ is the virtual displacement in the direction of the loading at the element of area of dimensions dx, dy, $m_{ult,x}$ and $m_{ult,y}$ are the yield moments per unit width in the x and y directions, θ_x and θ_y are the components of the virtual rotation of the slab segments in the x and y directions and x_0 and y_0 are the projected length of the yield lines in x and y directions of slab. The equation (1) contains terms C_1, C_2, $C_3$and C_4 which define the positions of the node points of the yield lines. The values of C_1, C_2, C_3 and C_4 to be used in the equation are those which give the minimum load to cause failure.

A computer program has been written to find the values of C_1, C_2, C_3 and C_4 (in terms of r_1, r_2, r_3 and r_4) corresponding to minimum load carrying capacity of the slab. For definitions of various parameters refer notations. Johansen[1] has proved that the yield line theory is an upper bound method, so care has been taken to examine all the possible yield line patterns for OLC slab to ensure that the most critical collapse mode is considered otherwise the load carrying capacity of the slab will be overestimated.

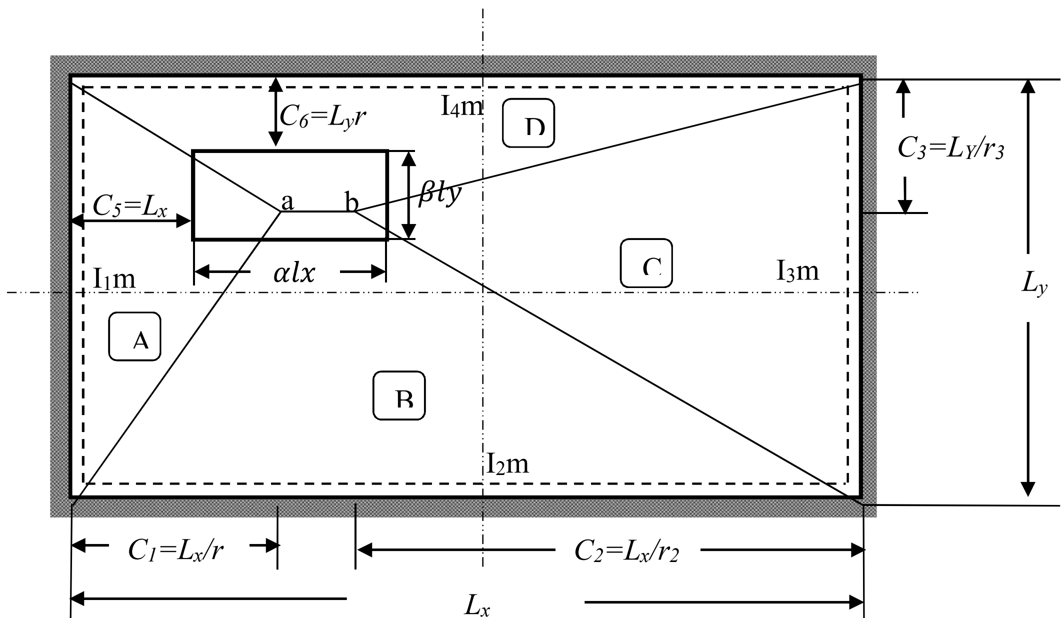
Formulation of Virtual Work Equations

There is several possible yield line patterns associated with different edge conditions of the slab. For the OLC edge condition of slab, the possible admissible failure yield line patterns are nineteen. These admissible failure yield line patterns are obtained basing on the yield line principle of Johansen K.W[1]. For the given configuration of the slab, these nineteen failure patterns and corresponding equations have been investigated depending upon the support condition of the slab using a computer program.

The generalized virtual work equations for continuous edge (CS) condition of slab are derived for the predicted possible admissible failure yield line patterns using the virtual work equation. To get the equations for other edge conditions of the slab, modification should be carried out in the numerators of the equations of each failure patterns. For OLC slab $I_1=I_2=I_3=0$ or $I_1= I_3=I_4= 0$.

Virtual Work Equations for One Long side Continuous (OLC) Slab

Nineteen possible failure patterns are predicted for this edge condition of the slab. Let δ be the virtual displacement at a & b (Fig. A), for the considered failure Pattern-1of a slab. Three unknown dimensions C_1, C_2, &C_3 are necessary to define the yield line propagation completely. All other admissible failure patterns are as shown in APPENDIX-A**.**



Fig–1

The derived equations below are for the continuously supported rectangular slab with interior corner opening.

PATTERN 1:

The external work done by segment A:-

$$=\left[W_{ult}\frac{1}{2}C_5y_1\frac{C_5}{3C_1}+W_{ult}C_5y_3\frac{C_5}{2C_1}+W_{ult}\frac{1}{2}C_8x_1\frac{x_1}{3C_1}\right]$$

$$=W_{ult}L_y{}^2r\left[\frac{r_1r_5^3}{6r_3}+\frac{r_1r_5^2}{2}\left(1-r_8-\frac{r_1r_5}{r_3}\right)+\frac{r_3^2r_8^3}{6r_1(r_3-1)^2}\right]$$

The external work done by segment B:-

$$=\left[W_{ult}\frac{1}{2}x_1C_8\frac{C_8}{3(L_y-C_3)}+W_{ult}x_3C_8\frac{C_8}{2(L_y-C_3)}+W_{ult}\frac{1}{2}x_2C_8\frac{C_8}{3(L_y-C_3)}\right]$$

$$=W_{ult}L_y^{\ 2}r\left[\frac{r_3^2 r_8^3}{6r_1(r_3-1)^2}+\frac{r_3^2 r_8^3}{2(r_3-1)}\left(1-\frac{r_3 r_8}{(r_3-1)}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\right)+\frac{r_3^2 r_8^3}{6r_2(r_3-1)^2}\right]$$

The external work done by segment C:-

$$=\left[W_{ult}\frac{1}{2}C_8 x_2\frac{x_2}{3C_2}+W_{ult}y_4 C_7\frac{C_7}{2C_2}+W_{ult}\frac{1}{2}y_2 C_7\frac{C_7}{3C_2}\right]$$

$$=W_{ult}L_y^{\ 2}r\left[\frac{r_3^2 r_8^3}{6r_2(r_3-1)^2}+\left(1-r_8-\frac{r_2 r_7}{r_3}\right)\frac{r_2 r_7^2}{2}+\frac{r_2^2 r_7^3}{6r_3}\right]$$

The external work done by segment D:-

$$=\left[W_{ult}\frac{1}{2}C_7 y_2\frac{y_2}{3C_3}+W_{ult}\alpha L_x C_6\frac{C_6}{2C_3}+W_{ult}\frac{1}{2}C_5 y_1\frac{y_1}{3C_3}\right]$$

$$=W_{ult}L_y^{\ 2}r\left[\frac{r_2^2 r_7^3}{6r_3}+\frac{r_3 r_6^2(1-r_5-r_7)}{2}+\frac{r_1^2 r_5^3}{6r_3}\right]$$

Total work done = work done by segment’s (A+B+C+D)

$$=W_{ult}L_y^{\ 2}r\left[\begin{array}{l}\left[\frac{r_1 r_5^3}{6r_3}+\frac{r_1 r_5^2}{2}\left(1-r_8-\frac{r_1 r_5}{r_3}\right)+\frac{r_3^2 r_8^3}{6r_1(r_3-1)^2}\right]+\left[\frac{r_3^2 r_8^3}{6r_1(r_3-1)^2}+\frac{r_3^2 r_8^3}{2(r_3-1)}\left(1-\frac{r_3 r_8}{(r_3-1)}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\right)+\frac{r_3^2 r_8^3}{6r_2(r_3-1)^2}\right]\\+\left[\frac{r_3^2 r_8^3}{6r_2(r_3-1)^2}+\left(1-r_8-\frac{r_2 r_7}{r_3}\right)\frac{r_2 r_7^2}{2}+\frac{r_2^2 r_7^3}{6r_3}\right]+\left[\frac{r_2^2 r_7^3}{6r_3}+\frac{r_3 r_6^2(1-r_5-r_7)}{2}+\frac{r_1^2 r_5^3}{6r_3}\right]\end{array}\right]$$

