

Supra Pairwise b-Locally open(closed) functions in Supra Bitopological Spaces

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Abstract

The purpose of this paper is to define and study supra pairwise b-Locally open(res. closed) functions in supra bitopological spaces and obtain some results and several characterizations concerning these concepts are discussed.

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1 Introduction

The concept of bitopological spaces was initiated by Kelly[4]. In 1983, Masshour[6] introduced the concept of supra topological spaces and discussed the S-continuous maps and S^* -continuous maps. The arbitrary union condition is enough to have a supra topological space. Gowri and Rajayal[2] are established the concept of supra bitopological spaces. Rajesh[8] introduced the notion of b-Locally closed sets in bitopological spaces and Tripathy and Sarma[11] study pairwise b-Locally open and pairwise b-Locally closed functions in bitopological spaces. In this paper, we investigate supra pairwise bL-open(res. closed) functions in supra bitopological spaces and also discuss various properties of these functions.

2 Preliminaries

Definition 2.1 [6] (X, S_τ) is said to be a supra topological space if it is satisfying these conditions:

- (1) $X, \emptyset \in S_\tau$
- (2) The union of any number of sets in S_τ belongs to S_τ .

Definition 2.2 [6] Each element $A \in S_\tau$ is called a supra open set in (X, S_τ) , and its complement is called a supra closed set in (X, S_τ) .

Definition 2.3 [2] If S_{τ_1} and S_{τ_2} are two supra topologies on a non-empty set X , then the triplet $(X, S_{\tau_1}, S_{\tau_2})$ is said to be a supra bitopological space.

Definition 2.4 [2] Each element of S_{τ_i} is called a supra τ_i -open sets(briefly S_{τ_i} -open sets) in $(X, S_{\tau_1}, S_{\tau_2})$. Then the complement of S_{τ_i} -open sets are called a supra τ_i -closed sets(briefly S_{τ_i} -closed sets), for $i = 1, 2$.

Definition 2.5 [2] If $(X, S_{\tau_1}, S_{\tau_2})$ is a supra bitopological space, $Y \subseteq X$, $Y \neq \emptyset$ then $(Y, S_{\tau_1}^*, S_{\tau_2}^*)$ is a supra bitopological subspace of $(X, S_{\tau_1}, S_{\tau_2})$ if $S_{\tau_1}^* = \{U \cap Y; U \text{ is a } S_{\tau_1} - \text{open in } X\}$ and $S_{\tau_2}^* = \{V \cap Y; V \text{ is a } S_{\tau_2} - \text{open in } X\}$.

Definition 2.6 [2] The S_{τ_i} -closure of the set A is denoted by $S_{\tau_i}\text{-cl}(A)$ and is defined as $S_{\tau_i}\text{-cl}(A) = \cap \{B : B \text{ is a } S_{\tau_i} - \text{closed and } A \subseteq B, \text{ for } i = 1, 2\}$.

Definition 2.7 [2] The S_{τ_i} -interior of the set A is denoted by $S_{\tau_i}\text{-int}(A)$ and is defined as $S_{\tau_i}\text{-int}(A) = \cup \{B : B \text{ is a } S_{\tau_i} - \text{open and } B \subseteq A, \text{ for } i = 1, 2\}$.

Definition 2.8 [3] Let A be a subset of a supra bitopological space $(X, S_{\tau_1}, S_{\tau_2})$, then A is said to be supra τ_{ij} -Locally-open (briefly, $S_{\tau_{ij}}$ -L-open) if $A = U \cup V$, where U is S_{τ_i} -closed and V is S_{τ_j} -open in X .
The family of all $S_{\tau_{ij}}$ -L-open sets of $(X, S_{\tau_1}, S_{\tau_2})$ is denoted by $S_{\tau_{ij}}\text{-L-O}(X)$, where $i, j = 1, 2$ and $i \neq j$.

Definition 2.9 [3] Let A be a subset of a supra bitopological space $(X, S_{\tau_1}, S_{\tau_2})$, then A is said to be supra τ_{ij} -Locally-closed (briefly, $S_{\tau_{ij}}$ -L-closed) if $A = U \cap V$, where U is S_{τ_i} -open and V is S_{τ_j} -closed in X .
The family of all $S_{\tau_{ij}}$ -L-closed sets of $(X, S_{\tau_1}, S_{\tau_2})$ is denoted by $S_{\tau_{ij}}\text{-L-C}(X)$, where $i, j = 1, 2$ and $i \neq j$.

