ON (1,2)*-(sp)*-CLOSED SETS

[1]S. GANESAN , [2]R. SELVAVINAYAGAM and [2]M. MEENA

[1]*Assistant Professor, PG & Research Department of Mathematics, Raja Doraisingam Government Arts College, Sivagangai-630561, Tamil Nadu, India.*

[2]*Assistant Professor, PG & Research Department of Computer Science, Raja Doraisingam Government Arts College, Sivagangai-630561, Tamil Nadu, India.*

[3]*Assistant Professor, Department of Mathematics, Madurai Sivakasi Nadars Pioneer Meenakshi Women's College, Poovanthi-630611, Tamil Nadu, India.*

ABSTRACT

In this paper, we introduce a new class of sets called (1,2)-(sp)*-closed sets in bitopological spaces. We prove that this class lies between the class of $\tau_{1,2}$-closed sets and the class of (1,2)*-g-closed sets. Also we find some basic properties of (1,2)*-(sp)*-closed sets. Applying these sets, we introduce a new space called $T_{(1,2)^*(sp)^*}$-space.*

2010 Mathematics Subject Classification: *54C10, 54C08, 54C05*

Key words and Phrases: *(1,2)*-(sp)*-closed sets, (1,2)*-(sp)*-open sets, $T_{(1,2)*-(sp)*}$-space*

1. INTRODUCTION

Ravi and Lellis Thivagar [6] introduced the concepts of (1,2)*-semi-open sets, (1,2)*- α-open sets, (1,2)*-generalized closed sets and (1,2)*-α -generalized closed sets in bitopological spaces. Jafari etal [2] introduced the notion of (1,2)*-$\alpha\hat{g}$ -closed sets and investigated its fundamental properties. In this chapter we introduce a new class of sets called (1,2)*-(sp)*-closed sets in bitopological spaces and prove that this class lies between the class of $\tau_{1,2}$-closed sets and the class of (1,2)*-g-closed sets. Further we introduce a new space called $T_{(1,2)*-(sp)*}$-space.

2. PRELIMINARIES

Throughout this paper, X and Y denote bitopological spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2), respectively, on which no separation axioms are assumed.

Definition 2.1 [6]

Let S be a subset of X. Then S is said to be $\tau_{1,2}$-open if S = A $\cup$ B where A $\in$ τ_1 and B $\in$ τ_2.

We call $\tau_{1,2}$-closed set is the complement of $\tau_{1,2}$-open.

Example 2.2

Let X = {a, b, c}, τ_1 = {ϕ, {a, c}, X} and τ_2 = {ϕ, {b, c}, X}. Then the sets in {ϕ, {a, c}, {b. c}, X} are called $\tau_{1,2}$-open and the sets in {ϕ, {a}, {b}, X} are called $\tau_{1,2}$-closed.

Definition 2.3 [6]

Let S be a subset of X. Then

(i) The $\tau_1\tau_2$-interior of S, denoted by $\tau_1\tau_2$-int(S) or $\tau_{1,2}$-int(S), is defined by $\cup$ {F : F $\subset$ S and F is $\tau_{1,2}$-open}.

(ii) The $\tau_1\tau_2$-closure of S, denoted by $\tau_1\tau_2$-cl(S) or $\tau_{1,2}$-cl(S), is defined by $\cap$ {F : S $\subset$ F and F is $\tau_{1,2}$-closed}.

Remark 2.4 [6]

Notice that $\tau_{1,2}$-open sets need not necessarily form a topology.

Remark 2.5 [3, 6]

Without proving we list the following properties for the bitopological space (X, τ_1, τ_2), where $\tau_{1,2}$-open subsets are defined as above.

(P0) If $S_1 \subset S_2 \subset X$, then $\tau_1\tau_2$-int(S_1) $\subset$ $\tau_1\tau_2$-int(S_2) and $\tau_1\tau_2$-cl(S_1) $\subset$ $\tau_1\tau_2$-cl(S_2).

(P1) (a) $\tau_1\tau_2$-int(S) is $\tau_{1,2}$-open for each S $\subset$ X;

(b) $\tau_1\tau_2$-cl(S) is $\tau_{1,2}$-closed for each S $\subset$ X.

(P2) (a) A set S $\subset$ X is $\tau_{1,2}$-open if and only if S = $\tau_1\tau_2$-int(S);

(b) A set S $\subset$ X is $\tau_{1,2}$-closed if and only if S = $\tau_1\tau_2$-cl(S).

(P3) (a) For any S $\subset$ X we have $\tau_1\tau_2$-int($\tau_1\tau_2$-int(S)) = $\tau_1\tau_2$-int (S);

(b) For any S $\subset$ X we have $\tau_1\tau_2$-cl($\tau_1\tau_2$-cl(S)) = $\tau_1\tau_2$-cl(S).

