Vibration analysis from out of plane modes of thick annular circular plate with different boundary conditions by finite element method

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Abstract: In this study, vibration analysis from out of plane modes of a uniform annular circular plate having different boundary conditions (F-F, C-C, C-F and F-C} are investigated by keeping the mass of the plate constant. FEM is used to determine the Eigen value of the plate. The present work is validated with the existing literature and good agreement of results is observed. Further, the effect of aspect ratio and radii ratio of thick annular plate is observed. Further, the solution for flexural response for actuation and suppression associated with different boundary conditions are suggested and finally the conclusion is drawn.

I. INTRODUCTION

In engineering applications, annular circular plates with uniform thickness are widely used in structural components i.e. diaphragms and deck plates in launch vehicles, diaphragms of turbines, telephone industry, aircrafts/missiles, naval structures, nuclear reactors, optical systems, constructions of ships, automobiles and other vehicles, space shuttle, sound emitters and receivers, circular ports and panels, pneumatic pumps, cochleae, gongs and cymbals etc. Several researchers have examined the vibration response of circular or annular plate with tapered or uniform thickness.

II. LITERATURE SURVEY

Wang et al. [1] determined the elastic buckling of tapered plate using shooting method and Rayleigh - Ritz approach. Gupta and Goyal [2] determined the forced asymmetric response of linearly tapered circular plate using classical plate theory. Laura et al. [3] used the optimized Rayleigh – Ritz method to determine the buckling of circular annular plates having non uniform thickness. Sharma et al. [4] used the Ritz method to determine the transverse vibration of free annular circular plate with linearly varying thickness. Wang [5] determined the vibration modes of concentrically supported free circular plate using asymptotic formula. Singh and Hassan [6] used the Rayleigh – Ritz method to determine the transverse vibration of a circular plate with arbitrary thickness variation. Duan et al. [7] determined the vibration modes of circular plates with free edge using Mindlin plate theory. Gupta et al. [8] determined the Vibration of non-homogeneous circular Mindlin plates with variable thickness using chebyshev collocation technique. Kang [9] determined the Three-dimensional vibration analysis of thick, circular and annular plates with nonlinear thickness variation using Ritz method. Leissa (10) determined mode and natural frequencies of plate at various geometry condition and boundary condition. Mindlin et al. (11) used sixth order thick plate theory to describe flexural vibration of thick circular disk. Mcgee et al. (12) used the modified Bessel function to solve free vibration of thick annular sector plate with simply supported radial edges. Lee et al. (13) determines the natural frequencies from the out of plane modes of annular circular plate by thick and thin plate theories. Irie et al. (14) studied out of plane vibration of annular Mindlin plate with varying thickness by transfer matrix approach. Prasad et al. (15) investigated the axisymmetric vibrations of circular plates of linearly varying thickness using Frobenius method. Luisoni et al. (16) investigated the antisymmetric modes of vibration of a circular plate elastically restrained against rotation and of linearly varying thickness using classical plate theory. Grossi and Laura (17) investigated the transverse vibrations of circular plates of linearly varying thickness using Galerkin's method. Singh and Sexana (18) determined the axisymmetric vibration of a circular plate with double linear variable thickness using Rayleigh – Ritz method. Laura and De Greco (19) have written a note on vibrations and elastic stability of circular plates with thickness varying in bilinear fashion using Rayleigh – Ritz method. Singh and Chakraverty (20) determined the transverse vibration of circular and elliptic plates with quadratically varying thickness using Rayleigh – Ritz method. Barakat and Baumann (21) investigated the axisymmetric vibrations of a thin circular plate having parabolic thickness variation using Ritz Galerkin method. Lenox and Conway (22) found an exact closed form solution for the flexural vibration of a thin annular plate having a parabolic thickness variation using Bessel function. Avalos et al. (23) investigated the transverse vibrations of a circular plate carrying an elastically mounted mass using Rayleigh – Ritz method. Gupta et al. (24) found the buckling and vibration of polar orthotropic annular plates of variable thickness using Spline technique. Chen (25) found the axisymmetric vibration of circular and annular plates with arbitrarily varying thickness using Finite Element Method. Gorman (26) investigated the natural frequencies of transverse vibration of polar orthotropic variable thickness annular plates using Finite Element Method.

