CALCULATIONOF NATURAL FREQUENCIES OF MDOF SYSTEM

A. Sivaji[1], G.Naveen Kumar[2]

[1, 2] Annamacharya Institute of Technology & Sciences (Autonomous),
Department of Civil Engineering
Rajampet, Kadapa Dist., AP, India - 516126

ABSTRACT

This paper deals with the calculation of natural frequency of the multi-degree of freedom system, for calculating natural frequency of system by manual is simple for Two-Degree of freedom system, Three-Degree of freedom system and there after it becomes strenuous, so in this we have used MATLAB programming sinceMATLAB is a multi-paradigm numerical computing environment and proprietary "programming" language,MATLAB stores the data in the matrices and allows matrix manipulationsMATLAB stores the data in the matrix form so it is very convenient to use the software. There are many approximate methods in these Stodola, Dunkerley,Rayleigh,Holzer and Influencecoefficient methods are considered and have calculated the natural frequency of the system by using the MATLAB. In this we can calculate the natural frequency of the system for Multi-Degree of freedom System.

Keywords: *Multi-degree of freedom, Natural frequency, MATLAB.*

INTRODUCTION:

Vibrations have become essential topics for assuring structural integrity and operational functionality in different engineering areas. Vibration is a mechanical phenomenon whereby oscillations occur about an equilibrium point. The word comes from Latin *vibrationem* ("shaking, brandishing"). The oscillations may be periodic, such as the motion of a pendulum or random. Vibration can be desirable: for example, the motion of a tuning fork, the reed in a woodwind instrument or harmonica, a mobile phone, or the cone of a loudspeaker.

In many cases, however, vibration is undesirable, wasting energy and creating unwanted disturbance. For example, the vibrational motions of engines, electric motors, or any mechanical device in operation are typically unwanted. Such vibrations could be caused by imbalances in the rotating parts, uneven friction, or the meshing of gear teeth. Careful designs usually minimize unwanted vibrations. Similarly vibration will be developed in structures which due to so many reasons. For understanding these vibration in structure we need done the analysis on the structure

Structural analysis is mainly concerned with finding out the behavior of a physical structure when subjected to force. This action can be in the form of load due to the weight of things such as people, furniture, wind, snow, etc. or some other kind of excitation such as an earthquake, shaking of the ground due to a blast nearby, etc. In essence all these loads are dynamic, including the self-weight of the structure because at some point in time these loads were not there. The distinction is made between the dynamic and the static analysis on the basis of whether the applied action has enough acceleration in comparison to the structure's natural frequency. If a load is applied sufficiently slowly, the inertia forces (Newton's first law of motion) can be ignored and the analysis can be simplified as static analysis. Structural dynamics, therefore, is a type of structural analysis which covers the behavior of structures subjected to dynamic (actions having high acceleration) loading. Dynamic loads include people, wind, waves, traffic, earthquakes, and blasts. Any structure can be subjected to dynamic loading. Dynamic analysis can be used to find dynamic displacements, time history, and analysis. A dynamic analysis is also related to the inertia forces developed by a structure when it is excited by means of dynamic loads applied suddenly (e.g., wind blasts, explosion, and earthquake).A static load is one which varies very slowly. A dynamic load is one which changes with time fairly quickly in comparison to the structure's natural frequency. If it changes slowly, the structure's response may be determined with static analysis, but if it varies quickly (relative to the structure's ability to respond), the response must be determined with a dynamic analysis. Dynamic analysis for simple structures can be carried out manually, but for complex structures finite element analysis can be used to calculate the mode shapes and frequencies.

Frequency is the number of occurrences of a repeating event per unit of time.[1] It is also referred to as temporal frequency, which emphasizes the contrast to spatial frequency and angular frequency. The period is the duration of time of one cycle in a repeating event, so the period is the reciprocal of the frequency. Frequency is an important parameter used in science and engineering to specify the rate of oscillatory and vibratory phenomena, such as mechanical vibrations, audio signals (sound), radio waves, and light.

Natural frequency is the frequency at which a system tends to oscillate in the absence of any driving or damping forceFree vibrations of an elastic body are called *natural vibrations* and occur at a frequency called the natural frequency. Natural vibrations are different from forced vibrations which happen at frequency of applied force (forced frequency). If forced frequency is equal to the natural frequency, the amplitude of vibration increases manifold. This phenomenon is known as resonance.

To make the structure safe it is desired to make the frequency of the structure not to match with the forcing frequency. We know the frequency of the forcing system, to the frequency of the structure we need to calculate the natural frequency of the system

TYPES OF VIBRATIONS:

Free Vibration: Vibration of a system because of its own elastic property. No external force is required for this vibration and only initiation of vibration may be necessary. When a system is initially disturbed by a displacement, velocity or acceleration, the system begins to vibrate with a constant amplitude and frequency depending on its stiffness and mass. This frequency is called as natural frequency, and the form of the vibration is called as mode shapes

Forced Vibration: A system that vibrates under an external force at the same frequency as that of external force. If an external force is applied to a system, the system will follow the force with the same frequency. However, when the forcing frequency is increased to the system's natural frequency, amplitudes will dangerously increase in this region. This phenomenon called as "Resonance"

Degree of freedom: The number of independent coordinate systems required to specify a motion. If the motion is in one direction due to the vibration of a single spring, then it is a Single degree of freedom system. If a particle is likely to vibrate in space, it will have six degrees of freedom, namely three translations and three rotations along three axis. A continuum can have infinite degrees of freedom.