Energy absorbed yield lines: -

$$=\left[\begin{array}{l}\left[m_{ult}K'_x(y_1+C_8)\frac{1}{C_1}+m_{ult}I_1 L_y\frac{1}{C_1}\right]+\left[m_{ult}K'_y(x_1+x_2)\frac{1}{(L_y-C_3)}+m_{ult}I_2 L_x\frac{1}{(L_y-C_3)}\right]\\+\left[m_{ult}K'_x(y_2+C_8)\frac{1}{C_2}+m_{ult}I_3 L_y\frac{1}{C_2}\right]+\left[m_{ult}K'_y(C_5+C_7)\frac{1}{C_3}+m_{ult}I_4 L_x\frac{1}{C_3}\right]\end{array}\right]$$

$$=m_{ult}\left[\begin{array}{l}\left[\frac{K_x^1 r_1}{r}\left(\frac{r_1 r_5}{r_3}+r_8\right)+\frac{I_1 r_1}{r}\right]+\left[\frac{K_y^1 r r_3^2 r_8}{(r_3-1)^2}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)+\frac{I_2 r r_3}{(r_3-1)}\right]+\left[\frac{K_x^1 r_2}{r}\left(\frac{r_2 r_7}{r_3}+r_8\right)+\frac{I_3 r_2}{r}\right]+\\\left[K_y^1 r r_3(r_5+r_7)+I_4 r r_3\right]\end{array}\right]$$

Equating total work done by the segments to energy absorbed by yield lines we get

$$\frac{W_{ult} l_y^2}{m_{ult}} = \frac{\left[\begin{array}{l}\left[\frac{K_x^1 r_1}{r}\left(\frac{r_1 r_5}{r_3}+r_8\right)+\frac{I_1 r_1}{r}\right]+\left[\frac{K_y^1 r r_3^2 r_8}{(r_3-1)^2}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)+\frac{I_2 r r_3}{(r_3-1)}\right]+\left[\frac{K_x^1 r_2}{r}\left(\frac{r_2 r_7}{r_3}+r_8\right)+\frac{I_3 r_2}{r}\right]+\\ \left[K_y^1 r r_3 (r_5+r_7)+I_4 r r_3\right]\end{array}\right]}{r\left[\begin{array}{l}\left[\frac{r_1 r_5^3}{6 r_3}+\frac{r_1 r_5^2}{2}\left(1-r_8-\frac{r_1 r_5}{r_3}\right)+\frac{r_3^2 r_8^3}{6 r_1 (r_3-1)^2}\right]+\left[\frac{r_3^2 r_8^3}{6 r_1 (r_3-1)^2}+\frac{r_3^2 r_8^3}{2(r_3-1)}\left(1-\frac{r_3 r_8}{(r_3-1)}\left(\frac{1}{r_1}+\frac{1}{r_2}\right)\right)+\frac{r_3^2 r_8^3}{6 r_2 (r_3-1)^2}\right.\\ \left.+\left[\frac{r_3^2 r_8^3}{6 r_2 (r_3-1)^2}+\left(1-r_8-\frac{r_2 r_7}{r_3}\right)\frac{r_2 r_7^2}{2}+\frac{r_2^2 r_7^3}{6 r_3}\right]+\left[\frac{r_2^2 r_7^3}{6 r_3}+\frac{r_3 r_6^2 (1-r_5-r_7)}{2}+\frac{r_1^2 r_5^3}{6 r_3}\right]\right.\end{array}\right]}$$

Equation 2 for Failure Pattern – 2

$$\frac{W_{ult} l_y^2}{m_{ult}} = \frac{\left[\left[\frac{K_x^1 r_5 r_1^2}{r}+\frac{I_1 r_1}{r}\right]+\left[\frac{K_y^1 r r_3}{(r_3-1)}(r_5+r_7)+\frac{I_2 r r_3}{(r_3-1)}\right]+\left[\frac{K_x^1 r_2^2 r_7}{r}+\frac{I_3 r_2}{r}\right]+\left[\frac{K_y^1 r_3}{r}(r_5+r_7)+\frac{I_4 r_3}{r}\right]\right]}{r*\left[\begin{array}{l}\left[\frac{r_1^2 r_5^3}{6 r_3}+(1-r_1 r_5)\frac{r_5^2 r_1}{2}+\frac{r_1^2 r_5^3 (r_3-1)}{6 r_3}\right]+\left[\frac{r_1^2 r_5^3 (r_3-1)}{6 r_3}+\frac{\alpha r_8^3 r_3}{2(r_3-1)}+\frac{r_2^2 r_7^3 (r_3-1)}{6 r_3}\right]+\\ \left[\frac{r_2^2 r_7^3 (r_3-1)}{6 r_3}+\frac{(1-r_7 r_2) r_2 r_7^3}{2}+\frac{r_2^2 r_7^3}{6 r_3}\right]+\left[\frac{r_2^2 r_7^3}{6 r_3}+\frac{\alpha r_6^2 r_3}{2}+\frac{r_1^2 r_5^3}{6 r_3}\right]\end{array}\right]}$$

Equation 3 for Failure Pattern –3

$$\frac{W_{ult} l_y^2}{m_{ult}} = \frac{\left[\begin{array}{l}\left[\frac{K_x^1 r_1}{r}\left(\frac{r_1 r_5}{r_3}+r_8\right)+\frac{I_1 r_1}{r}\right]+\left[\frac{K_y^1 r r_3}{(r_3-1)}\left(\frac{r_8 r_3}{r_1 (r_3-1)}+r_7\right)+\frac{I_2 r r_3}{(r_3-1)}\right]+\left[K_y^1 r r_3 (r_7+r_5)+I_4 r r_3\right]+\\ \left[\frac{K_x^1 r_2^2 r_7}{r}+\frac{I_3 r_2}{r}\right]\end{array}\right]}{r*\left[\begin{array}{l}\left[\frac{r_1^2 r_5^3}{6 r_3}+\left(1-r_8-\frac{r_1 r_5}{r_3}\right)\frac{r_1 r_5^2}{2}+\frac{r_8^3 r_3^2}{6 r_1 (r_3-1)^2}\right]+\left[\frac{r_8^3 r_3^2}{6 r_1 (r_3-1)^2}+\left(1-r_7-\frac{r_8 r_3}{r_1 (r_3-1)}\right)\frac{r_3 r_8^2}{2(r_3-1)}+\frac{r_2^2 r_7^3 (r_3-1)}{6 r_3}\right]+\\ \left[\frac{r_2^2 r_7^3 (r_3-1)}{6 r_3}+(1-r_2 r_7)\frac{r_2 r_7^2}{2}+\frac{r_2^2 r_7^3}{6 r_3}\right]+\left[\frac{r_2^2 r_7^3}{6 r_3}+\frac{\alpha r_3 r_6^2}{2}+\frac{r_1^2 r_5^3}{6 r_3}\right]\end{array}\right]}$$

Equation 4 for Failure Pattern – 4

$$\frac{W_{ult} l_y^2}{m_{ult}} = \frac{\begin{bmatrix} \left[\frac{K_x^1 r_1}{r}\left(r_6 + \frac{r_1 r_5 (r_3 - 1)}{r_3}\right) + \frac{I_2 r_1}{r}\right] + \left[\frac{K_y^1 r r_3}{(r_3 - 1)}(r_5 + r_7) + \frac{I_2 r r_3}{(r_3 - 1)}\right] + \left[\frac{K_x^1 r_2^2 r_7}{r} + \frac{I_3 r_2}{r}\right] + \\ \left[K_y^1 r r_3\left(\frac{r_6 r_3}{r_1} + r_7\right) + I_3 r r_3\right] \end{bmatrix}}{r * \begin{bmatrix} \left[\frac{r_3^2 r_6^2}{6 r_1} + \left(1 - r_6 - \frac{r_1 r_5 (r_3 - 1)}{r_3}\right)\frac{r_1 r_5^2}{2} + \frac{r_1^2 r_5^3 (r_3 - 1)}{6 r_3}\right] + \left[\frac{r_1^2 r_5^3 (r_3 - 1)}{6 r_3} + \frac{\alpha r_3 r_8^2}{2(r_3 - 1)} + \frac{r_2^2 r_7^3 (r_3 - 1)}{6 r_3}\right] + \\ \left[\frac{r_2^2 r_7^3 (r_3 - 1)}{6 r_3} + (1 - r_2 r_7)\frac{r_2 r_7^2}{2} + \frac{r_2^2 r_7^3}{6 r_3}\right] + \left[\frac{r_2^2 r_7^3}{6 r_3} + \left(1 - r_7 - \frac{r_6 r_3}{r_1}\right)\frac{r_3 r_6^2}{2} + \frac{r_3^2 r_6^2}{6 r_1}\right] \end{bmatrix}}$$