Review of the literature suggested that a comparative study of vibration of uniform annular circular plate with different boundary conditions is not reported. Therefore, in this paper an attempt has been made to find out the vibration response of a uniform annular circular plate with different boundary conditions by keeping the mass of the plate constant.

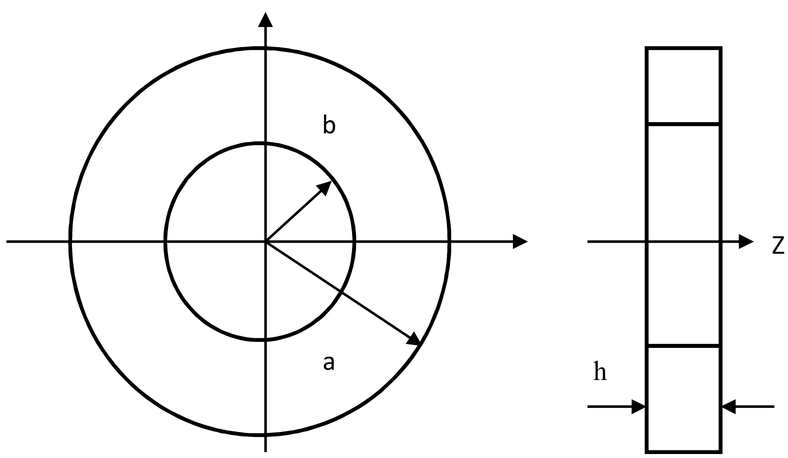

Fig.1. Geometry and co-ordinate system of annular circular plate

III. PROPOSED METHOD

2. Mathematical formulation

2.1. Plate free vibration

Natural frequency and modes shape of the plate to solve the eigen value problem for ω^2 is given by Eq. 1:

$$([k]-\omega^2[M])\psi_{mn} = 0 \quad (1)$$

where [k] is the stiffness matrix and [M] is the mass matrix while ψ_{mn} is the mode shape of structure and ω is the corresponding natural frequency of the plate in rad/sec.
The non-dimensional frequency parameter 'λ^2'is given by Eq.2:

$$\lambda^2 = \omega a^2 \sqrt{\frac{\rho\, h}{D}} \quad (2)$$

Where D, the flexure rigidity = $\frac{Eh^3}{12(1-\upsilon^2)}$, a = outer radius, E = Young's modulus of elasticity, υ = Poisson's ratio, h = thickness of the plate and ρ = density of plate.

In this paper, effect of non dimensional frequency parameter on flexural response due to out of plane modes for an uniform annular circular plate with different boundary conditions are investigated keeping the total mass of the plate constant . Four different boundary conditions (F-F, C-C, C-F and F-C) of plate are considered. The selections of different boundary conditions are such that in all four cases the total mass of the plate remains constant. The specification and the material properties of an annular circular plate are reported in Table 1. ANSYS has been used for numerical computation.

Table 1. The specification and the material properties of the annular circular plate

Dimension of the plate	Isotropic annular circular plate
Outer radius (a) m	0.1515
Inner radius (b) m	0.0825
Radii ratio, (b/a)	0.54
Thickness ratio, (h/a),	0.21
Density, ρ (kg/m^3)	7905.9
Young's modulus, E (GPa)	218
Poisson's ratio, υ	0.305

Table 2. Comparison and validation of frequency parameter λ^2 of uniform isotropic annular circular plate with F-F boundary conditions for taper ratio, $T_x = 0.00$ obtained in present work with that of Lee et al. [13].

