# IDEALS IN QUOTIENT TERNARY SEMIRING

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**ABSTRACT :** The main aim of this paper is that of extending some well-known theorems in the theory of quotient ternary semi-rings. In this paper, we will make an intensive study of the properties of quotient ternary semi-rings as compared to similar properties of quotient ring.

**Keywords : Quotient ternary semi-ring, Weakly prime ideals, Semi-domain like ternary semiring.**

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## I. INTRODUCTION

Algebraic structures play a prominent role in mathematics with wide ranging applications in many disciplines such as theoretical physics, computer sciences, control engineering, information sciences, coding theory, topological spaces, and the like. Semirings are natural topic in algebra to study because they are the algebraic structure of the set of natural numbers. In structure, semirings lie between semigroups and rings. P.J. Allen [ 1 ] introduced the notion of Q-ideal and a construction process was presented by which one can build one quotient structure of a semiring modulo a Q-ideal. If I is an ideal of a semiring R, then Golan has presented the notion quotient semiring R/I, but this definition is different from the definition of Allen. Here we follow the definition of Golan. The theory of ternary algebraic systems was introduced by D.H. Lehmer [3]. D. Madhusudanarao and G.Srinivasarao [4] investigated and studiedabout special elements in a ternary semirings.The main part of this paper is proving several well-known theorems in the theory of quotient ternary semiring.

## II. PRELIMINARIES

**DEFINITION 2.1 :** A nonempty set T together with a binary operation called addition and a ternary multiplication denoted by [ ] is said to be a ***ternary semi-ring*** if T is an additive commutative semi-group satisfying the following conditions :

i) $[[abc]de] = [a[bcd]e] = [ab[cde]]$,

ii) $[(a + b)cd] = [acd] + [bcd]$,

iii) $[a(b + c)d] = [abd] + [acd]$,

iv) $[ab(c + d)] = [abc] + [abd]$ for all $a; b; c; d; e \in$T.

Throughout Twill denote a ternary semi-ring unless otherwise stated.

**NOTE 2.1.2** : For the convenience we write $x_1x_2x_3$ instead of $\left[x_1x_2x_3\right]$

**NOTE 2.1.3** : Let T be a ternary semi-ring. If A,B and C are three subsets of T , we shall denote the set ABC = $\left\{\Sigma abc : a \in A, b \in B, c \in C\right\}$.

**NOTE 2.1.4** : Let T be a ternary semi-ring. If A,B are two subsets of T , we shall denote the set A + B = $\left\{a+b : a \in A, b \in B\right\}$.

**DEFINITION 2.5 :**A ternary semi-ring T is said to be ***commutativeternary semi-ring*** provided *abc* = *bca* = *cab* = *bac* = *cba* = *acb*for all *a,b,c*$\in$ T.

**DEFINITION 2.6 :**A nonempty subset A of a ternary semiring T is said to be ***ternary ideal*** or simply an ***ideal*** of T if

(1) *a, b* ∈ A implies *a* + *b*∈ A

(2) *b, c* $\in$ T , *a* $\in$A implies *bca*$\in$ A, *bac*$\in$A, *abc*$\in$A.

**NOTE 2.7** : Anonempty subset A of a ternary semiring T is an ideal of T if and only if it is left ideal, lateral ideal and right ideal of T.

**DEFINITION 2.8 :** An ideal A of a ternary semiring T is said to be a ***prime ideal*** of T provided X,Y,Z are ideals of T and XYZ $\subseteq$ A $\Rightarrow$ X $\subseteq$A or Y $\subseteq$A or Z $\subseteq$A.

**EXAMPLE 2.9** :Let T be the ternary semiring of non-negative integers where *a* + *b*= max{*a*, *b*}, [*abc*] = min{*a*, *b*, *c*} Then T does not an identity and every proper ideal in T is prime.