Equation 5 for Failure Pattern – 5

$$\frac{W_{ult} l_y^2}{m_{ult}} = \frac{\begin{bmatrix} \left[\frac{K_x^1 r_1}{r}(r_6 + r_8) + \frac{I_1 r_1}{r}\right] + \left[\frac{K_y^1 r r_3}{(r_3 - 1)}\left(\frac{r_8 r_3}{r_1 (r_3 - 1)} + r_7\right) + \frac{I_2 r r_3}{(r_3 - 1)}\right] + \left[\frac{K_x^1}{r}(r_2^2 r_7) + \frac{I_3 r_2}{r}\right] + \\ \left[K_y^1 r r_3\left(r_7 + \frac{r_3 r_6}{r_1}\right) + I_4 r r_3\right] \end{bmatrix}}{r * \begin{bmatrix} \left[\frac{r_3^2 r_6^3}{6 r_1} + \frac{\beta r_1 r_5^2}{2} + \frac{r_8^3 r_3^2}{6 r_1 (r_3 - 1)^2}\right] + \left[\frac{r_8^3 r_3^2}{6 r_1 (r_3 - 1)^2} + \left(1 - r_7 - \frac{r_3 r_8}{r_1 (r_3 - 1)}\right)\frac{r_6^2 r_3}{2(r_3 - 1)} + \frac{r_2^2 r_7^3 (r_3 - 1)}{6 r_3}\right] \\ + \left[\frac{r_2^2 r_7^3 (r_3 - 1)}{6 r_3} + \frac{r_2 r_7^2}{2}(1 - r_2 r_7) + \frac{r_2^2 r_7^3}{6 r_3}\right] + \left[\frac{r_2^2 r_7^3}{6 r_3} + \left(1 - r_7 - \frac{r_3 r_6}{r_1}\right)\frac{r_3 r_6^2}{2} + \frac{r_3^2 r_6^3}{6 r_1}\right] \end{bmatrix}}$$

Equation 6 for Failure Pattern – 6

$$\frac{W_{ult} l_y^2}{m_{ult}} = \frac{\begin{bmatrix} \left[\frac{K_x^1 r_1}{r}\left(r_6 + \frac{r_1 r_5 (r_3 - 1)}{r_3}\right) + \frac{I_1 r_1}{r}\right] + \left[\frac{K_y^1 r r_3}{(r_3 - 1)}(r_7 + r_5) + \frac{I_2 r r_3}{(r_3 - 1)}\right] + \left[\frac{K_x^1 r_2}{r} + \frac{I_3 r_2}{r}\right] + \\ \left[K_y^1 r_3 r\left(\frac{r_3 r_6}{r_1} + r_7\right) + I_4 r_3 r\right] \end{bmatrix}}{r * \begin{bmatrix} \left[\frac{r_3^2 r_6^3}{6 r_1} + \left(1 - r_6 - \frac{(r_3 - 1) r_5 r_1}{r_3}\right)\frac{r_1 r_5^2}{2} + \frac{r_1^2 r_5^3 (r_3 - 1)}{6 r_3}\right] + \left[\frac{r_1^2 r_5^3 (r_3 - 1)}{6 r_3} + \frac{\alpha r_3 r_8^2}{2(r_3 - 1)} + \frac{(r_3 - 1)(r_2 r_7 - 1)}{2 r_2 r_3} + \frac{(r_3 - 1)}{6 r_2 r_3}\right] \\ + \left[\frac{(r_3 - 1)}{6 r_2 r_3} + \frac{1}{6 r_2 r_3}\right] + \left[\frac{1}{6 r_2 r_3} + \frac{(r_7 r_2 - 1)}{2 r_3 r_2} + \left(1 - r_7 - \frac{r_6 r_3}{r_1}\right)\frac{r_3 r_6^2}{2} + \frac{r_3^2 r_6^3}{6 r_1}\right] \end{bmatrix}}$$

Equation 7 for Failure Pattern – 7

$$\frac{W_{ult} l_y^2}{m_{ult}} = \frac{\left[\left[\frac{K_x^1 r_1}{r}\left(r_8 + \frac{r_1 r_5}{r_3}\right) + \frac{I_1 r_1}{r}\right] + \left[\frac{K_y^1 r r_3}{(r_3-1)}\left(r_7 + \frac{r_8 r_3}{r_1(r_3-1)}\right) + \frac{I_2 r_3 r}{(r_3-1)}\right] + \left[\frac{K_x^1 r_2}{r} + \frac{I_3 r_2}{r}\right] + \left[K_y^1 r r_3 (r_5 + r_7) + I_4 r r_3\right]\right]}{r * \left[\left[\frac{r_1^2 r_5^3}{6 r_3} + \left(1 - r_8 - \frac{r_1 r_5}{r_3}\right)\frac{r_1 r_5^2}{2} + \frac{r_3^2 r_8^3}{6 r_1 (r_3-1)^2}\right] + \left[\frac{r_3^2 r_8^3}{6 r_1 (r_3-1)^2} + \left(1 - r_7 - \frac{r_3 r_8}{r_1(r_3-1)}\right)\frac{r_3 r_8^2}{2(r_3-1)} + \frac{(r_2 r_7 - 1)(r_3 - 1)}{2 r_3 r_2} + \frac{(r_3-1)}{6 r_2 r_3}\right] + \left[\frac{(r_3-1)}{6 r_2 r_3} + \frac{1}{6 r_3 r_2}\right] + \left[\frac{1}{6 r_3 r_2} + \frac{(r_2 r_7 - 1)}{2 r_3 r_2} + \frac{\alpha r_3 r_6^2}{2} + \frac{r_1^2 r_5^3}{6 r_3}\right]\right]}$$

Equation 8 for Failure Pattern –8

$$\frac{W_{ult} l_y^2}{m_{ult}} = \frac{\left[\left(\frac{K_x^1 r_1}{r}(r_6 + r_8) + \frac{I_1 r_1}{r}\right) + \left[\frac{K_y^1 r r_3}{(r_3-1)}\left(\frac{r_8 r_3}{r_1(r_3-1)} + r_7\right) + \frac{I_2 r r_3}{(r_3-1)}\right] + \left[\frac{K_x^1 r_2}{r} + \frac{I_3 r_2}{r}\right] + \left[K_y^1 r r_3\left(r_7 + \frac{r_3 r_6}{r_1}\right) + I_4 r r_3\right]\right]}{r * \left[\left[\frac{r_3^2 r_6^3}{6 r_1} + \frac{\beta r_1 r_5^2}{2} + \frac{r_8^3 r_3^2}{6 r_1 (r_3-1)^2}\right] + \left[\frac{r_8^3 r_3^2}{6 r_1 (r_3-1)^2} + \left(1 - r_7 - \frac{r_3 r_8}{r_1(r_3-1)}\right)\frac{r_8^2 r_3}{2(r_3-1)} + \frac{(r_2 r_7 - 1)(r_3 - 1)}{2 r_3 r_2} + \frac{(r_3-1)}{6 r_3 r_2}\right] + \left[\frac{(r_3-1)}{6 r_2 r_3} + \frac{1}{6 r_2 r_3}\right] + \left[\frac{1}{6 r_2 r_3} + \frac{(r_2 r_7 - 1)}{2 r_3 r_2} + \left(1 - r_7 - \frac{r_3 r_6}{r_1}\right)\frac{r_3 r_6^2}{2} + \frac{r_3^2 r_6^3}{6 r_1}\right]\right]}$$

