Application of Bianchi type-I model in string theory

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Abstract: The present day observations indicate that the universe at large scale is homogeneous and isotropic and the accelerating phase of universe. The late time accelerated expansion of the universe has attracted much attention in the recent years. The present study deals with a spatially homogeneous and anisotropic Bianchi type - I cosmological models representing massive strings. The Einstein's field equations have been solved by applying a variation law for generalized Hubble's parameter in Bianchi type - I space-time. The energy-momentum tensor, as formulated by Letelier (1983), has been used to construct massive string cosmological models for which we assume the expansion scalar in the models is proportional to one of the components of shear tensor. We have analysed a comparative study of accelerating and decelerating in the presence of string scenario. The strings eventually disappear from the universe for sufficiently large times, which is in agreement with current astronomical observations. The study reveals that massive strings dominate in the decelerating universe whereas strings dominate in the accelerating universe.

Keywords: String Cosmology- Bianchi type -I Universe - Accelerating Universe

1. Introduction

The present day observations indicate that the universe at large scale is homogeneous and isotropic and the accelerating phase of universe. As the observed universe is almost homogeneous and isotropic, space-time is usually described by a **Friedman- Lemaitre-Robertson-Walker** cosmology. But it is also believed that in the early universe the FLRW model does not give a correct matter description. The anomalies found in the cosmic microwave background and the large structure observations stimulated a growing interest in anisotropic cosmological model of the universe. Observations by the Differential Radiometers on NACA's Cosmic Background Explorer registered anisotropy in various angle scales. It is conjectured, that these anisotropies hide in their hearts the entire history of the cosmic evolution down to recombination, and they are considered to be indicative of the universe geometry and the matter composing the universe. It is expected, that much more will be known about anisotropy of cosmic microwave's background after the investigations of the microwave's anisotropy probe. There is a general agreement among cosmologists that cosmic microwave's background anisotropy in the small angle scale holds the key to the formation of the discrete structure. The theoretical argument (Misner 1968) and the modern experimental data support the existence of an anisotropic phase, which turns into an isotropic one. In recent years, there has been considerable interest in string cosmology. Cosmic strings are topologically stable objects which might be found during a phase transition in the early universe (Kibble 1976). Cosmic strings play an important role in the study of the early universe. These arise during the phase transition after the big bang explosion as the temperature goes down below some critical temperature as predicted by grand unified theories (Zel'dovich et al. 1975; Kibble 1976, 1980; Everett 1981; Vilenkin 1981a). It is believed that the existence of strings in the early universe gives rise to the density fluctuations which leads to the formation of galaxies (Zel'dovich 1980; Vilenkin 1981b, 1985; Hindmarsh and Kibble 1995; Turner and Tyson 1999). Massive closed loops of strings serve as seeds for the formation of large structures like galaxies and cluster of galaxies.

While matter is accreted onto loops, they oscillate violently and lose their energy by gravitational radiation and therefore they shrink and disappear. These cosmic strings have stress-energy and couple to the gravitational field. Therefore it is interesting to study the gravitational effects that arise from strings. The pioneering work in the formulation of the energy-momentum tensor for classical massive strings was done by Letelier (1979) who considered the massive strings to be formed by geometric strings with particle attached along its extension. Letelier (1983) first used this idea in obtaining cosmological solutions in Bianchi-I and Kantowski-Sachs space-times. Stachel (1980) has also studied massive string. Roy and Banerjee (1995) have dealt with LRS cosmological models of Bianchi type-II representing clouds of geometrical as well as massive strings. Wang (2003) studied the Letelier model in the context of LRS Bianchi type-II space-time. Bali and Dave (2001, 2003), Bali and Upadhaya (2003), Bali and Singh (2005), Bali and Anjali (2006), Bali and Pradhan (2007), Bali et al. (2007) have obtained Bianchi types I, III and IX string cosmological models in general relativity. Yadav et al. (2007a, 2007b) have studied some Bianchi type I viscous fluid string cosmological models with magnetic field. Recently Wang (2004a, 2004b, 2005, 2006) has also discussed LRS Bianchi type I and Bianchi type III cosmological models for a cloud string with bulk viscosity. Yadav et al. (2007a, 2007b) have obtained the integrability of cosmic string in Bianchi type III space-time in presence of bulk viscous fluid by applying a new technique. Reddy (2003, 2005), Reddy et al. (2005, 2007), Reddy and Naidu (2007), Rao et al. (2008a, 2008b, 2009), Rao and Vinutha (2010), Pradhan (2007, 2009), Pradhan and Mathur (2008) and Pradhan et al. (2008, 2010a, 2010b) have studied string cosmological models in different contexts. Recently, Belinchon (2009a, 2009b), Pradhan et al. (2009), Amirhashchi and Zainuddin (2010) and Tripathi et al. (2009, 2010) have obtained cosmic strings in different Bianchi type spacetimes. The simplest anisotropic models of the universe are Bianchi type-I homogeneous models whose spatial sections are flat but the expansion or contraction rate are directional dependent. The advantages of these anisotropic models are that they have a significant role in the description of the evolution of the early phase of the universe and they help in finding more general cosmological models than the isotropic FRW models. Recently, Saha and Visinescu (2010) and Saha et al. (2010) have studied Bianchi type-I models with cosmic string in presence of magnetic flux. Motivated by the above discussions, in this paper, the Einstein's field equations have been solved for massive string by applying a variation law for generalized Hubble's parameter in Bianchi-I space-time. The paper has the following structure. The metric and the field equations are presented in Sect. 2. In Sect. 3, we deal with an exact solution of the field equations with cloud of strings. Sections 3.1 and 3.2 deal with power-law and exponential-law solutions and their physical and geometric aspects respectively. Finally, in Sect. 4, we conclude the results and outline future prospects.