Definition 2.10 [3] Let A be a subset of a supra bitopological space $(X, S_{\tau_1}, S_{\tau_2})$, then A is said to be supra τ_{ij} -b-open (briefly, $S_{\tau_{ij}}$ -b-open) if $A \subseteq S_{\tau_j}\text{-cl}(S_{\tau_i}\text{-int}(A)) \cup S_{\tau_i}\text{-int}(S_{\tau_j}\text{-cl}(A))$. where $i, j = 1, 2$ and $i \neq j$.
The family of all $S_{\tau_{ij}}$ -b-open sets of $(X, S_{\tau_1}, S_{\tau_2})$ is denoted by $S_{\tau_{ij}}\text{-bO}(X)$, where $i, j = 1, 2$ and $i \neq j$.

Definition 2.11 [3] Let A be a subset of a supra bitopological space $(X, S_{\tau_1}, S_{\tau_2})$, then A is said to be supra τ_{ij} -b-closed (briefly, $S_{\tau_{ij}}$ -b-closed) if $S_{\tau_j}\text{-int}(S_{\tau_i}\text{-cl}(A)) \cap S_{\tau_i}\text{-cl}(S_{\tau_j}\text{-int}(A))$. where $i, j = 1, 2$ and $i \neq j$.
The family of all $S_{\tau_{ij}}$ -b-closed sets of $(X, S_{\tau_1}, S_{\tau_2})$ is denoted by $S_{\tau_{ij}}\text{-bC}(X)$, where $i, j = 1, 2$ and $i \neq j$.

Definition 2.12 [3] A subset A of a supra bitopological space $(X, S_{\tau_1}, S_{\tau_2})$ is called supra τ_{ij} -b-locally-open (briefly, $S_{\tau_{ij}}$ -bL-open) if $A = U \cup V$, where U is S_{τ_i} -b-closed and V is S_{τ_j} -b-open in $(X, S_{\tau_1}, S_{\tau_2})$.
The family of all $S_{\tau_{ij}}$ -bL-open sets of $(X, S_{\tau_1}, S_{\tau_2})$ is denoted by $S_{\tau_{ij}}\text{-bL-O}(X)$, where $i, j = 1, 2$ and $i \neq j$.

Definition 2.13 [3] A subset A of a supra bitopological space $(X, S_{\tau_1}, S_{\tau_2})$ is called supra τ_{ij} -b-locally-closed (briefly, $S_{\tau_{ij}}$ -bL-closed) if $A = U \cap V$, where U is S_{τ_i} -b-open and V is S_{τ_j} -b-closed in $(X, S_{\tau_1}, S_{\tau_2})$.

The family of all $S_{\tau_{ij}}$ -bL-closed sets of $(X, S_{\tau_1}, S_{\tau_2})$ is denoted by $S_{\tau_{ij}}$ -bL- $C(X)$, where $i, j = 1, 2$ and $i \neq j$.

Definition 2.14 [3] The $S_{\tau_{ij}}$ -bL-closure of the set A is denoted by $S_{\tau_{ij}}$ -bL-cl(A) and is defined as $S_{\tau_{ij}}$ -bL-cl(A) = $\cap \{B : B \text{ is a } S_{\tau_{ij}} - \text{bL} - \text{closed and } A \subseteq B, \text{ for } i, j = 1, 2\}$.

Definition 2.15 [3] The $S_{\tau_{ij}}$ -bL-interior of the set A is denoted by $S_{\tau_{ij}}$ -bL-int(A) and is defined as $S_{\tau_{ij}}$ -bL-int(A) = $\cup \{B : B \text{ is a } S_{\tau_{ij}} - \text{bL} - \text{open and } B \subseteq A, \text{ for } i, j = 1, 2\}$.

3 Supra pairwise b-Locally open mapping

In this section, we introduced supra pairwise b-Locally open mapping in supra bitopological spaces.