**DEFINITION 2.10 :** An ideal I of a ternary semiring T is called a **k-ideal** if *x* + *y* ∈ I ; *x* ∈ T, *y* ∈ I ⟹*x*∈ I.

**DEFINITION 2.11 :**A ternary semiringTis said to be ***zero divisor free*** (ZDF) iffor *a, b, c* ∈ *T*, [*abc*] = 0 implies that *a* = 0 or *b* = 0 or *c* = 0.

**DEFINITION 2.12 :** A commutative ternary semiring (ring) is called a ***ternary semi-integral*** **(*integral*,** resp.) ***domain***if it is zero divisor free.

**DEFINITION 2.13 :**An element *a* of a ternary semiring T is said to be a ***mid-unit*** provided *xayaz* = *xyz* for all *x*, *y*, *z*∈ T.

**DEFINITION 2.14 :**. An element *a* of a ternary semiring Tis said to be ***invertible*** in Tif there exists an element *b* in *T*(calledthe ***ternary semiring-inverse***of*a*) such that *abt*= *bat* = *tab* = *tba*= *t* for all *t* ∈T.

**DEFINITION 2.15 :**. A ternary semiring (ring) Twith $\left|S\right| \geq 2$ is said to be a ***ternarydivision semiring*****(*ring*,** resp.) if every non-zero element of Tis invertible.

**THEOREM 2.16 : Every ternary division semiring is regular ternary semiring.**

**DEFINITION 2.17 :** A commutative ternary division semiring (ring) is said to be a ***ternary semifield*****(*field*,** resp.), i.e. a commutative ternary semiring (ring) Twith $\left|T\right| \geq 2$, is a ternary semifield

(field) if for every non-zero element *a*of T, there exists an element *b* in Tsuch that $abx= x$ for all $x$ ∈T.

**NOTE 2.18 :**A ternary semi-field T has always an identity.

**EXAMPLE 2.19 :** Denote by $R_0^-$, $Q_0^-$ and $Z_0^-$ the sets of all non-positive real numbers, non-positive rational numbers and non-positive integers, respectively.Then $R_0^-$ and $Q_0^-$ form ternary semi-fields with usual binary addition andternary multiplication and $Z_0^-$ forms only a ternary semi-integral domain but not a ternary semi-field.

**NOTE 2.20 :**The collection of all zero divisors of a ternary semi-ring T will be denoted by Z(T).

**DEFINITION 2.21 :**The subset $\{a \in T : a^n = 0$, for n is an odd positive integer$\}$ of Z(T) consisting of the nilpotent elements of a ternary semiring T will be denoted by nil(T), and it read as nilradical of T.

**DEFINITION 2.22 :** An ideal I of a ternary semiring T will be called a ***partitioning ideal*** (=Q –ideal) if there exists a subset Q of T such that (i) $T = U\{q + I : q \in Q\}$ (ii) if $q_1, q_2 \in Q$ then $(q_1 + I) \cap (q_2 + I) \neq \emptyset \Leftrightarrow q_1 = q_2$

**LEMMA 2.23 :**If *I* is a partitioning ideal of a ternary semiring T, then there exists a unique $q_0 \in Q$ such that $I = q_0 + I$ [ ].

**LEMMA 2.24 :**Let I be a partitioning ideal of a ternary semiring T. If $x \in T$, then there exists a unique $q \in Q$ such that $x + I \subseteq q + I$. Hence $x = q + a$ for some $a \in I$ [ ].

**DEFINITIION 2.25 :**If T and $T^1$ are ternary semi-rings, then a function f from T to $T^1$ is a ternary semi-ring homomorphism if and only if (i) $f(a+b) = f(a) + f(b)$ (ii) $f(abc) = f(a).f(b).f(c)$, $\forall a, b, c \in T$.