(P4) (a) $\tau_1\tau_2$-int (X – S) = X – $\tau_1\tau_2$-cl (S) for any S $\subset$ X;

(b) $\tau_1\tau_2$-cl (X – S) = X – $\tau_1\tau_2$-int (S) for any S $\subset$ X.

(P5) (a) $\tau_1\tau_2$-int (S) = int τ_1 (S) $\cup$ int τ_2 (S) for any S $\subset$ X;

(b) $\tau_1\tau_2$-cl (S) = cl τ_1 (S) $\cap$ cl τ_2 (S) for any S $\subset$ X.

(P6) For any family $\{S_i / i \in I\}$ of subsets of X we have :

(a_1) $\bigcup_i \tau_1\tau_2\text{-int}(S_i) \subset \tau_1\tau_2\text{-int}(\bigcup_i S_i)$;

(b_1) $\bigcup_i \tau_1\tau_2\text{-cl}(S_i) \subset \tau_1\tau_2\text{-cl}(\bigcup_i S_i)$;

(a_2) $\tau_1\tau_2\text{-int}(\bigcap_i S_i) \subset \bigcap_i \tau_1\tau_2\text{-int}(S_i)$;

(b_2) $\tau_1\tau_2\text{-cl}(\bigcap_i S_i) \subset \bigcap_i \tau_1\tau_2\text{-cl}(S_i)$.

We recall the following definitions which are useful in the sequel.

Definition 2.6

A subset A of a bitopological space (X, τ_1, τ_2) is called

(1) (1,2)*-semi-open [10] if A $\subseteq$ $\tau_{1,2}$-cl($\tau_{1,2}$-int(A)).

(2) (1,2)*-α-open [4, 10] if A $\subseteq$ $\tau_{1,2}$-int($\tau_{1,2}$-cl($\tau_{1,2}$-int(A))).

(3) (1,2)*-β-open [11] if A $\subseteq$ $\tau_{1,2}$-cl($\tau_{1,2}$-int($\tau_{1,2}$-cl(A))).

The complement of a (1, 2)*-semi-open (resp. (1,2)*-α-open, (1,2)*-β-open) set is called (1,2)*-semi-closed (resp. (1,2)*-α-closed, (1,2)*-β-closed).

The (1,2)*-α-closure [2, 7] (resp. (1,2)*-semi-closure [2, 7], (1,2)*-β-closure [16, 17]) of a subset A of X, denoted by (1,2)*-α cl(A) (resp. (1,2)*-scl(A), (1,2)*-β cl(A)) is defined to be the intersection of all (1,2)*-α-closed (resp. (1,2)*-semi-closed, (1,2)*-β-closed) sets of X containing A. It is known that (1,2)*-α cl(A) (resp. (1,2)*-scl(A), (1,2)*-β cl(A)) is a (1,2)*-α-closed (resp. (1,2)*-semi-closed, (1,2)*-β-closed) sets. For any subset A of an arbitrarily chosen bitopological space, the (1,2)*-α-interior [2, 7] (resp. (1,2)*-semi-interior [2, 7], (1,2)*-β-interior [16, 17]) of A, denoted by (1,2)*-α int(A) (resp. (1,2)*-sint(A), (1,2)*-β int(A)) is defined to be the union of all (1,2)*-α-open (resp. (1,2)*-semi-open, (1,2)*-β-open) sets of X contained in A.

Definition 2.7

A subset A of a bitopological space (X, τ_1, τ_2) is called

(1) (1,2)*-g-closed [12, 14] if $\tau_{1,2}$-cl(A) ⊆ U whenever A ⊆ U and U is $\tau_{1,2}$-open in X. Then complement of (1,2)*-g-closed set is called (1,2)*-g-open set.

(2) (1,2)*-gsp.closed [15, 17] if $\tau_{1,2}$-β cl(A) ⊆ U whenever A ⊆ U and U is $\tau_{1,2}$-open. Then complement of (1,2)*-gsp-closed set is called (1,2)*-gsp-open set.

(3) (1,2)*-gs-closed [10] if $\tau_{1,2}$-scl(A) ⊆ U whenever A ⊆ U and U is $\tau_{1,2}$-open in X. Then complement of (1,2)*-gs-closed set is called (1,2)*-gs-open set.

(4) (1,2)*-$\hat{g}$-closed [1, 17] if $\tau_{1,2}$-cl(A) ⊆ U whenever A ⊆ U and U is (1,2)*-semi-open in X. Then complement of (1,2)*-$\hat{g}$-closed set is called (1,2)*-$\hat{g}$-open set.