Definition 3.1 A function $f:(X, S_{\tau_1}, S_{\tau_2}) \rightarrow (Y, S_{\tau'_1}, S_{\tau'_2})$ is said to be supra pairwise continuous (briefly, S_{τ} -p-continuous) if the image of each S_{τ_i} -open set in X is $S_{\tau'_i}$ -open set in Y , where $i = 1, 2$.

Definition 3.2 Let $(X, S_{\tau_1}, S_{\tau_2})$ and $(Y, S_{\tau'_1}, S_{\tau'_2})$ be two supra bitopological spaces. A function $f:(X, S_{\tau_1}, S_{\tau_2}) \rightarrow (Y, S_{\tau'_1}, S_{\tau'_2})$ is called supra pairwise bL-open (briefly, S_{τ} -p-bL-open) mapping if the image of each $S_{\tau_{ij}}$ -bL-open set in X is $S_{\tau'_i}$ -open set in Y , where $i = 1, 2$.

Example 3.3 Let $X = \{a, b, c, d\}$, $Y = \{p, q, r, s\}$,
 $S_{\tau_1} = \{\emptyset, X, \{a, d\}, \{b, d\}, \{a, b, d\}\}$,
 $S_{\tau_2} = \{\emptyset, X, \{a, b\}, \{b, c\}, \{a, b, c\}\}$,
 $S_{\tau'_1} = \{\emptyset, Y, \{q\}, \{r\}, \{q, r\}, \{r, s\}, \{q, s\}, \{q, r, s\}\}$,
 $S_{\tau'_2} = \{\emptyset, Y, \{s\}, \{q, r\}, \{q, s\}, \{r, s\}, \{q, r, s\}\}$.

Consider the function $f:(X, S_{\tau_1}, S_{\tau_2}) \rightarrow (Y, S_{\tau'_1}, S_{\tau'_2})$ defined by $f(a) = r$, $f(b) = q$,
 $f(c) = r$, $f(d) = s$. Here

$S_{\tau_{12}}$ -bL- $O(X) = \{\emptyset, X, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$
and image of each $S_{\tau_{12}}$ -bL- $O(X)$ are $S_{\tau'_1}$ -open.

Also $S_{\tau_{21}}$ -bL- $O(X) = \{\emptyset, X, \{d\}, \{a, b\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$
and image of each $S_{\tau_{21}}$ -bL- $O(X)$ are $S_{\tau'_2}$ -open.

Hence f is S_{τ} -p-bL-open mapping.

Theorem 3.4 Let $f:(X, S_{\tau_1}, S_{\tau_2}) \rightarrow (Y, S_{\tau'_1}, S_{\tau'_2})$ be a mapping between two supra bitopological spaces. Then the following are equivalent.

(i) f is S_{τ} -p-bL-open mapping. (ii) $f(S_{\tau_{ij}}$ -bL-int(B)) $\subseteq S_{\tau'_i}$ -int($f(B)$), for every subset B of X , where $i = 1, 2$.

Proof: (i) \implies (ii) Since $S_{\tau_{ij}}$ -bL-int(B) is a $S_{\tau_{ij}}$ -bL-open set in X for any subset B of X and $S_{\tau_{ij}}$ -bL-int(B) $\subseteq B$. Thus, we have $f(S_{\tau_{ij}}$ -bL-int(B)) $\subseteq f(B)$. Since f is S_{τ} -p-bL-open mapping, therefore $f(S_{\tau_{ij}}$ -bL-int(B)) is $S_{\tau'_i}$ -open. Hence $f(S_{\tau_{ij}}$ -bL-int(B)) $\subseteq S_{\tau'_i}$ -int($f(B)$).

(ii) \implies (i) Let B be a $S_{\tau_{ij}}$ -bL-open set in X. Then we get $S_{\tau'_i}$ -int($f(B)$) \subseteq $f(B) \rightarrow$ (1)
 By hypothesis $f(S_{\tau_{ij}}$ -bL-int(B)) \subseteq $S_{\tau'_i}$ -int($f(B)$) \implies $f(B) \subseteq S_{\tau'_i}$ -int($f(B)$) \rightarrow (2).
 By (1) and (2) we have $f(A)$ is $S_{\tau'_i}$ -open in Y.
 Therefore f is S_{τ} -p-bL-open mapping. □

Definition 3.5 A subset U is called an $S_{\tau_{ij}}$ -bL-open neighbourhood of a point x of $(X, S_{\tau_1}, S_{\tau_2})$ if there exists a $S_{\tau_{ij}}$ -bL-open set V such that $x \in V \subseteq U$.