Plate	Mode	Non dimensional frequency parameter, λ^2	
		H. Lee et al.[13]	Present work
Parabolic plate			
	1	3.82	3.83
b/a = 0.54	2	8.85	8.82
h/a = 0.21	3	10.59	10.02
	4	15.42	13.70

Table 3. Comparison and validation of frequency parameter λ^2 of uniform isotropic annular circular plate with C-F boundary conditions for taper ratio, $T_x = 0.00$ obtained in present work with that of Lee et al. [13].

Plate	Mode	Non dimensional frequency parameter, λ^2	
		H. Lee et al.[13]	Present work
Parabolic plate			
	1	13.61	13.49
b/a = 0.54	2	13.43	13.50
h/a = 0.21	3	15.28	14.12
	4	16.81	16.67

IV. RESULTS AND DISCUSSION

3.1. Validation of modal frequency

For validation of modal frequency of thick annular isotropic plate, the published result of Lee et al. [13] is taken as reported in Table 2 and 3. From Table 2 &Table 3 it is clear that the results obtained in this paper matches well with the published results [13].

3.2. Flexural response of plate with different boundary conditions

The effects on natural frequency parameter (λ2) on flexural response due to out of plane modes for an uniform annular circular plate with different boundary conditions are investigated by keeping the mass of the plate constant. Table 4 compares the first four natural frequency parameter λ^2 of uniform plate with different boundary conditions using FEM. It is investigated from the Table 4 that the natural frequency parameter increases as the number of modes increases for all cases of boundary condition.. This is because with increase in the number of modes the stiffness increases resulting in the increase of natural frequency parameter associated with theses modes. Further from the Table 4 it is investigate that the natural frequency parameter of C-C and C-F boundary condition does not increases much with the modes. This may be due to the higher stiffness associated with this two boundary condition. Table 5 compares the first four natural frequency parameter of uniform plate with different boundary condition for different radii ratio with different modes. From the Table 5 it is investigated that with increasing radii ratio with different modes the natural frequency parameter of plate increases for C-C, C-F and F-C boundary condition except for F-F boundary condition. This is because with this C-C, C-F and F-C boundary condition the stiffness increases resulting in higher frequency parameter. However with F-F boundary condition the stiffness of modes decreases with increasing radii ratio resulting in lower natural frequency parameter. Fig.2 shows the numerical comparison for the effect of all modes with different radii ratio (β) for uniform plate with free - free and clamped – clamped boundary condition. It is shown from Fig.2 that for F-F boundary condition the natural frequency parameter decreases for all modes except 3rd mode. This may be the less stiffness associated with 3rd mode. However for CC boundary condition due to the more stiffness the natural frequency parameter increases with all modes for increasing radii ratio. Fig.3 compares the numerical comparison for the effect of all modes with different radii ratio (β) for uniform plate with clamped - free and free – clamped boundary conditions. It is clear from Fig.3 that due to more stiffness, the natural frequency parameter of all the modes increases with increasing radii ratio associated with clamped - free boundary condition and free - clamped boundary condition. Table 6 compares the first four natural frequency parameter of uniform plate with different boundary condition for different aspect ratio with different modes.

Table 4. Comparison of first four frequency parameter λ^2 of uniform isotropic annular circular plate with different boundary condition for β (b/a) = 0.54, h/a = 0.21.

Plate	Mode	Non dimensional frequency parameter, λ^2			
		F-F	C-C	C-F	F-C
Uniform b/a = 0.54 h/a = 0.21	1	3.83	53.43	13.49	17.37
	2	8.82	53.78	13.50	19.40
	3	10.02	54.98	14.15	24.80
	4	13.70	57.40	16.67	32.51

Table 5. Comparison of first four frequency parameter λ^2 of uniform isotropic annular circular plate with different boundary condition for different radii ratio β (b/a) and for h/a = 0.21.