(5) (1,2)*-αg-closed [2, 6] if $\tau_{1,2}$-α cl(A) ⊆ U whenever A ⊆ U and U is $\tau_{1,2}$-open in X. Then complement of (1,2)*- αg-closed set is called (1,2)*-αg-open set.

(6) (1,2)*-$\alpha\hat{g}$-closed [2] if $\tau_{1,2}$-α cl(A) ⊆ U whenever A ⊆ U and U is (1,2)*-$\hat{g}$-open in X. Then complement of (1,2)*- $\alpha\hat{g}$-closed set is called (1,2)*- $\alpha\hat{g}$-open set.

Remark 2.8

The collection of all (1,2)*-g-closed (resp. (1,2)*-gsp.closed, (1,2)*-gs-closed, (1,2)*-$\hat{g}$-closed, (1,2)*-αg-closed, (1,2)*-$\alpha\hat{g}$-closed) sets is denoted by (1,2)*-gc(X) (resp. (1,2)*-gspc(X), (1,2)*-gsc(X), (1,2)*-$\hat{g}$ c(X), (1,2)*-αg c(X), (1,2)*-$\alpha\hat{g}$ c(X)).

Definition 2.9

A bitopological space (X, τ_1, τ_2) is called

(1) $(1,2)*\text{-}T_{1/2}$-space [5, 14] if every (1,2)*-g-closed set in it is $\tau_{1,2}$-closed.

(2) $T_{(1,2)^{*}b}$-space [2] if every (1,2)*-gs-closed set in it is $\tau_{1,2}$-closed.

(3) $\alpha T_{(1,2)^{*}b}$-space [2] if every (1,2)*-αg-closed set in it is $\tau_{1,2}$-closed.

(4) $T_{(1,2)^{*}-\alpha\hat{g}}$-space [2] if every (1,2)*-$\alpha\hat{g}$-closed set in it is (1,2)*-α -closed.

Proposition 2.10 [2]

(1)Every $\tau_{1,2}$ -closed set is (1,2)*- α -closed but not conversely.

(2)Every (1,2)*- α -closed set is (1,2)*- $\alpha\hat{g}$ -closed but not conversely.

(3)Every (1,2)*- $\alpha\hat{g}$ -closed set is (1,2)*- αg -closed but not conversely.

(4)Every (1,2)*- $\alpha\hat{g}$ -closed set is (1,2)*- gs-closed but not conversely.

Remark 2.11 [11]

We have the following implication for properties of subsets

(1,2)*- α -closed → (1,2)*-semi-closed → (1,2)*- β -closed

3.BASIC PROPERTIES OF (1,2)*-(sp)*-CLOSED SETS

Definition 3.1

A subset A of a bitopological space (X, τ_1, τ_2) is said to be (1,2)*-(sp)*-closed. If $\tau_{1,2}$ -cl(A) ⊆ U whenever A ⊆ U and U is (1,2)*- β -open in X.

The class of all (1,2)*-(sp)*-closed subset of X is denoted by (1,2)*-(sp)*c(X).

Proposition 3.2

Every $\tau_{1,2}$ -closed set is (1,2)*-(sp)*-closed.

Proof follows from the definition.

Proposition 3.3

Every (1,2)*-(sp)*-closed set is (1,2)*-gsp-closed.

Proof

Let A be a (1,2)*-(sp)*-closed. Let A ⊆ U and U be $\tau_{1,2}$ -open. Then A ⊆ U and U is (1,2)*- β -open and $\tau_{1,2}$ -cl(A) ⊆ U, since A is (1,2)*-(sp)*-closed. Then $\tau_{1,2}$ - β cl(A) ⊆ $\tau_{1,2}$ -cl(A) ⊆ U. Therefore A is (1,2)*-gsp-closed.

The converse of the above proposition is not true as seen in the following example.

Example 3.4

Let X = {a, b, c}, τ_1 = {ϕ, {a, b}, X} and τ_2 = {ϕ, {b, c}, X}. Then the sets in {ϕ, {a, b}, {b. c}, X} are called $\tau_{1,2}$-open and the sets in {ϕ, {a}, {c}, X} are called $\tau_{1,2}$-closed. Then (1,2)*-(sp)*c(X) = {ϕ, {a}, {c}, X} and (1,2)*-gspc(X) = {ϕ, {a}, {b}, {c}, {a, c}, X}. Clearly {a, c} is an (1,2)*-gsp-closed but not (1,2)*-(sp)*-closed in X .

Proposition 3.5

Every (1,2)*-(sp)*-closed set is (1,2)*-g-closed.