Theorem 3.6 Let $f:(X, S_{\tau_1}, S_{\tau_2}) \rightarrow (Y, S_{\tau'_1}, S_{\tau'_2})$ be a function. Then the following are equivalent.

(i) f is S_{τ} -p-bL-open mapping. (ii) For each $x \in X$ and each $S_{\tau_{ij}}$ -bL-open neighbourhood U of x in X , then there exists a $S_{\tau'_i}$ -neighbourhood D of $f(x)$ such that $D \subseteq f(U)$, where $i = 1, 2$.

Proof: (i) \implies (ii) Let U be a $S_{\tau_{ij}}$ -bL-open neighbourhood of x and $x \in X$. Then there exists a $S_{\tau_{ij}}$ -bL-open set V in X such that $x \in V \subseteq U$. Since f is S_{τ} -p-bL-open mapping, by Theorem (3.4), we have $f(S_{\tau_{ij}}$ -bL-int(V)) \subseteq $S_{\tau'_i}$ -int($f(V)$). Therefore $f(V)$ is $S_{\tau'_i}$ -open set such that $f(x) \in f(V) \subseteq f(U)$. Put $f(V) = D$, then we have D is a $S_{\tau'_i}$ -open set such that $D \subseteq f(U)$.

(ii) \implies (i) Let U be a $S_{\tau_{ij}}$ -bL-open set in X and $x \in X$. By hypothesis, for each $f(x) \in f(U)$, then there exists a $S_{\tau'_i}$ -neighbourhood V of $f(x)$ such that $V \subseteq f(U)$. Since V is $S_{\tau'_i}$ -neighbourhood of $f(x)$, then there exists a $S_{\tau'_i}$ -open set D such that $f(x) \in D \subseteq V$.

Now $f(V) = \cup D: f(x) \in f(U)$. It is clear that $f(U)$ is $S_{\tau'_i}$ -open.
 Therefore f is S_{τ} -p-bL-open mapping. □

Theorem 3.7 Let $f:(X, S_{\tau_1}, S_{\tau_2}) \rightarrow (Y, S_{\tau'_1}, S_{\tau'_2})$ and $g:(Y, S_{\tau'_1}, S_{\tau'_2}) \rightarrow (Z, S_{\tau''_1}, S_{\tau''_2})$ be two mappings. If $g \circ f: X \rightarrow Z$ is S_{τ} -p-bL-open mapping and g is S_{τ} -p-continuous injection, then f is S_{τ} -p-bL-open mapping.

Proof: Let C be an S_{τ} -p-bL-open set in X . Since $g \circ f$ is S_{τ} -p-bL-open mapping, therefore $(g \circ f)(c)$ is $S_{\tau''_i}$ -open. Further g is S_{τ} -p-continuous and injective, then $g^{-1}(g(f(c))) = f(c)$ is $S_{\tau'_i}$ -open. Therefore f is S_{τ} -p-bL-open mapping. □

Theorem 3.8 A function $f:(X, S_{\tau_1}, S_{\tau_2}) \rightarrow (Y, S_{\tau'_1}, S_{\tau'_2})$ is S_{τ} -p-bL-open iff for any subset B of Y and for any $S_{\tau_{ij}}$ -bL-closed set C in X such that $f^{-1}(B) \subseteq C$, then there exists a $S_{\tau'_i}$ -closed set D containing B such that $f^{-1}(D) \subseteq C$, where $i = 1, 2$.

Proof: Let E be a $S_{\tau_{ij}}$ -bL-open set in $(X, S_{\tau_1}, S_{\tau_2})$. Putting $B = Y - f(E) \implies f^{-1}(B) = f^{-1}(Y) - E = X - E \rightarrow$ (1).
 Then $X - E$ is a $S_{\tau_{ij}}$ -bL-closed set in X such that $f^{-1}(B) \subseteq X - E$. By hypothesis, there exists a $S_{\tau'_i}$ -closed set D containing B such that $f^{-1}(D) \subseteq X - E \implies D \subseteq f(x) - f(E) = Y - f(E) \implies f(E) \subseteq Y - D \rightarrow$ (2).
 Since $B \subseteq D$, we have $Y - D \subseteq Y - B = f(E)$ by (1) $\implies Y - D \subseteq f(E) \rightarrow$ (3).