Equation 9 for Failure Pattern – 9

$$\frac{W_{ult} l_y^2}{m_{ult}} = \frac{\left[\left[\frac{K_x^1 r_1}{r}\left(\frac{r_1 r_5}{r_3} + \frac{r_1 r_5 (r_3 - 1)}{r_3}\right) + \frac{I r_1}{r}\right] + \left[\frac{K_y^1 r r_3}{(r_3-1)}(r_7 + r_5) + \frac{I_2 r r_3}{(r_3-1)}\right] + \left[\frac{K_x^1 r_2}{r} + \frac{I_3 r_2}{r}\right] + \left[K_y^1 r r_3 (r_7 + r_5) + I_4 r r_3\right]\right]}{r * \left[\left[\frac{r_1^2 r_5^3}{6 r_3} + (1 - r_1 r_5)\frac{r_1 r_5^2}{2} + \frac{r_1^2 r_5^3 (r_3 - 1)}{6 r_3}\right] + \left[\frac{r_1^2 r_5^3 (r_3-1)}{6 r_3} + \frac{\alpha r_8^2 r_3}{2(r_3-1)} + \frac{(r_2 r_7 - 1)(r_3 - 1)}{2 r_3 r_2} + \frac{(r_3-1)}{6 r_2 r_3}\right] + \left[\frac{(r_3-1)}{6 r_2 r_3} + \frac{1}{6 r_2 r_3}\right] + \left[\frac{1}{6 r_2 r_3} + \frac{(r_2 r_7 - 1)}{2 r_3 r_2} + \frac{\alpha r_6^2 r_3}{2} + \frac{r_1^2 r_5^3}{6 r_3}\right]\right]}$$

Equation 10 for Failure Pattern – 10

$$\frac{W_{ult} l_y^2}{m_{ult}} = \frac{\left[\left[\frac{K_x^1 r_1}{r}\left(r_6 + \frac{r_5 r_1}{r_4}\right) + \frac{I_1 r_1}{r}\right] + \left[K_y^1 r r_4 (r_5 + r_7) + I_2 r r_4\right] + \left[\frac{K_x^1}{r}\frac{r_1}{r_1 - 1}\left(\frac{r_1 r_7}{r_4(r_1 - 1)} + r_6\right) + \frac{I_3}{r}\frac{r_1}{r_1 - 1}\right] + \left[K_y^1 r (r_3^2 r_6) + I_4 r r_3\right]\right]}{r * \left[\left[\frac{r_3^2 r_6^3}{6 r_1} + \frac{(r_4 - r_5 r_1 - r_4 r_6) r_1 r_5^2}{2 r_4} + \frac{r_1^2 r_5^3}{6 r_4}\right] + \left[\frac{r_1^2 r_5^3}{6 r_4} + \frac{\alpha r_8^2 r_4}{2} + \frac{r_1^2 r_7^3}{6 r_4 (r_1 - 1)^2}\right] + \left[\frac{r_1^2 r_7^3}{6 r_4 (r_1 - 1)^2} + \frac{r_1 r_7^2}{2(r_1 - 1)}\left(1 - \frac{r_1 r_7}{r_4 (r_1 - 1)} - r_6\right) + \frac{r_3^2 r_6^3 (r_1 - 1)}{6 r_1}\right] + \left[\frac{r_3^2 r_6^3 (r_1 - 1)}{6 r_1} + r_3 r_6\left(\frac{r_1 - r_3 r_6 - (r_1 - 1) r_3 r_6}{2 r_1}\right) + \frac{r_3^2 r_6^3}{6 r_1}\right]\right]}$$

Equation 11 for Failure Pattern – 11

$$\frac{W_{ult} l_y^2}{m_{ult}} = \frac{\left[\left[\frac{K_x^1 r_1}{r_1}\left(\frac{r_1 r_5 + r_3 r_8}{r_3}\right) + \frac{I_1 r_1}{r}\right] + \left[K_y^1 r r_4\left(\frac{r_4 r_8 + (r_4 r_8)(r_1 - 1)}{r_1}\right) + I_2 r r_4\right] + \left[\frac{K_x^1 r_1}{r(r_1 - 1)}\left(\frac{r_1 r_7}{r_3(r_1 - 1)} + r_8\right) + \frac{I_3 r_1}{r(r_1 - 1)}\right] + \left[K_y^1 r r_3 (r_5 + r_7) + I_4 r r_3\right]\right]}{r * \left[\left[\frac{r_1^2 r_5^3}{6 r_3} + \frac{(r_3 - r_1 r_5 - r_3 r_8) r_1 r_5^2}{2 r_3} + \frac{r_4^2 r_8^3}{6 r_1}\right] + \left[\frac{r_4^2 r_8^3}{6 r_1} + \frac{(r_1 - r_4 r_8 - r_4 r_8 (r_1 - 1)) r_4 r_8^2}{2 r_1} + \frac{r_4^2 r_8^3 (r_1 - 1)}{6 r_1}\right] + \left[\frac{r_4^2 r_8^3 (r_1 - 1)}{6 r_1} + \left(1 - \frac{r_1 r_7}{r_3 (r_1 - 1)} - r_8\right)\frac{r_1 r_7^2}{2 r_3 (r_1 - 1)^2} + \frac{r_1^2 r_7^3}{6 r_3 (r_3 - 1)^2}\right] + \left[\frac{r_1^2 r_7^3}{6 r_3 (r_3 - 1)^2} + \frac{\alpha r_6^2 r_3}{2} + \frac{r_1^2 r_5^3}{6 r_3}\right]\right]}$$

Equation 12 for Failure Pattern – 12

$$\frac{W_{ult} l_y^2}{m_{ult}} = \frac{\left[\begin{array}{l} \left[\frac{K_x^1 r_1}{r}\left(\frac{r_5 r_1}{r_3}+\frac{r_1 r_5}{r_4}\right)+\frac{I_1 r_1}{r}\right]+\left[K_y^1 r r_4 (r_7 + r_5) + I_2 r r_4\right]+ \\ \left[\frac{K_x^1}{r}\left(\frac{r_1^2 r_7}{r_4 (r_1-1)^2}+\frac{r_1^2 r_7}{r_3 (r_1-1)^2}\right)+\frac{I_1 r_1}{r(r_1-1)}\right]+\left[K_y^1 r r_3 (r_5 + r_7) + \frac{I_y r_3}{r}\right] \end{array}\right]}{R * \left[\begin{array}{l} \left[\frac{r_1^2 r_5^3}{6 r_3}+\frac{r_1 r_5^2}{2}\left(1-\frac{r_1 r_5}{r_3}-\frac{r_1 r_5}{r_4}\right)+\frac{r_1^2 r_5^3}{6 r_4}\right]+\left[\frac{r_1^2 r_5^3}{6 r_4}+\frac{\alpha r_4 r_8^2}{2}+\frac{r_1^2 r_7^3}{6 r_4 (r_1-1)^2}\right]+ \\ \left[\frac{r_1^2 r_7^3}{6 r_4 (r_1-1)^2}+\frac{r_1 r_7^2}{2(r_1-1)}\left(1-\frac{r_1 r_7}{r_3 (r_1-1)}-\frac{r_1 r_7}{r_4 (r_1-1)}\right)+\frac{r_1^2 r_7^3}{6 r_3 (r_1-1)^2}\right]+ \\ \left[\frac{r_1^2 r_7^3}{6 r_3 (r_1-1)^2}+\frac{\alpha r_3 r_6^2}{2}+\frac{r_1^2 r_5^3}{6 r_3}\right] \end{array}\right]}$$