Let A be a (1,2)*-(sp)*-closed set and U be any $\tau_{1,2}$-open set containing A. Since A is (1,2)*-(sp)*-closed and every $\tau_{1,2}$-open set is (1,2)*-β-open, $\tau_{1,2}$-cl(A) ⊆ U. Hence A is (1,2)*-g-closed.

The converse of the above proposition is not true as seen in the following example.

Example 3.6

Let X = {a, b, c}, τ_1 = {ϕ, {a}, {a, c}, X} and τ_2 = {ϕ, {b, c}, X}. Then the sets in {ϕ, {a}, {a, c}, {b. c}, X} are called $\tau_{1,2}$-open and the sets in {ϕ, {a}, {b}, {b, c}, X} are called $\tau_{1,2}$-closed. Then (1,2)*-(sp)*c(X) = {ϕ, {a}, {b}, {b, c}, X} and (1,2)*-gc(X) = {ϕ, {a}, {b}, {a, b}, {b, c}, X}. Clearly {a, b} is an (1,2)*-g-closed but not (1,2)*-(sp)*-closed in X .

Proposition 3.7

Every (1,2)*-(sp)*-closed set is (1,2)*-gs-closed.

Proof

Let A be a (1,2)*-(sp)*-closed set and U be any $\tau_{1,2}$-open set containing A. Since A is (1,2)*-(sp)*-closed and every $\tau_{1,2}$-open set is (1,2)*-β-open, $\tau_{1,2}$-scl(A) ⊆ $\tau_{1,2}$-cl(A) ⊆ U. Hence A is (1,2)*-gs-closed.

The converse of the above proposition is not true in general as it can be seen from the following example.

Example 3.8

Let X = {a, b, c}, τ_1 = {ϕ, {a, b}, X} and τ_2 = {ϕ, {a, c}, X}. Then the sets in {ϕ, {a, b}, {a. c}, X} are called $\tau_{1,2}$-open and the sets in {ϕ, {b}, {c}, X} are called $\tau_{1,2}$-closed. Then (1,2)*-(sp)*c(X) = {ϕ, {b}, {c}, X} and (1,2)*-gsc(X) = {ϕ, {b}, {c}, {b, c}, X}. Clearly {b, c} is an (1,2)*-gs-closed but not (1,2)*-(sp)*-closed in X .

Proposition 3.9

Every (1,2)*-(sp)*-closed set is (1,2)*- $\hat{g}$ -closed.

Proof

Let A be a (1,2)*-(sp)*-closed set and U be any (1,2)*-semi-open set containing A. Since A is (1,2)*-(sp)*-closed and every (1,2)*-semi-open set is (1,2)*- β -open, $\tau_{1,2}$ -cl(A) ⊆ U. Hence A is (1,2)*- $\hat{g}$ -closed.

The converse of the above proposition is not true in general as it can be seen from the following example.

Example 3.10

Let X and τ_1, τ_2 be a defined as in example 3.6. Then (1,2)*- $\hat{g}$ c(X) = {ϕ, {a}, {b}, {a, b}, {b, c}, X}. Clearly {a, b} is (1,2)*- $\hat{g}$ -closed but not (1,2)*-(sp)*-closed in X.

Proposition 3.11

Every (1,2)*-(sp)*-closed set is (1,2)*- αg -closed.

Proof

Let A be a (1,2)*-(sp)*-closed set and U be any $\tau_{1,2}$ -open set containing A. Since A is (1,2)*-(sp)*-closed and every $\tau_{1,2}$ -open set is (1,2)*- β -open, $\tau_{1,2}$ - α cl(A) ⊆ $\tau_{1,2}$ -cl(A) ⊆ U. Hence A is a (1,2)*- αg -closed.

The following example supports that the converse of the above proposition is not true.

Example 3.12

Let X and τ_1, τ_2 be a defined as in example 3.8. Then (1,2)*- αg c(X) = {ϕ, {b}, {c}, {b, c}, X}. Clearly {b, c} is (1,2)*- αg -closed but not (1,2)*-(sp)*-closed in X.

Proposition 3.13

Every (1,2)*-(sp)*-closed set is (1,2)*- $\alpha\hat{g}$ -closed.

Proof

Let A be a (1,2)*-(sp)*-closed set and U be any (1,2)*- $\hat{g}$ -open containing A. Since A is (1,2)*-(sp)*-closed and every (1,2)*- $\hat{g}$ -open set is (1,2)*- β -open, $\tau_{1,2}$ - α cl(A) ⊆ $\tau_{1,2}$ -cl(A) ⊆ U. Hence A is a (1,2)*- $\alpha\hat{g}$ -closed.

The following example supports that the converse of the above proposition is not true.