Then from (2) and (3), we get $f(E) = Y - D$ and so $f(E)$ is $S_{\tau'_i}$ -open. Since D is $S_{\tau'_i}$ -closed. Therefore, f is S_{τ} -p-bL-open mapping.

Conversely, let B be any subset of Y and C be a $S_{\tau_{ij}}$ -bL-closed set of X such that $f^{-1}(B) \subseteq C$. Suppose that f is S_{τ} -p-bL-open mapping. Let $D = Y - f(X - C)$. Since C is $S_{\tau_{ij}}$ -bL-closed set in X and so $X - C$ is $S_{\tau_{ij}}$ -bL-open set in X . Since f is S_{τ} -p-bL-open, therefore $f(X - C)$ is $S_{\tau'_i}$ -open.

$$\implies Y - f(X - C) \text{ is } S_{\tau'_i}\text{-closed.}$$

$$\implies D \text{ is } S_{\tau'_i}\text{-closed.}$$

Now, $D = Y - f(X - C)$

$$\implies f^{-1}(D) = X - (X - C) \subseteq C.$$

$$\implies f^{-1}(D) \subseteq C.$$

Since $f^{-1}(B) \subseteq C$

$$\implies X - f^{-1}(B) \supseteq X - C$$

$$\implies Y - B \supseteq f(X - C)$$

$$\implies B \subseteq Y - f(X - C) = D$$

$$\implies B \subseteq D.$$

Then there exists a $S_{\tau'_i}$ -closed set D such that $f^{-1}(D) \subseteq C$. □

Theorem 3.9 *Let $f:(X, S_{\tau_1}, S_{\tau_2}) \rightarrow (Y, S_{\tau'_1}, S_{\tau'_2})$ be a mapping. Then the following are equivalent.*

(i) *f is S_{τ} -p-bL-open mapping.*

(ii) *$f^{-1}(S_{\tau'_i}\text{-cl}(B)) \subseteq S_{\tau_{ij}}\text{-bL-cl}(f^{-1}(B))$ for every subset B of Y , where $i = 1, 2$.*

Proof: (i) \implies (ii) Let B be a subset of Y . Then we have $f^{-1}(B) \subseteq S_{\tau_{ij}}\text{-bL-cl}(f^{-1}(B))$ and $S_{\tau_{ij}}\text{-bL-cl}(f^{-1}(A))$ is $S_{\tau_{ij}}$ -bL-closed set in X . Since f is S_{τ} -p-bL-open mapping, by Theorem 3.8, then there exists a $S_{\tau'_i}$ -closed set C such that $B \subseteq C$ and $f^{-1}(C) \subseteq S_{\tau_{ij}}\text{-bL-cl}(f^{-1}(B))$. Also $B \subseteq C$.

$$\implies f^{-1}(B) \subseteq f^{-1}(C) \subseteq S_{\tau_{ij}}\text{-bL-cl}(f^{-1}(B))$$

$$\implies f^{-1}(S_{\tau'_i}\text{-cl}(B)) \subseteq S_{\tau_{ij}}\text{-bL-cl}(f^{-1}(B)).$$

(ii) \implies (i) Let B be a subset of Y and C be a $S_{\tau_{ij}}$ -bL-closed set in X such that $f^{-1}(B) \subseteq C$. Then we get $B \subseteq S_{\tau'_i}\text{-cl}(B)$ and $S_{\tau'_i}\text{-cl}(B)$ is $S_{\tau'_i}$ -closed. Hence by hypothesis, $f^{-1}(S_{\tau'_i}\text{-cl}(B)) \subseteq S_{\tau_{ij}}\text{-bL-cl}(f^{-1}(B)) \subseteq S_{\tau_{ij}}\text{-bL-cl}(C) = C \implies f^{-1}(S_{\tau'_i}\text{-cl}(B)) \subseteq C$.