**DEFINITION 2.26 :**Let I be a partitioning ideal of a ternary semiring T. Let $T/I_{(Q)} = \{q + I : q \in Q\}$ forms a ternary semi-ring under the following addition " ⊕ " and a ternary multiplication " ⊙ " as $(q_1 + I) \oplus (q_2 + I) = q^1 + I$ where $q^1 \in Q$ is a unique element such that $q_1 + q_2 + I \subseteq q^1 + I$ and $(q_1 + I) \odot (q_2 + I) \odot (q_3 + I) = q_4 + I$ where $q_4 \in Q$ is a unique element such that $q_1 . q_2 .q_3 + I \subseteq q_4 + I$. This ternary semi-ring will be called q ***quotient ternary semi-ring*** of T and denoted by $(T/I_{(Q)}, \oplus, \odot)$ or simply as $T/I_{(Q)}$ [ ].

## III.IDEAL IN A QUOTIENT TERNARY SEMIRING

**LEMMA 3.1 : Let I be an ideal of a ternary semiring I. Then the following hold:**

**(i) If $a \in I$, then $a + I = I$**

**(ii) If *I* is a k-ideal of T and $a \in I$, then $a + I = b + I$ for every $b \in T$ if and only if $b \in I$. In particular, $c + I = I$ if and only if $c \in I$.**

**PROOF :**(i)Since $a + 0 = 0 + a$, we have $a \sim 0$. Hence $a + I = I$.
Suppose I is a k-ideal of T. Assume $a + I = b + I$ for every $b \in T$.$\Rightarrow a + x = b + y$ for some $x, y \in I$. Since I is a k-ideal, we have $b \in I$. Again assume that $b \in I$. Now we show that $a + I = b + I$. It is easy to prove by using condition (i) and other conditions also.

**LEMMA 3.2 : Let I and J be ideals of a ternary semiring T with $I \subseteq J$. Then the following hold:**
**(i) $J/I = \{a + I : a \in J\}$ is an ideal of T/I. In particular , if J is a k-ideal of T, then J/I is a k-ideal**

**of T/I. (ii) If 1 + I∈ J/I, then T/I = J/I (iii) If *a* + I is invertible element of T/I with *a* + I ∈ J/I, then T/I = J/I.**

**PROOF :**(i) Clearly 0 + I ∈ J/I ⟹ J/I ≠ ∅ and is a subset of T/I.

Let *a* + I, *b* + I ∈ J/I and r + I ∈ J/I. Since I is an ideal of T and *a, b* are in T ⟹*a* + *b* ∈ T ⟹ (*a* + *b*) + I ∈ J/I. Since I is an ideal of a ternary semi-ring T, r ∈ J, we have *aar, ara*and *raa* are in T ⟹ (*a* + I ) ( *r* + I ) (*a* + I ) ∈ J/I, (*a* + I ) (*a* + I ) ( *r* + I ) ∈ J/I and ( *r* + I ) (*a* + I )(*a* + I ) ∈ J/I ⟹ J/I is a lateral, right and left of T ⟹ J/I is an ideal of T/I. Assume J be a k-ideal of T. Let us suppose *u* + I ∈ J/I and (*u* + I) + (*v* + I) = (*u* + *v* ) + I ∈ J/I, where *u* ∈ J and *v* ∈ T. Then *u* + *v* + *x* = *c* + *y* for some *x, y* ∈I and *c*∈ *J.* Since J is a k-ideal, we have *v* ∈J. Hence *v* + I ∈J. Thus J/I is a k-ideal of T/I.

(ii)Suppose 1 + I∈J/I. Now we show that T/I = J/I

Since J is an ideal of T, we have J/I ⊆T/I. Suppose *x* + I ∈ T/I. Clearly (*x* + I) (1 + I) (1 + I) ∈ J/I, because J/I is an ideal of T/I. Implies (*x*11 + I) ∈ J/I implies *x* + I ∈ J/I i.e. T/I ⊆ J/I . Hence T/I = J/I .