Example 3.14

Let X and τ_1, τ_2 be a defined as in example 3.6. Then (1,2)*-$\alpha\hat{g}$ c(X) = {ϕ, {a}, {b}, {a, b}, {b, c}, X}. Clearly {a, b} is (1,2)*- $\alpha\hat{g}$ -closed but not (1,2)*-(sp)*-closed in X.

Proposition 3.15

Let (X, τ_1, τ_2) be a bitopological space and A ⊂ X. Then the following are true.

(1) If A is (1,2)*-g-closed, then A is (1,2)*-gsp-closed.
(2) If A is (1,2)*-αg -closed, then A is (1,2)*-gsp-closed.
(3) If A is (1,2)*-gs-closed, then A is (1,2)*-gsp-closed.

Proof

(1), (2), (3): Since (1,2)*-β cl(A) ⊂ (1,2)*scl(A) ⊂ (1,2)*-α cl(A) ⊂ $\tau_{1,2}$ -cl(A), the proof is clear.

Remark 3.16

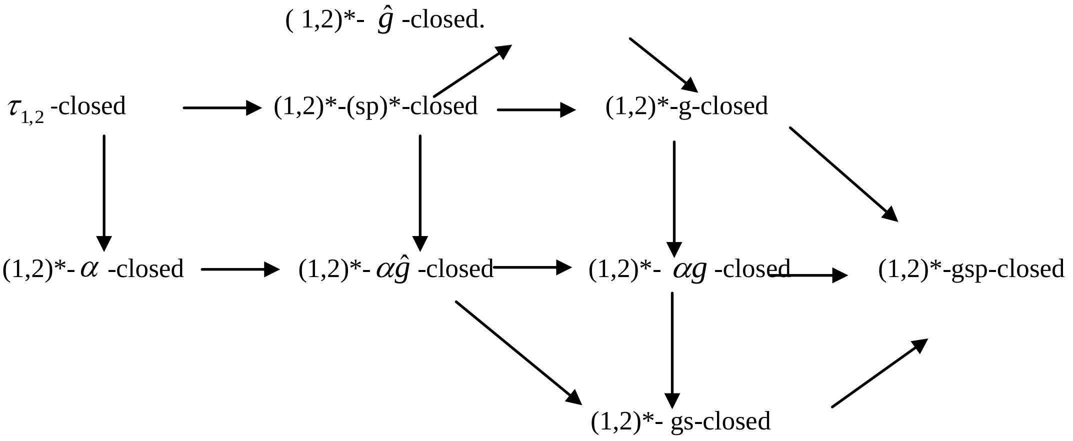
The converse of proposition 3.15 is not true. For,

Example 3.17

Let X and τ_1, τ_2 be a defined as in example 3.4. Then (1,2)*-gc(X) = (1,2)*-αg c (X) = (1,2)*-gsc(X) = {ϕ, {a}, {c}, {a, c}, X}. Clearly the set {b} is (1,2)*-gsp-closed but it is not (1,2)*-g-closed (resp. (1,2)*-αg -closed, (1,2)*-gs-closed).

Remark 3.18

From the above discussions and known results in [2] we obtain the following diagram where A ⟶ B represents A implies B, but not conversely.



None of the above implications is reversible as shown in the remaining examples and in the related paper [2].

Remark 3.19

The union of two (1,2)*-(sp)*-closed sets need not be (1,2)*-(sp)*-closed as shown in the following example.

Example 3.20

Let X = {a, b, c}, τ_1 = {ϕ, {c}, {a, c}, {b, c}, X} and τ_2 = {ϕ, {a}, {a, b}, X}. Then the sets in {ϕ, {a}, {c}, {a, b}, {a, c}, {b. c}, X} are called $\tau_{1,2}$-open and the sets in {ϕ, {a}, {b}, {c}, {a, b}, {b, c}, X} are called $\tau_{1,2}$-closed. Then (1,2)*-(sp)*c(X) = {ϕ, {a}, {b}, {c}, {a, b}, {b, c}, X}. Clearly {a} and {c} are (1,2)*-(sp)*-closed but {a, c} is not (1,2)*-(sp)*-closed.

Proposition 3.21

If a set A is (1,2)*-(sp)*-closed, then $\tau_{1,2}$-cl(A) – A contains no nonempty $\tau_{1,2}$-closed.

Proof

Let A be is (1,2)*-(sp)*-closed and F a $\tau_{1,2}$-closed subset of $\tau_{1,2}$-cl(A) –A. Then $A \subset F^c$, F^c is $\tau_{1,2}$-open and hence (1,2)*-β-open. Since (1,2)*-(sp)*-closed , $\tau_{1,2}$-cl(A) $\subset F^c$. Consequently $F \subset \tau_{1,2}$-cl(A) $\cap$ (($\tau_{1,2}$-cl(A))^c = ϕ.