MODELLING OF THE SYSTEM:

All mechanical systems can be modeled by containing three basic components Spring, Damper, Mass When these components are subjected to force, they response with a displacement, velocity and acceleration

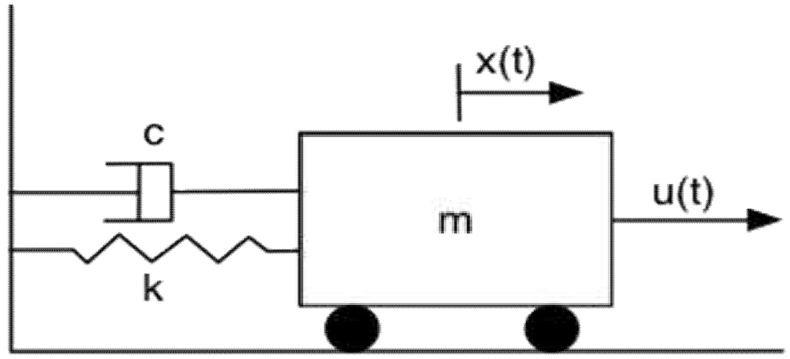


Fig.1 schematic of mass, spring and damper of SDOF System

Degree of freedom: The number of independent coordinate systems required to specify a motion with respect to the inertia. If the motion is in one direction due to the vibration of a single spring, then it is a Single degree of freedom system. If a particle is likely to vibrate in space, it will have six degrees of freedom, namely three translations and three rotations along three axis. A continuum can have infinite degrees of freedom.

MULTI-DEGREE OF FREEDOM SYSTEM:

Multi-degree-of-freedom (multi-DOF) systems are defined as those requiring two or more coordinates to describe their motion of the structure

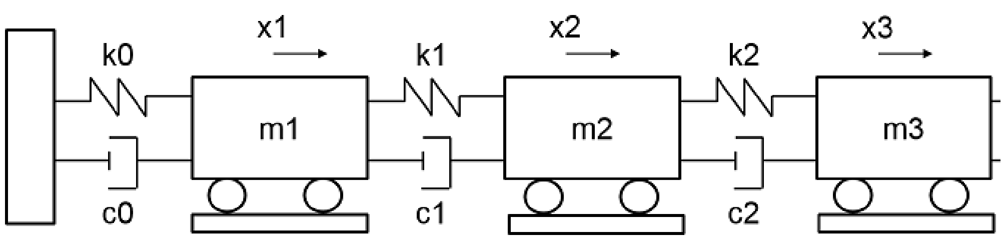


Fig.2 Multi-degree of freedom system

APPROXIMATE METHODS:

For calculating the natural frequency of the system it easy for a single degree of freedom system, if we know the mass (M) and stiffness (K) of the system by using formulae

$$\omega_n = \sqrt{\frac{k}{M}} \quad \text{rads/s}$$

$$f_n = \frac{1}{2.\pi.} \sqrt{\frac{k}{M}} \quad \text{cycles/s}$$

When coming to the multi-degree of freedom system it becomes strenuous, and need for programing approach is required.

There are many approximate methods for the valuating the natural frequency of the structure and by using some method we can obtain higher order frequency and mode shape. But in this paper we are limiting to the natural frequency and we are using five approximate methods for determining the frequency of system by using MATLAB software.

1. Stodola method
2. Dunkerley method
3. Holzer method
4. Influence coefficient method
5. Rayleigh Ritz method

MATLAB:

MATLAB (*matrix laboratory*) is a multi-paradigm numerical computing environment and proprietary "programming" languagedeveloped by MathWorks. MATLAB allows matrix manipulations, plotting of functions and data, implementation of algorithms, creation of user interfaces, and interfacing with programs written in other languages.

Although MATLAB is intended primarily for numerical computing, an optional toolbox uses the MuPAD symbolic engine, allowing access to symbolic computing abilities. An additional package, Simulink, adds graphical multi-domain simulation and model-based design for dynamic and embedded systems. The MATLAB application is built around the MATLAB scripting language. Common usage of the MATLAB application involves using the Command Window as an interactive mathematical shell or executing text files containing MATLAB code.

MATLAB has structure data types. Since all variables in MATLAB are arrays, a more adequate name is "structure array", where each element of the array has the same field names. In addition, MATLAB supports dynamic field names. When creating a MATLAB function, the name of the file should match the name of the first function in the file. Valid function names begin with an alphabetic character, and can contain letters, numbers, or underscores. Functions are often case sensitive.