Equation 13 for Failure Pattern – 13

$$\frac{W_{ult} l_y^2}{m_{ult}} = \frac{\left[\begin{array}{l} \left[\frac{k_x^1 r_1}{r}\left(r_8+\frac{r_1 r_5}{r_3}\right)+\frac{I_1 r_1}{r}\right]+\left[K_y^1 r r_4\left(\frac{r_4 r_8}{r_1}+r_7\right)+I_2 r r_4\right]+ \\ \left[\frac{k_x^1 r_1^2 r_7}{r(r_1-1)^2}\left(\frac{1}{r_3}+\frac{1}{r_4}\right)+\frac{I_3 r_1}{r(r_1-1)}\right]+\left[K_y^1 r r_3 (r_5 + r_7) + + I_4 r r_3\right] \end{array}\right]}{r * \left[\begin{array}{l} \left[\frac{r_1^2 r_5^3}{6 r_3}+\left(1-r_5-\frac{r_1 r_5}{r_3}\right)\frac{r_1 r_5^2}{2}+\frac{r_4^2 r_8^3}{6 r_1}\right]+\left[\frac{r_4^2 r_8^3}{6 r_1}+\left(1-r_7-\frac{r_4 r_8}{r_1}\right)\frac{r_4 r_8^2}{2}+\frac{r_1^2 r_7^3}{6 r_4 (r_1-1)^2}\right]+ \\ \left[\frac{r_1^2 r_7^3}{6 r_4 (r_1-1)^2}+\left(1-\frac{r_1 r_7}{(r_1-1)}\left(\frac{1}{r_3}+\frac{1}{r_4}\right)\right)\frac{r_1 r_7^2}{2(r_1-1)}+\frac{r_1^2 r_7^3}{6 r_3 (r_1-1)^2}\right]+\left[\frac{r_1^2 r_7^3}{6 r_3 (r_1-1)^2}+\frac{\alpha r_3 r_6^2}{2}+\frac{r_1^2 r_5^3}{6 r_3}\right] \end{array}\right]}$$

Equation 14 for Failure Pattern – 14

$$\frac{W_{ult} l_y^2}{m_{ult}} = \frac{\left[\begin{array}{l} \left[\frac{k_x^1 r_1}{r}\left(r_6 + \frac{r_1 r_5}{r_4}\right) + \frac{I_1 r_1}{r}\right] + \left[K_y^1 r r_4 (r_5 + r_7) + I_2 r r_4\right] + \\ \left[\frac{k_x^1 r_1^2 r_7}{r(r_1-1)^2}\left(\frac{1}{r_3} + \frac{1}{r_4}\right) + \frac{I_3 r_1}{r(r_1-1)}\right] + \left[K_y^1 r r_3 \left(\frac{r_3 r_6}{r_1} + r_7\right) + + I_4 r r_3\right] \end{array}\right]}{r * \left[\begin{array}{l} \left[\frac{r_3^2 r_6^3}{6 r_1} + \left(1 - r_6 - \frac{r_1 r_5}{r_4}\right)\frac{r_1 r_5^2}{2} + \frac{r_1^2 r_5^3}{6 r_4}\right] + \left[\frac{r_1^2 r_5^3}{6 r_4} + \frac{\alpha r_4 r_8^2}{2} + \frac{r_1^2 r_7^3}{6 r_4 (r_1-1)^2}\right] + \\ \left[\frac{r_1^2 r_7^3}{6 r_4 (r_1-1)^2} + \left(1 - \frac{r_1 r_7}{(r_1-1)}\left(\frac{1}{r_3} + \frac{1}{r_4}\right)\right)\frac{r_1 r_7^2}{2(r_1-1)} + \frac{r_1^2 r_7^3}{6 r_3 (r_1-1)^2}\right] + \\ \left[\frac{r_1^2 r_7^3}{6 r_3 (r_1-1)^2} + \left(1 - -r_7 - \frac{r_3 r_6}{r_1}\right)\frac{r_3 r_6^2}{2} + \frac{r_3^2 r_6^3}{6 r_1}\right] \end{array}\right]}$$

Equation 15 for Failure Pattern – 15

$$\frac{W_{ult} l_y^2}{m_{ult}} = \frac{\left[\begin{array}{l} \left[\frac{K_x^1 r_1}{r}(r_8 + r_6) + \frac{I_1 r_1}{r}\right] + \left[K_y^1 r r_4 \left(r_7 + \frac{r_4 r_8}{r_1}\right) + I_2 r r_4\right] + \\ \left[\frac{K_x^1 r_1^2 r_7}{r(r_1-1)^2}\left(\frac{1}{r_4} + \frac{1}{r_3}\right) + \frac{I_3 r_1}{r(r_1-1)}\right] + \left[K_y^1 r r_3 \left(r_7 + \frac{r_3 r_6}{r_1}\right) + I_4 r r_3\right] \end{array}\right]}{r * \left[\begin{array}{l} \left[\frac{r_3^2 r_6^3}{6 r_1} + \frac{\beta r_1 r_5^2}{2} + \frac{r_4^2 r_8^3}{6 r_1}\right] + \left[\frac{r_4^2 r_8^3}{6 r_1} + \left(1 - r_7 - \frac{r_4 r_8}{r_1}\right)\frac{r_4 r_8^2}{2} + \frac{r_7^3 r_1^2}{6 r_4 (r_1-1)^2}\right] + \\ \left[\frac{r_7^3 r_1^2}{6 r_4 (r_1-1)^2} + \frac{r_7^2 r_1}{2(r_1-1)}\left(1 - \frac{r_7 r_1}{(r_1-1)}\left(\frac{1}{r_4} + \frac{1}{r_3}\right)\right) + \frac{r_7^3 r_1^2}{6 r_3 (r_1-1)^2}\right] + \\ \left[\frac{r_7^3 r_1^2}{6 r_3 (r_1-1)^2} + \left(1 - -r_7 - \frac{r_3 r_6}{r_1}\right)\frac{r_3 r_6^2}{2} + \frac{r_3^2 r_6^3}{6 r_1}\right] \end{array}\right]}$$

Equation 16 for Failure Pattern– 16

$$\frac{W_{ult} l_y^2}{m_{ult}} = \frac{\left[\left[\frac{K_x^1 r_1}{r}(r_6 + r_8) + \frac{I_1 r_1}{r}\right] + \left[K_y^1 r r_4^2 r_8 + I_2 r r_4\right] + \left[\frac{K_x^1 r_1}{r(r-1)}(r_6 + r_8) + \frac{I_4 r_1}{r(r-1)}\right] + \left[K_y^1 r r_3^2 r_6 + I_4 r r_3\right]\right]}{r * \left[\begin{array}{l} \left[\frac{r_3^2 r_6^3}{6 r_1} + \frac{\beta r_1 r_5^2}{2} + \frac{r_4^2 r_8^3}{6 r_1}\right] + \left[\frac{r_4^2 r_8^3}{6 r_1} + \frac{r_8^2 r_4}{2}(1 - r_4 r_8) + \frac{r_8^3 r_4^2 (r_1-1)}{6 r_1}\right] + \left[\frac{r_8^3 r_4^2 (r_1-1)}{6 r_1} + + \frac{\beta r_7^2 r_1}{2(r_1-1)} + \frac{r_3^2 r_6^3 (r_1-1)}{6 r_1}\right] + \\ \left[\frac{r_3^2 r_6^3 (r_1-1)}{6 r_1} + \frac{r_6^2 r_3}{2}(1 - r_3 r_6) + \frac{r_3^2 r_6^3}{6 r_1}\right] \end{array}\right]}$$

Equation 17 for Failure Pattern– 17

$$\frac{W_{ult} l_y^2}{m_{ult}} = \frac{\left[\left[\frac{K_x^1 r_1}{r}\left(r_6 + r_8\right) + \frac{I_1 r_1}{r}\right] + \left[K_y^1 r r_4 + I_2 r r_4\right] + \left[\frac{K_x^1 r_1}{r\left(r_1 - 1\right)}\left(r_6 + r_8\right) + \frac{I_3 r_1}{r\left(r_1 - 1\right)}\right] + \left[K_y^1 r\left(r_3^2 r_6\right) + I_4 r r_3\right]\right]}{r * \left[\begin{array}{l}\left[\frac{r_3^2 r_6^3}{6 r_1} + \frac{\beta r_5^2 r_1}{2} + \frac{\left(r_4 r_8 - 1\right)}{2 r_4 r_1} + \frac{1}{6 r_1 r_4}\right] + \left[\frac{1}{6 r_1 r_4} + \frac{\left(r_1 - 1\right)}{6 r_1 r_4}\right] + \\ \left[\frac{\left(r_1 - 1\right)}{6 r_1 r_4} + \frac{\left(r_4 r_8 - 1\right)\left(r_1 - 1\right)}{2 r_4 r_1} + \frac{\beta r_1 r_7^2}{2\left(r_1 - 1\right)} + \frac{r_3^2 r_6^3\left(r_1 - 1\right)}{6 r_1}\right] + \left[\frac{r_3^2 r_6^2\left(r_1 - 1\right)}{6 r_1} + (1 - r_3 r_6)\frac{r_6^2 r_3}{2} + \frac{r_3^2 r_6^3}{6 r_1}\right]\end{array}\right]}$$