The converse of proposition 3.21 need not be true.

Example 3.22

Let X and τ_1, τ_2 be a defined as in example 3.4. Let A = {a, c}, $\tau_{1,2}$-cl(A) –A contains no nonempty $\tau_{1,2}$-closed set. However A is not (1,2)*-(sp)*-closed.

Proposition 3.23

If a set A is (1,2)*-(sp)*-closed, then $\tau_{1,2}$-cl(A) - A contain no non empty (1,2)*-β-closed set.

Proof

Let A be is (1,2)*-(sp)*-closed and S be a (1,2)*-β-closed subset of $\tau_{1,2}$-cl(A) - A. Then $A \subseteq S^c$ and S^c is (1,2)*-β-open. So $\tau_{1,2}$-cl(A) $\subseteq S^c$. Hence $S \subseteq$ ($\tau_{1,2}$-cl(A))^c. Thus, $S \subseteq \tau_{1,2}$-cl(A) $\cap$ (($\tau_{1,2}$-cl(A))^c = ϕ.

Proposition 3.24

Let A be a (1,2)*-(sp)*-closed subset of (X, τ_1, τ_2). If A ⊆ B ⊆ $\tau_{1,2}$-cl(A) then, B is also a (1,2)*-(sp)*-closed subset of (X, τ_1, τ_2).

Proof

Let U be a (1,2)*-β-open set of (X, τ_1, τ_2) such that B ⊆ U. Since A ⊆ B, we have A ⊆ U, since A is (1,2)*-(sp)*-closed set, $\tau_{1,2}$-cl(A) ⊆ U. Also since B ⊆ $\tau_{1,2}$-cl(A), $\tau_{1,2}$-cl(B) ⊆ $\tau_{1,2}$-cl($\tau_{1,2}$-cl(A)) = $\tau_{1,2}$-cl(A) ⊆ U. Thus $\tau_{1,2}$-cl(B) ⊆ U. Hence B is also (1,2)*-(sp)*-closed subset of (X, τ_1, τ_2).

Proposition 3.25

If A is a (1,2)*-β-open and (1,2)*-(sp)*-closed subset of (X, τ_1, τ_2) then , A is a $\tau_{1,2}$-closed subset of (X, τ_1, τ_2).

Proof

Since A is (1,2)*-β-open and (1,2)*-(sp)*-closed, $\tau_{1,2}$-cl(A) ⊆ A. Hence A is $\tau_{1,2}$-closed.

4. APPLICATIONS

Definition 4.1

A subset A of (X, τ_1, τ_2) is called (1,2)*-(sp)*-open if and only if A^c is (1,2)*-(sp)*-closed in (X, τ_1, τ_2).

Remark 4.2

For a subset A of (X, τ_1, τ_2), $\tau_{1,2}$-cl(A^c) = [$\tau_{1,2}$-int(A)]^c

Theorem 4.3

A subset A of (X, τ_1, τ_2) is (1,2)*-(sp)*-open if and only if F ⊆ $\tau_{1,2}$-int(A) whenever F is (1,2)*-β-closed and F ⊂ A.

Proof

Necessity: Let A be (1,2)*-(sp)*-open set in (X, τ_1, τ_2). Let F be (1,2)*- β -closed and F $\subset$ A. Then $F^c \supseteq A^c$ and F^c is (1,2)*- β -open. Since A^c is (1,2)*-(sp)*-closed, $\tau_{1,2}$ -cl(A^c) $\subseteq F^c$. By remark 4.2 $\tau_{1,2}$ -int(A)$]^c \subseteq F^c$. That is F $\subset \tau_{1,2}$ -int(A).

Sufficiency: Let $A^c \subseteq$ U where U is (1,2)*- β -open. Then $U^c \subset$ A where U^c is (1,2)*- β -closed. By the hypothesis $U^c \subseteq \tau_{1,2}$ -int(A). That is [$\tau_{1,2}$ -int(A)$]^c \subseteq$ U. By remark 4.2, $\tau_{1,2}$ -cl(A^c) $\subseteq$ U. This implies A^c is (1,2)*-(sp)*-closed. Hence A is (1,2)*-(sp)*-open.

Proposition 4.4

If $\tau_{1,2}$ -int(A) $\subseteq$ B $\subseteq$ A and A is (1,2)*-(sp)*-open then B is (1,2)*-(sp)*-open.

Proof

$\tau_{1,2}$ -int(A) $\subseteq$ B $\subseteq$ A implies $A^c \subseteq B^c \subseteq$ [$\tau_{1,2}$ -int(A)$]^c$. By remark 4.2, $A^c \subseteq B^c \subseteq$ [$\tau_{1,2}$ -cl(A^c)]. Also A^c is (1,2)*-(sp)*-closed. By proposition 3.22, B^c is (1,2)*-(sp)*-closed. Hence B is (1,2)*-(sp)*-open.