Fig.3 MATLAB Software

MATLAB CODE FOR STODOLA METHOD:

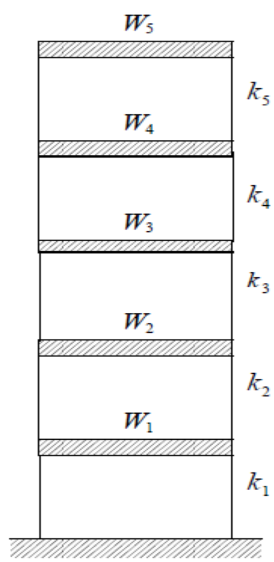
```
function [] = stodola()
dof = input('Enter the number of degree of freedom');
```

```
mass= ones(1,dof);
stiffness= ones(1,dof);
for k = 1:dof
mass(k) = input('Enter the masses of the system \n');
end
for k = 1:dof
stiffness(k) = input('Enter the stiffness of the system\n');
end
ad = ones(1,dof);
for z=1:1000
inertia = ad .* mass;
t=dof-1;
sforce=inertia;
while t>0
sforce(1,t) = sforce(1,t)+sforce(1,t+1);
   t=t-1;
end
sdef = sforce;
for i = 1:dof
sdef(1,i) = sdef(1,i)./ stiffness(1,i);
end
caldef=sdef;
for i=2:dof
caldef(1,i)= caldef(1,i) + caldef(1,i-1);
end
scale=caldef;
svalue=scale(1,1);
for i=1:dof
   scale(1,i)= scale(1,i) ./ svalue(1,1);
end
if scale==ad
break;
end
ad=scale;
end
freq1=0;
for i=1:dof
    freq1 = freq1 + caldef(1,i);
end
freq2=0;
```

```
for i=1:dof
    freq2 = freq2 + scale(1,i);
end
freqf = sqrt((freq2/freq1));
fprintf('\nfrequency of the system is: %0.9f \n\n', freqf);
disp('Mode shape of the system is = '); disp(scale);
end
```

EXAMPLE PROBLEM:

Consider a five storey structure weight of each floor W1 = W2 = W3 = W4 = W5 = 500 KNand stiffness of the floor is K1 = 500 KN/cm,K2 = 500 KN/cm,K3 = 500 KN/cm,K4 = 500 KN/cm,K5 = 500 KN/cm



INPUTDATA:

Enter the number of degree of freedom

5

Enter the masses of the system M1

.5

Enter the masses of the system M2

.5

Enter the masses of the system M3

.5

Enter the masses of the system M4

.5

Enter the masses of the system M5

.5

Enter the stiffness of the system K1

500

Enter the stiffness of the system K2

450

Enter the stiffness of the system K3

400

Enter the stiffness of the system K4

350

Enter the stiffness of the system K5

300

OUTPUT FROM DIFFERENT METHODS:

Frequency of the system by Stodola method is: 8.381470347

Mode shape of the system by Stodola method is =

1.0000 2.0331 3.0167 3.8382 4.3471

Frequency of the system by Holzer method is: 8.381567087

Mode shape of the system by Holzer method is =

1.0000 2.0331 3.0168 3.8383 4.3472

Frequency of the system by influence coefficient method is: 8.381470347

Mode shape of the system influence coefficient method is =

1.0000 2.0331 3.0167 3.8382 4.3471

Frequency of the system by Rayleigh Ritz method is: 8.381470347

Mode shape of the systemby Rayleigh Ritz method is =

1.0000 2.0331 3.0167 3.8382 4.3471

Frequency of the system by Dunkerley Method is: 7.512587421

CONCLUSION:

Calculation of Natural frequency for the multi-degree of freedom system (MDOF) system is mandatory for many design problems. By manual calculation it is time consuming, by simple coding in MATLAB can give the result in no time.

Natural frequency is calculated for a problem of five stories by using different methods in MATLAB and the solution i.e. natural frequency and mode shapes (displacements) are obtained, it is noted that all the valves of natural frequency and mode shapes are same upto 9 decimal points except for the dunkerley's method and this method did not provide mode shapes.

This can be extended by doing higher order frequency of the Multi-degree of freedom system (MDOF) system, where we can obtain frequency greater than natural frequency and its respective mode shapes.

REFERENCES:

[1] S.R Damodaraswamy, S. Kavitha, *Basics of Structural Dynamics and Aseismic Design 5th Edition*, (Phi Learning Pvt. Ltd)

[2] M. Paz, *Structural Dynamics: Theory and Computation* (Springer Science & Business Media, Berlin, 2012).

[3] A.K. Chopra, *Dynamics of Structures* (Prentice Hall, New Jersey, 1995).

[4] S.S. Rao and F.F. Yap, *Mechanical Vibrations* (Addison-Wesley, Boston, 1995), 3rd ed.

[5] B. Balachandran and E. Magrab, *Vibrations* (Cengage Learning, Toronto, 2008), 2nd ed.

[6] S. Bhattacharya, A. Krishna and D. Lombard, *Soil Dynamics and Earthquake Engineering*, (2012)