Equation 18 for Failure Pattern– 18

$$\frac{W_{ult} l_y^2}{m_{ult}} = \frac{\left[\left[\frac{K^1 r_1}{r}\left(r_6 + r_8\right) + \frac{I_1 r_1}{r}\right] + \left[K_y^1 r r_4^2 r_8 + I_2 r r_4\right] + \left[\frac{K_x^1 r_1}{r\left(r_1 - 1\right)}\left(r_8 + r_6\right) + \frac{I_3 r_1}{r\left(r_1 - 1\right)}\right] + \left[K_y^1 r r_3 + I_y r r_3\right]\right]}{r * \left[\begin{array}{l}\left[\frac{1}{6 r_1 r_3} + \left(\frac{r_6 r_3 - 1}{2 r_3 r_1}\right) + \frac{\beta r_1 r_5^2}{2} + \frac{r_4^2 r_8^3}{6 r_1} + \frac{r_4^2 r_8^3}{6 r_1}\right] + \left[\frac{r_4^2 r_8^3}{6 r_1} + \frac{\left(r_8 r_4 - 1\right) r_4 r_8^2}{2} + \frac{\left(r_1 - 1\right) r_4^2 r_8^3}{6 r_1}\right] + \\ \left[\frac{\left(r_1 - 1\right) r_4^2 r_8^3}{6 r_1} + \frac{\beta r_1 r_7^2}{2\left(r_1 - 1\right)} + \frac{\left(r_6 r_3 - 1\right)\left(r_1 - 1\right)}{2 r_3 r_1} + \frac{\left(r_1 - 1\right)}{3 r_3 r_1}\right] + \left[\frac{\left(r_1 - 1\right)}{6 r_3 r_1} + \frac{1}{6 r_3 r_1}\right]\end{array}\right]}$$

Equation 19 for Failure Pattern– 19

$$\frac{W_{ult} l_y^2}{m_{ult}} = \frac{\left[\begin{array}{l}\left[\frac{K_x^1 r_1}{r}\left(r_6 + r_8\right) + \frac{I_1 r_1}{r}\right] + \left[K_y^1 r r_4\left(r_7 + \frac{r_4 r_8}{r_1}\right) + I_2 r r_4\right] + \left[\frac{K_x^1 r_1}{r\left(r_1 - 1\right)}\left(r_6 + \frac{r_7 r_1}{r_4\left(r_1 - 1\right)}\right) + \frac{I_3 r_1}{r\left(r_1 - 1\right)}\right] + \\ \left[K_y^1 r r_3 + I_4 r r_3\right]\end{array}\right]}{r * \left[\begin{array}{l}\left[\frac{1}{6 r_3 r_1} + \frac{\left(r_3 r_6 - 1\right)}{2 r_1 r_3} + \frac{\beta r_1 r_5^2}{2} + \frac{r_4^2 r_8^3}{6 r_1}\right] + \left[\frac{r_4^2 r_8^3}{6 r_1} + \left(1 - r_7 - \frac{r_4 r_8}{r_1}\right)\frac{r_8^2 r_4}{2} + \frac{r_1^2 r_7^3}{6 r_4\left(r_1 - 1\right)^2}\right] + \\ \left[\frac{r_1^2 r_7^3}{6 r_4\left(r_1 - 1\right)^2} + \left(1 - r_6 - \frac{r_1 r_7}{r_4\left(r_1 - 1\right)}\right)\frac{r_1 r_7^2}{2\left(r_1 - 1\right)} + \frac{\left(r_6 r_3 - 1\right) r_1}{2 r_3\left(r_1 - 1\right)} + \frac{\left(r_1 - 1\right)}{6 r_3 r_1}\right] + \left[\frac{\left(r_1 - 1\right)}{6 r_3 r_1} + \frac{1}{6 r_3 r_1}\right]\end{array}\right]}$$

Minimization of the virtual work equations

The value $W_{ult}L_y^2/m_{ult}$ of these equations consists of the unknown non dimensional parameters r_1, r_2, r_3 and r_4 which define the positions of the yield lines. A computer program has been developed for various values of the non dimensional parameters r_1, r_2, r_3 and r_4 within their allowable ranges in order to find the minimum value of $W_{ult}L_y^2/m_{ult}$ for the yield line failure patterns considered. In this computer program, the values of r_1, r_2, r_3 and r_4 were varied at increments of 0.1. Using the above equations, one can develop useful charts basing on orthogonality which may be used either for design or analysis in general.

A. Analysis Problem:

Determine the safe uniformly distributed load on a rectangular two way slab with interior corner opening one long side continuous slab for the following data:

A slab 6m x4m with interior corner opening of size 1.2m x0.4m at a distance of 1.2m from long edge and 0.8m from short edge is reinforced with 10mm diameter bars @ 200mm c/c perpendicular to long span and 8mm diameter bars @150mm c/c perpendicular to short span. Two meshes are used one at top long side continuous and one at bottom, thickness of the slab is 120mm. Characteristic strength of concrete is 20MPa and yield stress of steel is 415MPa.

Solution:

According to IS 456:2000, $m_{ult} = 0.87 f_y A_{st} z$,where $z = d\left(1 - \frac{f_y A_{st}}{f_{ck} bd^2}\right)$------------(1)

Assuming effective depth of slab in short span direction=100.00 mm

Area of the steel perpendicular to long span=374 mm^2

Area of the steel perpendicular to short span=314 mm^2

The ultimate moments in short and long span directions can be found using the expression (1).

Therefore, m_{ult} parallel to long span=13.489 kNm/m

m_{ult} parallel to short span=10.192kNm/m

For aspect ratio of slab, r= (6.0/4.0) = 1.5and taking m_{ult} = 13.489 kNm/m,

The orthogonal coefficients will be $K`_x$=0.755, $I_1$=$I_3$=0 and $K`_y$=I_2=1.0, I_4=0 with these orthogonal coefficients and for α=0.2, β=0.1, C_5=1.2m, C_6=0.8m.

Nineteen predicted failure patterns are evaluated by using computer program to find the governing failure pattern and the final results are as follows.

$W_{ult}L_y^2/m_{ult}$=19.67773, r_1=3.46191, r_2=3.15185, r_3=3.46191 and the failure pattern is 7

W_{ult}=(19.67773 x 13.489/4^2)= 16.5895kN/m^2

W_{dl}=(dead load including finishing)= (0.12 x 25) + 0.5=3.5 kN/m^2

Wult=1.5(W_{ll}+W_{dl}) =16.5895kN/m^2

W_{ll}=(16.5895/1.5)-3.5=7.56kN/m^2

The intensity of live load on the slab is 7.56kN/m^2

B. Design Problem:

Design one long side continuous slab of 6.5 m x5.0 m with interior corner openings of size 1.3m x1.0m at a distance of 1.3m from long edge and 1.0m from short edge to carry a uniformly distributed live load of 5kN/m^2 .Use M20 mix and Fe 415 grade steel.

Given data: Aspect ratio of slab(r)=6.5/5.0=1.3, αL_x=1.3 m, βL_y= 1.0m.

$\therefore$ α=0.2,β=0.2, C_5=1.3m, C_6=1.0mand by assuming K'_x= 3.2, I_1=I_3=0,K'_y=I_2=0.4, I_4=0.

Nineteen predicted failure patterns are evaluated by using computer program to find the governing failure pattern and the final results are as follows.