As an application of (1,2)*-(sp)*-closed sets we introduce the following definition.

Definition 4.5

A space (X, τ_1, τ_2) is called a $T_{(1,2)*-(sp)*}$ -space if every (1,2)*-(sp)*-closed set in it is $\tau_{1,2}$ -closed.

Example 4.6

Let X and τ_1, τ_2 be a defined as in example 3.6. Thus (X, τ_1, τ_2) is a $T_{(1,2)*-(sp)*}$ -space

Proposition 4.7

Every $(1,2)*$ - $T_{1/2}$ -space is $T_{(1,2)*-(sp)*}$ -space but not conversely.

Proof

Follow from proposition 3.5

The converse of proposition 4.7 need not be true as seen from the following example.

Example 4.8

Let X and τ_1, τ_2 be a defined as in example 3.8, (1,2)*-gc(X) = {ϕ, {b}, {c}, {b, c}, X}. Thus (X, τ_1, τ_2) is a $T_{(1,2)*-(sp)*}$-space but not a $(1,2)*$-$T_{1/2}$-space.

Proposition 4.9

Every $T_{(1,2)^* b}$-space is $T_{(1,2)*-(sp)*}$-space but not conversely.

Proof

Follow from proposition 3.7

The converse of proposition 4.9 need not be true as seen from the following example.

Example 4.10

Let X and τ_1, τ_2 be a defined as in example 3.6, (1,2)*-gsc(X) = {ϕ, {a}, {b}, {a, b}, {b, c}, X}. Thus (X, τ_1, τ_2) is a $T_{(1,2)*-(sp)*}$-space but not a $T_{(1,2)^* b}$.space.

Proposition 4.11

Every $\alpha T_{(1,2)^* b}$-space is $T_{(1,2)*-(sp)*}$-space but not conversely.

Proof

Follow from proposition 3.11

The converse of Proposition 4.11 need not be true as seen from the following example.

Example 4.12

Let X and τ_1, τ_2 be a defined as in example 3.4, (1,2)*-αg c(X) = {ϕ, {a}, {c}, {a, c}, X}. Thus (X, τ_1, τ_2) is a $T_{(1,2)*-(sp)*}$-space but not a $\alpha T_{(1,2)^* b}$.-space.

Proposition 4.13

Every $T_{(1,2)^*-\alpha\hat{g}}$-space is $T_{(1,2)*-(sp)*}$-space but not conversely.

Proof

Let A be a (1,2)*-(sp)*-closed. Then A is (1,2)*-$\alpha\hat{g}$-closed. Since (X, τ_1, τ_2) is $T_{(1,2)^*-\alpha\hat{g}}$-space, A is (1,2)*-$\alpha$-closed. It is true that every (1,2)*-α-closed is (1,2)*-β-closed. Therefore X is $T_{(1,2)*-(sp)*}$-space.

The converse of Proposition 4.13 need not be true as seen from the following example.

Example 4.14

Let X and τ_1, τ_2 be a defined as in example 3.14. Thus (X, τ_1, τ_2) is a $T_{(1,2)*-(sp)*}$-space but not a $T_{(1,2)^*-\alpha\hat{g}}$-space.

Theorem 4.15

For a space (X, τ_1, τ_2) the following conditions are equivalent:

(1) (X, τ_1, τ_2) is a $T_{(1,2)*-(sp)*}$-space.

(2) Every singleton subset of (X, τ_1, τ_2) is either (1,2)*-β-closed or $\tau_{1,2}$-open.

Proof

(1) $\rightarrow$ (2). Let x $\in$ X. Suppose {x} is not a (1,2)*-β-closed set of (X, τ_1, τ_2). Then X – {x} is not a (1,2)*-β-open set. So X is the only (1,2)*-β-open set containing X – {x}. So X – {x} is a (1,2)*-(sp)*-closed set of (X, τ_1, τ_2). Since (X, τ_1, τ_2) is a $T_{(1,2)*-(sp)*}$-space , X – {x} is a $\tau_{1,2}$-closed set of (X, τ_1, τ_2) or equivalently {x} is a $\tau_{1,2}$-open set of (X, τ_1, τ_2).

(2) $\rightarrow$ (1). Let A be a (1,2)*-(sp)*-closed subset of (X, τ_1, τ_2). Trivially A $\subset$ $\tau_{1,2}$-cl(A). Let x $\in$ $\tau_{1,2}$-cl(A) By (2) {x} is either (1,2)*-β-closed or $\tau_{1,2}$-open.