Therefore, by Theorem 3.8, then we get f is S_{τ} -p-bL-open mapping. □

4 Supra pairwise b-Locally closed mapping

In this section, supra pairwise b-Locally closed mappings are introduced and investigate their properties.

Definition 4.1 *Let $(X, S_{\tau_1}, S_{\tau_2})$ and $(Y, S_{\tau'_1}, S_{\tau'_2})$ be two supra bitopological spaces. A function $f:(X, S_{\tau_1}, S_{\tau_2}) \rightarrow (Y, S_{\tau'_1}, S_{\tau'_2})$ is called supra pairwise bL-closed (briefly, S_{τ} -p-bL-closed) mapping if the image of each $S_{\tau_{ij}}$ -bL-closed set in X is $S_{\tau'_i}$ -closed set in Y , where $i = 1, 2$.*

Example 4.2 Let $X = \{a, b, c, d\}$, $Y = \{p, q, r, s\}$,
 $S_{\tau_1} = \{\emptyset, X, \{a, d\}, \{b, d\}, \{a, b, d\}\}$,
 $S_{\tau_2} = \{\emptyset, X, \{a, b\}, \{b, c\}, \{a, b, c\}\}$,
 $S_{\tau'_1} = \{\emptyset, Y, \{p, q\}, \{p, r\}, \{p, s\}, \{p, q, r\}, \{p, q, s\}, \{p, r, s\}\}$,
 $S_{\tau'_2} = \{\emptyset, Y, \{p\}, \{p, q\}, \{p, s\}, \{p, q, r\}, \{p, q, s\}, \{p, r, s\}\}$.

Consider the function $f: (X, S_{\tau_1}, S_{\tau_2}) \rightarrow (Y, S_{\tau'_1}, S_{\tau'_2})$ defined by $f(a) = r$, $f(b) = q$,
 $f(c) = r$, $f(d) = s$.

Here $S_{\tau_{12}}\text{-bL-C}(X) = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, c, d\}\}$
and image of each $S_{\tau_{12}}\text{-bL-C}(X)$ are $S_{\tau'_1}$ -closed.

Also $S_{\tau_{21}}\text{-bL-C}(X) = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{b, c\}, \{a, c\}, \{c, d\}, \{a, b, c\}\}$
and image of each $S_{\tau_{21}}\text{-bL-C}(X)$ are $S_{\tau'_2}$ -closed.

Hence f is S_{τ} -p-bL-closed mapping.

Theorem 4.3 Let $f: (X, S_{\tau_1}, S_{\tau_2}) \rightarrow (Y, S_{\tau'_1}, S_{\tau'_2})$ be a mapping between two supra bitopological spaces. Then the following are equivalent.

(i) f is S_{τ} -p-bL-closed mapping. (ii) $S_{\tau'_i}\text{-cl}(f(B)) \subseteq f(S_{\tau_{ij}}\text{-bL-cl}(B))$, for each subset B of X , where $i = 1, 2$.

Proof: (i) \implies (ii) Let f be S_{τ} -p-bL-closed mapping. Then for any subset B of X . We get $B \subseteq S_{\tau_{ij}}\text{-bL-cl}(B)$ and $S_{\tau_{ij}}\text{-bL-cl}(B)$ is $S_{\tau_{ij}}\text{-bL-closed}$ set in X . Then $f(B) \subseteq f(S_{\tau_{ij}}\text{-bL-cl}(B))$. By assumption we obtain $f(S_{\tau_{ij}}\text{-bL-cl}(B))$ is $S_{\tau'_i}$ -closed. Therefore $S_{\tau'_i}\text{-cl}(f(B)) \subseteq f(S_{\tau_{ij}}\text{-bL-cl}(B))$.

(ii) \implies (i) Let B be a $S_{\tau_{ij}}\text{-bL-closed}$ set in X . Then we have $f(B) \subseteq f(S_{\tau_{ij}}\text{-bL-cl}(B))$. By hypothesis, $S_{\tau'_i}\text{-cl}(f(B)) \subseteq f(B)$.

Therefore $f(B)$ is $S_{\tau'_i}$ -closed in Y and hence f is S_{τ} -p-bL-closed mapping. \square

Theorem 4.4 Let $f: (X, S_{\tau_1}, S_{\tau_2}) \rightarrow (Y, S_{\tau'_1}, S_{\tau'_2})$ be a mapping. Then the following are equivalent.