2. The metric and field equations

We consider totally anisotropic Bianchi type -I line element, given by

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 dy^2 + C^2 dz^2, \quad \ldots\ldots(1)$$

where the metric potentials A, B and C are functions of t alone. This ensures that the model is spatially symmetric and homogeneous.

The energy-momentum tensor for a cloud of massive string and perfect fluid distribution is taken as

$$T_i^j = (\rho + p) v_j v^j + p g_i^j - \lambda x_j x^j \ldots\ldots(2)$$

Where ρ is the proper energy density for a cloud string with particles attached to them; λ is the string tension density; p is the isotropic pressure; $v^i = (0, 0, 0, 1)$ is the four velocity of the particles, and x^i is a unit space-like vector representing the direction of string.
The vectors v^i and x^i satisfy the following conditions

$$v_i v^i = -x_i x^i = -1, \; v^i x_i = 0 \ldots\ldots (3)$$

Choosing x^i parallel to $\partial/\partial x$, we have

$$x^i = (A^{-1}, 0, 0, 0) \ldots\ldots (4)$$

If the particle density of the configuration is denoted by ρ_p, then

$$\rho = \rho_p + \lambda \ldots\ldots (5)$$

The Einstein's field equations with varying $\wedge$ in suitable units are
(in gravitational units $8\pi G = 1, c = 1$.)

$$R_i^j - \frac{1}{2} R g_i^j = T_i^j + \wedge g_i^j \ldots\ldots(6)$$

in case of the metric (1) and perfect fluid distribution equation (2) in the commoving system of coordinates, lead to the following set of independent differential equations(field equations):

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\frac{\dot{C}}{C} = -p + \wedge \quad \ldots\ldots (7)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\frac{\dot{C}}{C} = -p + \wedge \quad \ldots\ldots (8)$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}=-p+\wedge \quad \ldots\ldots (9)$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{C}\dot{A}}{CA}=p+\wedge \quad \ldots\ldots (10)$$

In view of the vanishing divergence of the Einstein tensor, we have

$$\dot{\rho}+(\rho+p)\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)=-p+\wedge \quad \ldots\ldots (11)$$

We assume that the matter content obeys an equation of state,

$$p=\omega\rho,\ 0\leq\omega\leq 1 \ldots\ldots (12)$$

Spatial volume V as an average scale factor of the model (1) may be defined as

$$V=R^3=ABC \ldots\ldots (13)$$

Hubble parameter H in anisotropic models may be defined as

$$H=\frac{\dot{R}}{R}=\frac{1}{3}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)= \ldots\ldots (14)$$

Where a dot stands for ordinary time derivative of the concerned quantity

$$H=\frac{1}{3}\left(H_1+H_2+H_3\right)= \ldots\ldots (15)$$

Where, $H_1=\frac{\dot{A}}{A}\ H_2=\frac{\dot{B}}{B}\ H_3=\frac{\dot{C}}{C} \ldots\ldots (16)$

are directional Hubble factors in the x, y, and z directions, respectively.

SOLUTION OF THE FIELD EQUATIONS

The non-vanishing component of shear tensor σ_{ij} defined by

$\sigma_{ij}=u_{i,j}-u_{j,i}-\frac{2}{3}g_{i,j}\,u_k^k$ are obtained as

$$\sigma_1^1=\frac{4}{3}\frac{\dot{A}}{A}-\frac{2}{3}\left(\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right) \ldots\ldots (17)$$

$$\sigma_2^2 = \frac{4}{3}\frac{\dot{B}}{B} - \frac{2}{3}\left(\frac{\dot{C}}{C} + \frac{\dot{A}}{A}\right) \qquad (18)$$

$$\sigma_3^3 = \frac{4}{3}\frac{\dot{C}}{C} - \frac{2}{3}\left(\frac{\dot{A}}{A} + + \frac{\dot{B}}{B}\right) \qquad (19)$$