Plate	Mode	Non dimensional frequency parameter, λ^2			
		F-F	C-C	C-F	F-C
b/a = 0.3 h/a = 0.21	1	4.76	38.17	7.46	11.02
	2	11.79	39.09	12.57	17.83
	3	8.18	42.57	6.25	29.50
	4	16.75	49.76	6.48	43.13
b/a = 0.4 h/a = 0.21	1	4.41	44.05	9.56	12.68
	2	11.21	44.68	13.66	17.07
	3	8.27	46.95	8.42	26.46
	4	14.99	51.62	8.51	38.11
b/a = 0.5 h/a = 0.21	1	4.02	50.31	12.31	15.55
	2	10.43	50.72	15.31	18.17
	3	8.60	52.20	11.59	24.77
	4	13.97	55.16	11.61	33.76

b/a = 0.6 h/a = 0.21	1	3.60	58.13	16.49	20.27
	2	9.47	58.39	19.02	21.70
	3	9.14	59.32	16.48	25.75
	4	13.45	61.19	16.47	31.91
b/a = 0.7 h/a = 0.21	1	3.13	70.53	24.91	28.25
	2	8.39	70.70	26.15	28.94
	3	9.75	71.29	24.58	31.07
	4	12.92	72.43	24.56	34.64

Table 6. Comparison of first four frequency parameter λ^2 of uniform isotropic annular circular plate with different boundary condition for different aspect ratio (h/a) and for b/a = 0.54.

Plate	Mode	Non dimensional frequency parameter, λ^2			
		F-F	C-C	C-F	F-C
h/a = 0.1	1	4.02	82.79	16.42	19.65
	2	10.80	83.33	18.20	22.56
	3	9.56	85.11	15.34	30.16
	4	16.08	88.47	15.14	41.03
h/a = 0.15	1	3.94	66.95	15.38	18.67
	2	10.46	67.37	18.20	21.15
	3	9.24	68.79	14.54	27.66
	4	14.99	71.58	11.92	36.95
h/a = 0.2	1	3.85	55.01	14.29	17.55
	2	10.09	55.36	16.87	19.64
	3	8.89	56.59	13.64	25.18
	4	13.88	59.04	13.63	33.09
h/a = 0.25	1	3.75	46.22	13.24	16.39
	2	9.66	46.54	15.65	18.16
	3	8.48	47.64	12.72	22.89
	4	12.76	49.87	12.77	29.68
h/a = 0.3	1	3.63	39.66	12.29	15.26
	2	9.22	39.94	14.57	16.77
	3	8.06	40.96	11.83	20.87
	4	11.68	43.02	11.92	26.76