(i) f be S_{τ} -p-bL-closed mapping. (ii) For any subset B of Y and for any $S_{\tau_{ij}}\text{-bL-open}$ set C in X such that $f^{-1}(B) \subseteq C$, then there exists a $S_{\tau'_i}$ -open set D containing B such that $f^{-1}(D) \subseteq C$, where $i = 1, 2$.

Proof: The proof is similarly to proof of Theorem 3.8. \square

Definition 4.5 A function $f: (X, S_{\tau_1}, S_{\tau_2}) \rightarrow (Y, S_{\tau'_1}, S_{\tau'_2})$ is called supra pairwise bL-closed (briefly, S_{τ} -p-bL-closed) irresolute if $f^{-1}(B) \in S_{\tau_{ij}}\text{-bL-C}(X)$ for every $B \in S_{\tau'_{i,j}}\text{-bL-C}(Y)$

Theorem 4.6 Let $f: (X, S_{\tau_1}, S_{\tau_2}) \rightarrow (Y, S_{\tau'_1}, S_{\tau'_2})$ and $g: (Y, S_{\tau'_1}, S_{\tau'_2}) \rightarrow (Z, S_{\tau''_1}, S_{\tau''_2})$ be two mappings. If $g \circ f: X \rightarrow Z$ is S_{τ} -p-bL-closed mapping. If f is S_{τ} -p-bL-closed irresolute surjection, then g is S_{τ} -p-bL-closed mapping.

Proof: Let us assume that, B be $S_{\tau_{ij}}\text{-bL-closed}$ set in Y . since f is S_{τ} -p-bL-closed irresolute surjection, then $f^{-1}(B)$ is $S_{\tau_{ij}}\text{-bL-closed}$ set in X . Since $g \circ f$ is S_{τ} -p-bL-closed map and f is surjection, then we have $(g \circ f)(f^{-1}(B))$ is $S_{\tau''_i}$ -closed. This implies $g(B)$ is $S_{\tau''_i}$ -closed. Therefore g is S_{τ} -p-bL-closed mapping. \square

5 Conclusion

The notion of S_τ -p-bL-open(closed) functions in supra bitopological space has been introduced. Applying these notions we obtain and investigate many properties to supra bitopological spaces.

References

- [1] D. Andrijevic, On b-open sets, Mat Vesnik, 48, (1996), 59-64.
- [2] R. Gowri and A. K. R. Rajayal, On supra bitopological spaces, IOSR-JM, Vol 13, (2017), 55-58.
- [3] R. Gowri and A. K. R. Rajayal, Supra b-open(closed) sets and supra b-Locally open set in supra bitopological spaces, International Jr. of Research and Analytical Reviews (IJRAR), Vol 5, No 4, (2018), 1054-1059.
- [4] J. C. Kelly, Bitopological spaces, Proc. London Math. Soc.No.(3), 13(1963), 71-89.
- [5] S. Lal, Pairwise concepts in bitopological spaces, Aust.Jr. Math. soc (ser.A), 26(2015),241-250.
- [6] A. S. Mashhour, A.A. Allam, F.S. Mahmoud and F.H. Khedr, On Supra topological spaces, Indian Jr.Pure and Appl.Math.No.4(14)(1983), 502-510.
- [7] C. W. Patty, Bitopological spaces, Pure math. Jr. 34(1967), 387-392.
- [8] N. Rajesh, on b-Locally closed sets in bitopological spaces, Jour.Ind.Acad.Math.,30,(2008) 551-556.
- [9] M. S. Sarsak and N. Rajesh, Special Functions on bitopological spaces, International Mathematical Forum, Vol 4, (2009), 1775-1782.
- [10] O. R. Sayed and T. R. Noiri, on supra b-open sets and supra b-continuity, Eur.Jr.Appl.Matth., 3(2),(2010), 295-302.
- [11] B. C. Tripathy and D. J. Sarma, on pairwise b-Locally open and b-Locally closed functions in bitopological spaces,Tamkang Jr. of Mathematics, Vol 43, (2012), 533-539.