Case (a) Suppose that {x} is (1,2)*-(sp)*-closed. If x $\notin$ A, then $\tau_{1,2}$-cl(A) –A contains a nonempty (1,2)*-β-closed set {x}. By proposition 3.23 we arrive at a contradiction. Thus x $\in$ A.

Case (b) Suppose that {x} is $\tau_{1,2}$-open. Since x $\in$ $\tau_{1,2}$-cl(A), {x} $\cap$ A $\neq \phi$. This implies that x $\in$ A. Thus in any case, x $\in$A. So $\tau_{1,2}$-cl(A) $\subset$ A. Therefore $\tau_{1,2}$-cl(A) = A or equivalently A is a $\tau_{1,2}$-closed. Hence (X, τ_1, τ_2) is a $T_{(1,2)*-(sp)*}$-space.

REFERENCES

[1] Z. Duszynski, M. Jeyaraman, M. Sajan Joseph, M. Lellis Thivagar, and O. Ravi, A new generalization of closed sets in bitopology, *South Asian Journal of Mathematics, 4(5), (2014), 215-224.*

[2] S. Jafari, M. Lellis Thivagar, and Nirmala Mariappan, On (1,2)*- $\alpha\hat{g}$ -closed sets, *J. Adv. Math. Studies, 2(2) (2009), 25-34.*

[3] K. Kayathri, O. Ravi, M. Lellis Thivagar, and M. Joseph Israel, Decompositions of (1,2)*- rg-continuous maps in bitopological spaces, *Antarctica J. Math., 6(1) (2009), 13-23.*

[4] M. Lellis Thivagar, O. Ravi, and M.E. Abd El-Monsef, Remarks on bitopological (1,2)*-quotient mappings, *J. Egypt Math. Soc, 16(1) (2008), 17-25.*

[5] M. Lellis Thivagar, M. Joseph Israel, and O. Ravi, On preserving (1,2)*-g-closed set in bitopology, *International Journal of Mathematics Archive, 2(3), (2011), 382-384.*

[6] O. Ravi, and M. Lellis Thivagar, On stronger forms of (1,2)*- quotient mappings in bitopological spaces, *Internat. J. Math. Game Theory and Algebra, 14(6) (2004), 481- 492.*

[7] O. Ravi, M. Lellis Thivagar, and E. Hatir, Decomposition of (1,2)*-continuity and (1,2)*- α -continuity, Miskolc Mathematical Notes, 10(2) (2009), 163-171.

[8] O. Ravi, E. Ekici, and M. Lellis Thivagar, On (1,2)*-sets and decompositions of bitopological (1,2)*-continuous mappings, *Kochi J. Math., 3 (2008), 181-189.*

[9] O. Ravi, M. Lellis Thivagar, and E. Ekici, Decomposition of (1,2)*-continuity and complete (1,2)* -continuity in bitopological spaces, *Analele niversitatii Din Oradea. Fasc. Matematica Tom XV (2008), 29-37.*

[10] O. Ravi, and M. Lellis Thivagar, A bitopological (1,2)*-semi-generalized continuous maps, *Bull. Malays. Math. Sci. Soc, (2), 29(1) (2006), 79-88.*

[11] O. Ravi, and M. Lellis Thivagar, Remarks on λ-irresolute functions via (1,2)*- sets, *Advances in App. Math. Analysis, 5(1) (2010), 1-15.*

[12] O. Ravi, K. Kayathri, M. Lellis Thivagar, and M. Joseph Israel, Mildly (1,2)*- normal spaces and some bitopological functions, *Mathematica Bohemica, 135(1) (2010), 1-13.*

[13] O. Ravi, S. Pious Missier, and T. Salai Parkunan, On bitopological (1,2)*-generalized homeomorphisms, *Int J. Contemp. Math. Sciences, 5(11) (2010), 543-557.*

[14] O. Ravi, M. Lellis Thivagar, and Jinjinli, Remarks on extensions of (1,2)*-g-closed maps, *Archimedes J. Math, 1(2) (2011), 177-187.*

[15] O. Ravi, M. Kamaraj, A.Pandi, and K. Kumaresan, (1,2)*- $\ddot{g}$ -closed and (1,2)*- $\ddot{g}$ -open maps in bitopological spaces, *International Journal of Mathematical Archive., 3(2), (2012), 586-594.*

[16] O. Ravi, M. Lellis Thivagar, and A. Nagarajan, (1,2)*- αg -closed sets and (1,2)*- $g\alpha$ -closed sets (submitted).

[17] O. Ravi, K..M Dharmalingam, and A. Thamilisai, $\tilde{g}(1,2)*$ -closed sets in bitopological spaces, *International Journal of Mathematics and Its Applications., 3(3A), (2015), 53-61.*