The shear scalar σ is given by

$$\sigma^2 = \frac{1}{3}\left(\frac{\dot{A}^2}{A} + \frac{\dot{B}^2}{B} + \frac{\dot{C}^2}{C} - \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{\dot{B}}{B}\frac{\dot{C}}{C} - \frac{\dot{C}}{C}\frac{\dot{A}}{A}\right) \qquad (20)$$

$$\frac{\dot{\sigma}}{\sigma} = -\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = -3H \qquad (21)$$

The Einstein's field equations (7) - (10) in terms of Hubble parameter H, shear scalar σ and declaration parameter q can be written as

$$H^2(2q - 1) - \sigma^2 = p - \Lambda \qquad (22)$$

$$3H^2 - \sigma^2 = \rho + \Lambda \qquad (23)$$

Where $$q = -\frac{R\ddot{R}}{\dot{R}^2} \qquad (24)$$

From equations (7), (8) and (9) and integrating the equations, we get

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{d_1}{R^3} \qquad (25)$$

$$\frac{\dot{A}}{A} - \frac{\dot{C}}{C} = \frac{d_2}{R^3} \qquad (26)$$

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = \frac{d_3}{R^3} \qquad (27)$$

Here, d_1, d_2, d_3 are constants of integration. We now assume energy conservation equation Tij = 0 yields

$$\dot{\rho} + \rho(1+\omega)\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right) = 0 \qquad (28)$$

Using the equations (13) and (24) , we get

$$\rho = \frac{d_4}{R^{3(1+\omega)}} \quad \ldots\ldots (29)$$

Where, d_4 is the constant of integration.

Showing that the rate of volume expansion decreases during time evolution and the presence of positive Λ shows down the rate of decrease whereas a negative Λ would promote it. From equations (23) and (24),

$$\wedge = (2-q)H^2 - \frac{(1-\omega)\rho}{2} \quad \ldots\ldots (30)$$

Which implies $\wedge \le 0$ for $q \ge 2$.

Let us consider $\wedge = \alpha H^2$ ……(31)

Where, α is the positive constant. On simplifying, we get the values

$$A = [(3-\alpha)(C_1 t + C_2)]^{\frac{1}{3-\alpha}} \, exp\left(\frac{2d_1 + d_2}{6\{(3-\alpha)(C_1 t + C_2)\}^{\frac{3}{3-\alpha}}}\right)$$

$$B = [(3-\alpha)(C_1 t + C_2)]^{\frac{1}{3-\alpha}} \, exp\left(\frac{d_2 - d_1}{3\{(3-\alpha)(C_1 t + C_2)\}^{\frac{3}{3-\alpha}}}\right)$$

$$C = [(3-\alpha)(C_1 t + C_2)]^{\frac{1}{3-\alpha}} \, exp\left(\frac{2d_2 - d_1}{2\{(3-\alpha)(C_1 t + C_2)\}^{\frac{3}{3-\alpha}}}\right)$$

Spatial volume $V = R^3 = \{(3-\alpha)(C_1 t + C_2)\}^{\frac{3}{3-\alpha}}$

A, B, C values substituting in the metric (1)

$ds^2 = -dt^2 + A^2 dx^2 + B^2 dy^2 + C^2 dz^2$, which implies

$$ds^2 = -dt^2 + [(3-\alpha)(C_1 t + C_2)]^{\frac{2}{3-\alpha}} \left\{ exp\left(\frac{2d_1 + d_2}{3\{(3-\alpha)(C_1 t + C_2)\}^{\frac{3}{3-\alpha}}}\right) dx^2 + exp\left(\frac{2(d_2 - d_1)}{3\{(3-\alpha)(C_1 t + C_2)\}^{\frac{3}{3-\alpha}}}\right) dy^2 + exp\left(\frac{2d_2 - d_1}{\{(3-\alpha)(C_1 t + C_2)\}^{\frac{3}{3-\alpha}}}\right) dz^2 \right\}$$

Concluding Remarks

The main purpose of this paper is to study some Bianchi type-I cosmological model in metric String Cosmology. We find exact solution of the vacuum field equations for Bianchi type- I space times. Initially, the field equations look complicated but lead to a solution using some assumptions. The first assumption is that the expansion scalar θ is proportional to the shear scalar σ^2. The spatial volume V is zero at $t = -\frac{C_2}{C_1}$ and expansion scalar is infinite at $t = -\frac{C_2}{C_1}$. It shows that the Universe starts evolving with zero volume with an infinite rate of expansion. The scale factor R is also zero at $t = -\frac{C_2}{C_1}$, which means that during initial age the space time exhibits a point type singularity. At $t = -\frac{C_2}{C_1}$, ρ→∞, σ→∞. As time increases, the scale factor R and spatial volume V increases, but the expansion scalar decreases.

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