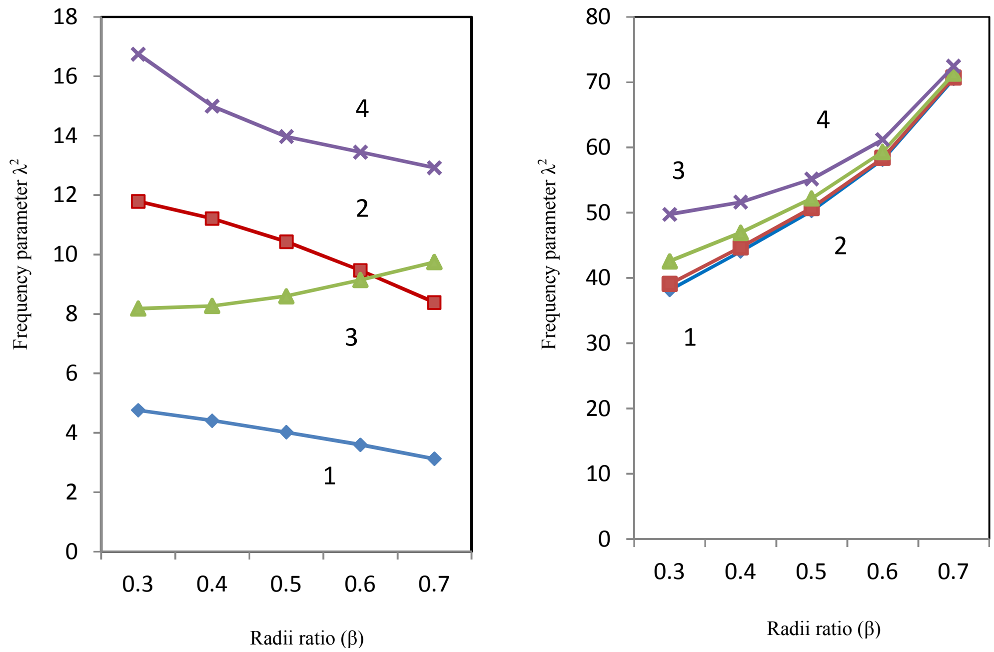

Fig. 2. Numerical comparison for the effect of all modes with different radii ratio (β) for uniform plate with (a) Free - free (b) Clamped – clamped boundary condition.

From the Table 6 it is investigated that with increasing Aspect ratio with different modes the natural frequency parameter of plate decreases for F-F, C-C, C-F and F-C boundary condition. This is because of the higher stiffness associated with all these boundary condition and as a consequence resulting in higher frequency parameter. Fig.4 shows the numerical comparison for the effect of all modes with different Aspect ratio (h/a) for uniform plate with free - free and clamped – clamped boundary condition. It is clear from fig.4 that with

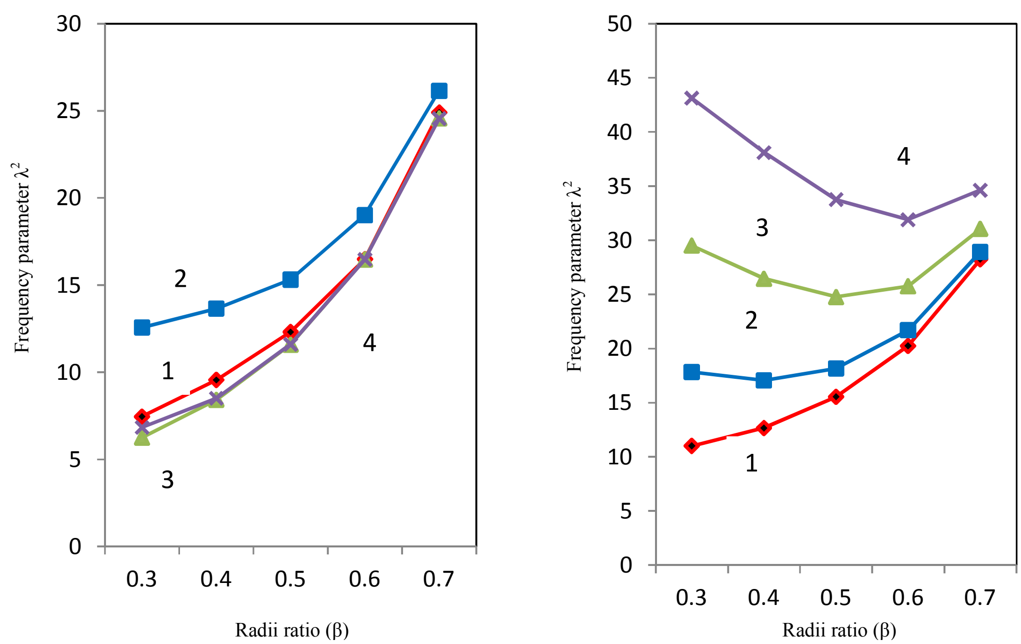

Fig. 3. Numerical comparison for the effect of all modes with different radii ratio (β) for uniform plate with (a) Clamped - free (b) Free – clamped boundary conditions