$W_{ult}L_y^2/m_{ult}$=27.77518, r_1=2.64546, r_3=3.94546, r_4=3.45185and the failure pattern is 8

Assuming overall thickness of slab =150 mm

Dead load of slab=150 X 25=3.75 kN

Dead loads including finishing's=4.25 kN/m^2

Total load=9.25kN/m^2

Ultimate total load=1.5 x 9.25=13.875kN/m^2

m_{ult}=(13.875 x 5.0^2/27.77518)= 12.489kNm/m

The orthogonal moments are $K^1{}_x m_{ult}$=$I_1 m_{ult}$=3.2x12.489=39.964kNm/m

$K^1{}_y m_{ult}$=$I_2 m_{ult}$= 0.4x12.489= 4.995kNm/m

Effective depth, $d = \sqrt{\dfrac{39.964\times10^6}{(0.138\times20\times1000)}}$ =120.33 mm

Adopt effective depth as 125 mm and overall depth as 150 mm

Minimum area of steel required along short span = 204.82 mm^2

Area of Steel (A_{st}):

Provide positive mesh 10mmØ @ 200 c/c and 12mmØ @ 100 c/c parallel to shortspan and long span respectively.

Provide negative mesh 10mmØ @ 200 c/c parallel to short span.

Two meshes i.e., positive mesh at bottom and negative mesh at top are used. The above reinforcement can be curtailed which crosses opening. Further the negative mesh reinforcement can also be curtailed as per different codes and other researchers.

Conclusions:

1. Nineteen possible admissible yield line patterns are presented for the one long side continuous slab subjected to uniformly distributed load (udl).
2. The virtual work equations for orthotropic slabs with interior corner opening with all sides continuous whose aspect ratio of opening is different from the aspect ratio of slab subjected to udl are presented.
3. Numerical examples are presented based on analysis and design of orthotropic slabs with interior corner openings.

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Appendix-A

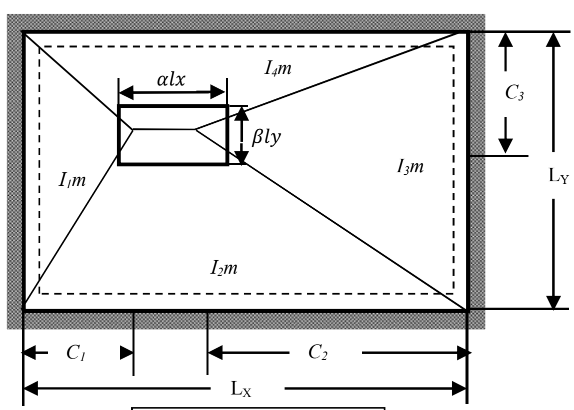


FIG – 2 (PATTERN – 2)

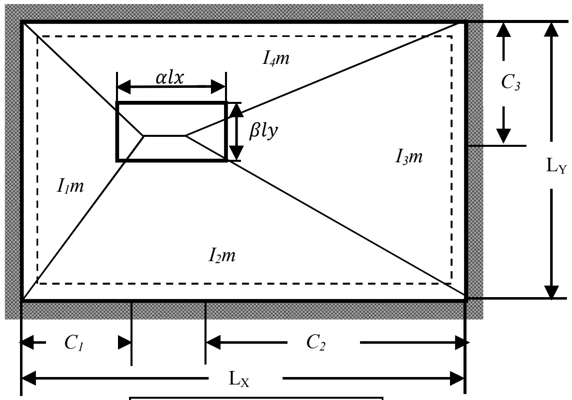
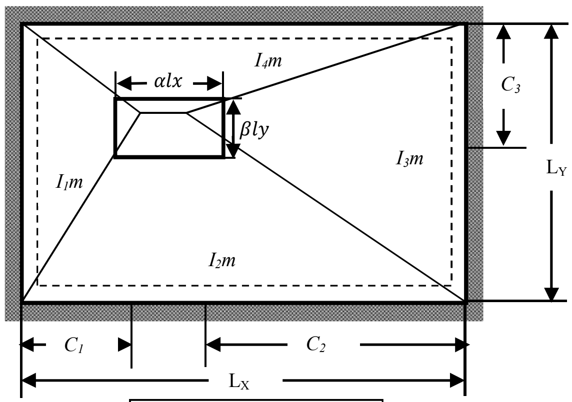


FIG - 3 (PATTERN – 3)



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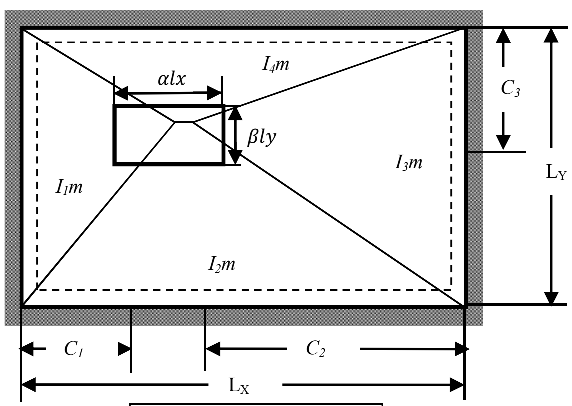

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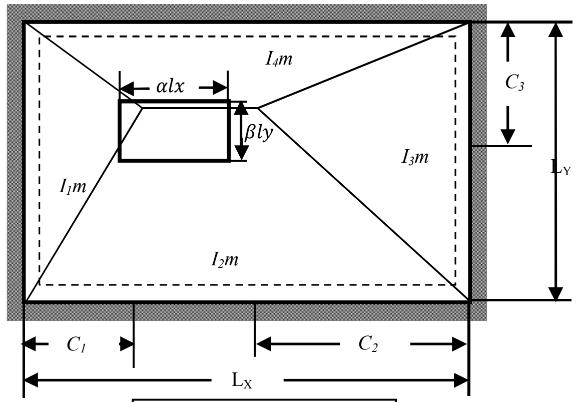


FIG – 6 (PATTERN – 6)

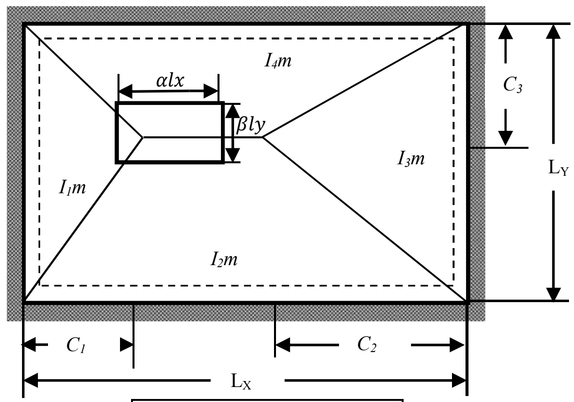


FIG – 7 (PATTERN – 7)

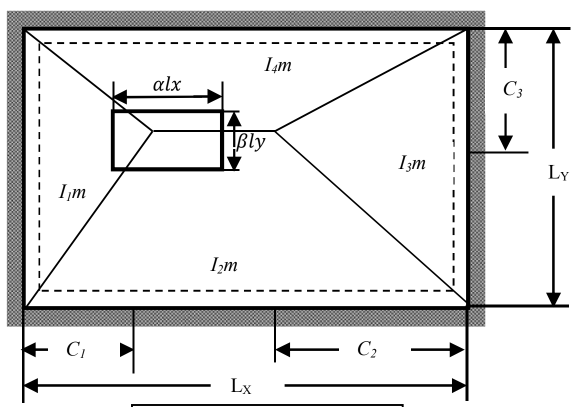


FIG – 8 (PATTERN – 8)

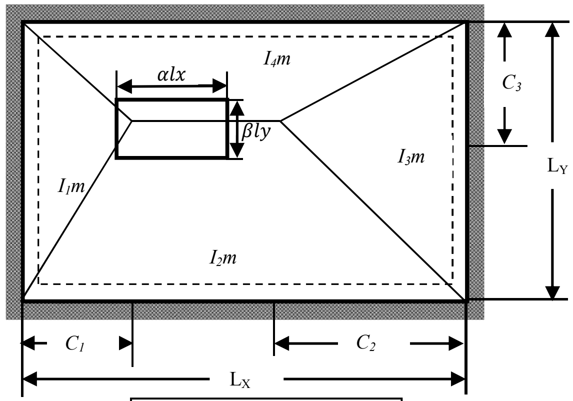


FIG – 9 (PATTERN – 9)