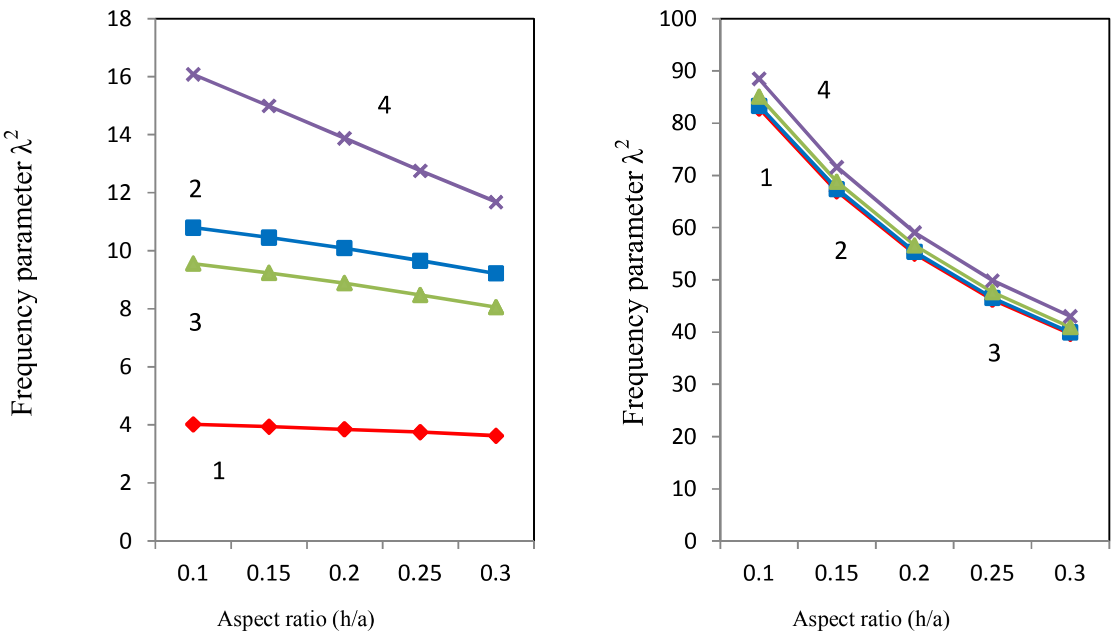

Fig. 4. Numerical comparison for the effect of all modes with different Aspect ratio (h/a) for uniform plate with (a) Free - free (b) Clamped – clamped boundary condition.

increasing Aspect ratio the natural frequency parameter decreases for all cases of boundary conditions. This is because with increase in the aspect ratio the plate becomes thick and hence the stiffness will be high and as a result the natural frequency parameter decreases with the higher modes. It is interesting to note that in comparison to F-F boundary condition, the natural frequency parameter of plate reduces more in C-C boundary condition as shown in Fig.4. Hence the curve is steeper in this case. Further from Fig.4 it is clear for F-F boundary condition that due to more stiffness the 4^{th} mode decreases more steeply than 1^{st} mode with increase Aspect ratio. Fig.5 compares the numerical comparison for the effect of all modes with different Aspect ratio (h/a) for uniform plate with clamped - free and free – clamped boundary condition. From Fig.5 it is clear that with increasing aspect ratio with modes the natural frequency of plate decreases for all cases of boundary condition. This may be the higher stiffness associated with all these modes. Hence it may be clear that mode variation has significant effect on flexural response of plate. Further this mode variation is highly depends on stiffness of the plate. As a consequence the flexural response depends on stiffness of the plate.

Hence it may be concluded that plate with F-F boundary condition will shows the highest resistance to flexural response than C-C, F-C and C-F boundary condition of plate due to mode variation. Further, modes with stiffness variation for plate with different cases of boundary condition provide a solution to flexural response. As for example, for flexural response actuation, plate with uniform thickness C-C, F-C and C-F boundary condition may be the option while for flexural response suppression, plate with F-F boundary condition may be the another alternative solution.