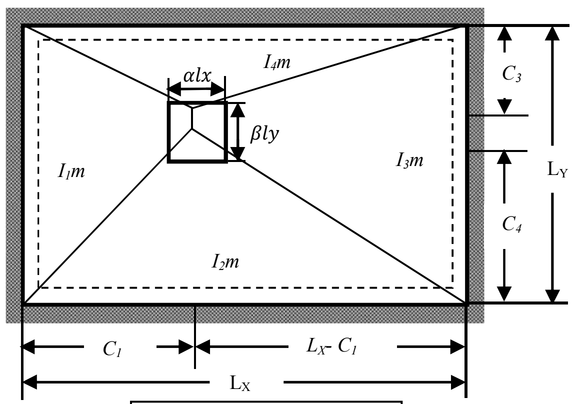


FIG – 10 (PATTERN – 10)

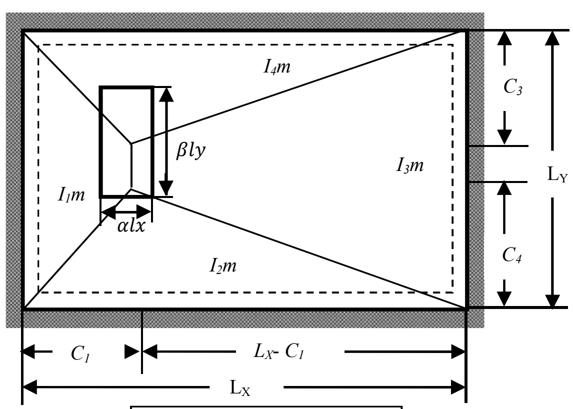
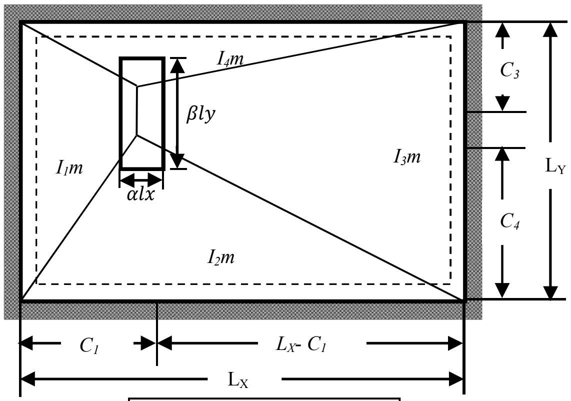


FIG – 11 (PATTERN – 11)



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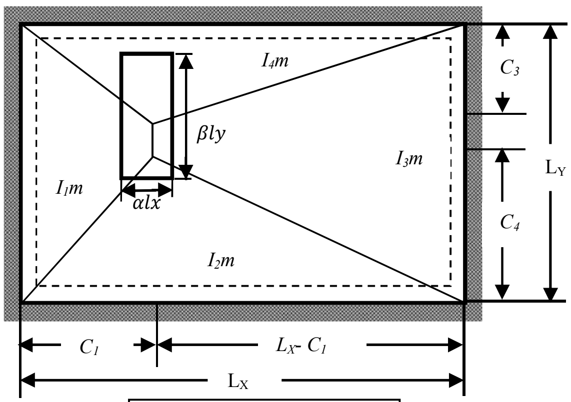


FIG – 13 (PATTERN –13)

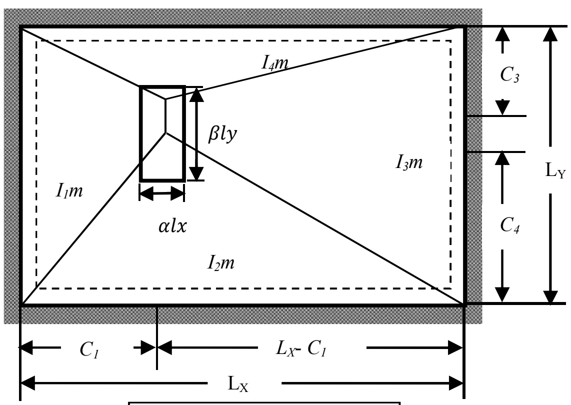
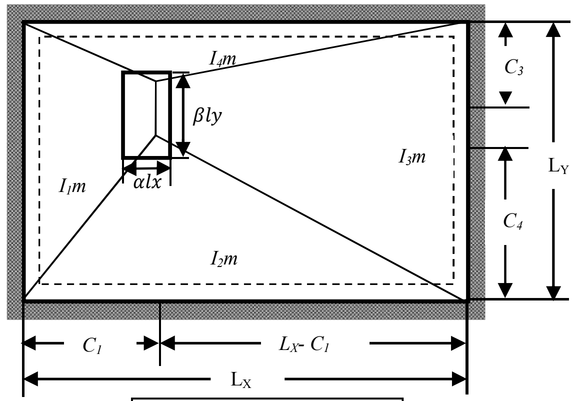


FIG – 14 (PATTERN – 14)



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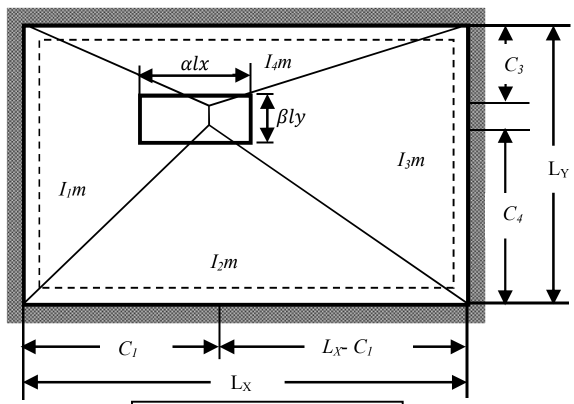

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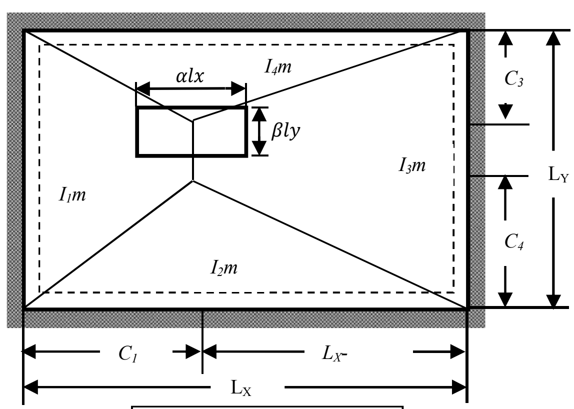

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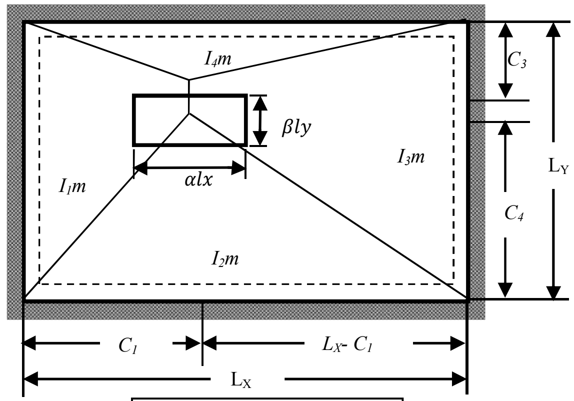

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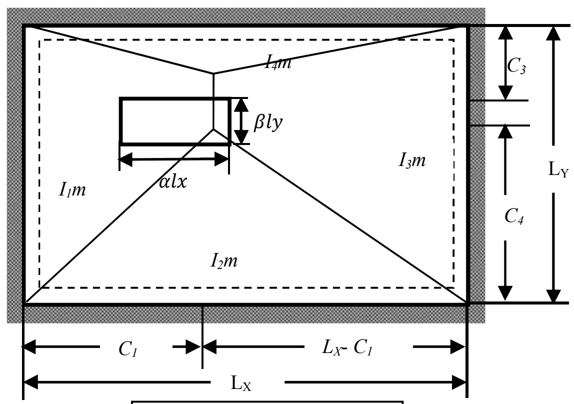

FIG – 19 (PATTERN – 19)