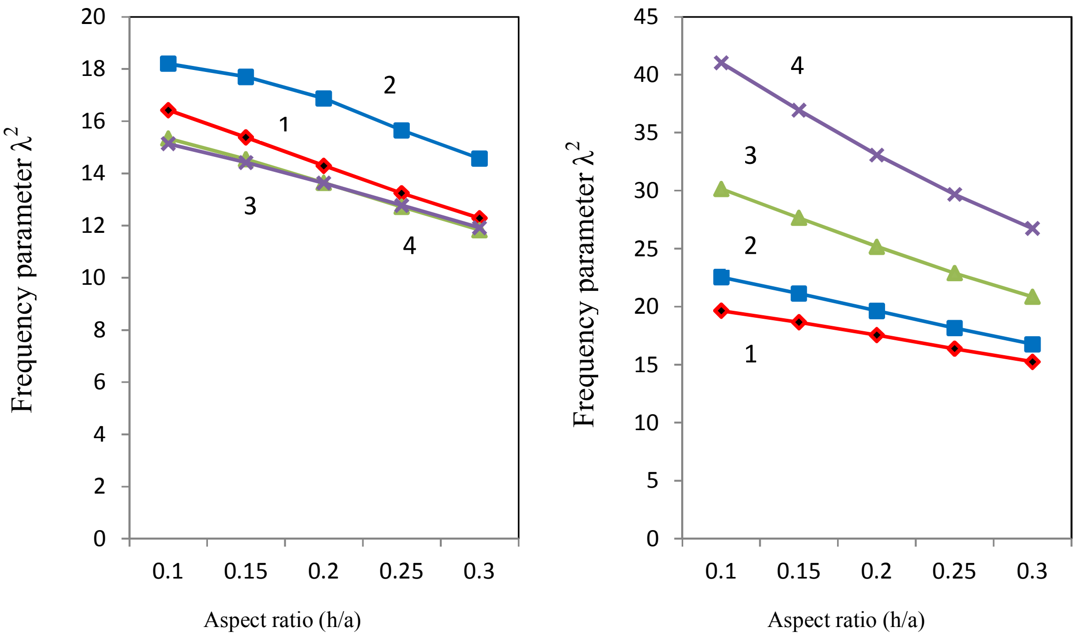

Fig. 5. Numerical comparison for the effect of all modes with different Aspect ratio (h/a) for uniform plate with (a) Clamped - free (b) Free – clamped boundary conditions.

IV. CONCLUSION

In this study, vibration analysis from out of plane modes of a uniform annular circular plate having different boundary conditions (F-F, C-C, C-F and F-C} are investigated by keeping the mass of the plate constant. It is investigated that the natural frequency parameter increases as the number of modes increases for all cases of boundary condition. Further It is investigated that due to the higher stiffness associated the natural frequency parameter of C-C and C-F boundary condition does not increases much with the modes. Further on increasing radii ratio with different modes the natural frequency parameter of plate increases for C-C, C-F and F-C boundary condition except for F-F boundary condition. This is because with this C-C, C-F and F-C boundary condition the stiffness increases resulting in higher frequency parameter. However with F-F boundary condition the stiffness of modes decreases with increasing radii ratio resulting in lower natural frequency parameter. Further it is investigated that with increasing Aspect ratio with different modes the natural frequency parameter of plate decreases for F-F, C-C, C-F and F-C boundary condition. Finally it may be concluded that plate with F-F boundary condition will shows the highest resistance to flexural response than C-C, F-C and C-F boundary condition of plate due to mode variation. Further, modes with stiffness variation for plate with different cases of boundary condition provide a solution to flexural response. As for example, for flexural response actuation, plate with uniform thickness C-C, F-C and C-F boundary condition may be the option while for flexural response suppression, plate with F-F boundary condition may be the another alternative solution.

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