(iii)Now we show that T/I = J/I. Since J is an ideal of T, we have J/I ⊆ T/I . Suppose *a* + I is an invertible element of T/I and*a* + I ∈ J/I . Suppose *x* + I ∈ T/I . Since *a* + I is an invertible element of T/I, then there exists 1 + I ∈T/I such that (*a* + I) (*x* + I) (1 + I) = *x* + I ⇒(*ax*1 + I) = (*x* + I) ∈ J/I hence T/I ⊆ J/I. Thus T/I = J/I .

**THEOREM 3.3 : Let I anan ideal of a ternary semiring T. Then the following hold:**
**(i)If L an ideal of T/I, then L = J/I for some ideal J of T.(ii)If P is a k-ideal of T with I ⊆ P, then P is a prime ideal of T if and only if P/I is a prime ideal of T/I.(iii)I is a prime k-ideal of T if and only if T/I is a semi-domain. In particular, (0) is prime if and only if only T is a semi-domain.**

**PROOF :**Given I is an ideal of a ternary semi-ring T.

(i)Let us suppose L be an ideal of T/I and Assume that J = {*r*∈ T: *r* + I ∈ L} and let *a* ∈ I. By lemma 3.1, *a* + I = 0 + I ∈ L then I ⊆ J.Let *a, b* ∈ J and *r* ∈ T. Clearly (*a* + I ) + (*b* + I) = (*a* + *b*) + I ∈ L ⇒*a* + *b* ∈ J. It is easy to prove that *aar,* ara*and raa* are in J. Hence J is an ideal of T. From the set construction J, L ⊆ J/I . Since L is an ideal of T/I, we have J/I ⊆ L. Hence L = J/I .

(ii)Given that P is a k-ideal of T with I ⊆ P. Suppose P be a prime ideal of a ternary semiring T. Clearly P/I is an ideal of T/I . Let (*a* + I ), (*b* + I), (*c* + I) ∈ T/I such that (*a* + I ) (*b* + I) (*c* + I) ∈ P/I where *a, b, c* ∈ T. Then *(abc* + I ) = *x* + I for some *x* ∈ P. Also *abc*∈ P. Since P is a k-ideal and P is a prime, we have either *a*∈ P or *b* ∈ P or *c* ∈ P implies (*a* + I ) ∈ P/I or (*b* + I) ∈ P/I or (*c* + I) ∈ P/I by lemma 3.2.

Conversely, suppose that P/I is prime. Let *a, b, c*∈ T such that *abc*∈ P. By lemma 3.1 (*a* + I ) (*b* + I) (*c* + I) = *abc* + I = 0 + I ∈ P/I. Since P/I is a prime ideal, we have (*a* + I ) ∈ P/I or (*b* + I) ∈ P/I or (*c* + I) ∈ P/I ⟹ either *a*∈ P or *b* ∈ P or *c* ∈ P ⟹ P is a prime ideal of T.

(iii)Let I be a prime ideal of T and let (*a* + I ), (*b* + I), (*c* + I) ∈ T/I such that (*a* + I ) (*b* + I) (*c* + I) = *abc*+ I = 0 + I, where *a, b, c* ∈ T. By lemma 3.1, (*a* + I ) = I or (*b* + I) = I or (*c* + I) = I. Hence T/I is a semi-domain or integral domain. The proof of the other implication is similar.

**THEOREM 3.4 : Let P be a proper k-ideal of a ternary semiring T. Then the following hold: (i) P is a maximal k-ideal of T if and only if T/P is a ternarysemifield.**
**(ii)If I is an ideal of T with I ⊆ P, then P is a maximal k-ideal of T if and only if P/I is a maximal ideal of T/I .**

**PROOF :**(i) Let P be a maximal k-ideal of a ternary semiring T. Now we show that T/P is a ternary semifield. It is enough if we show that every non-zero element *a* + P of T/P is invertible. Since *a* + P ≠ P, we have *a*∉ P. Thus P + T*aa* = T then there exists *t* ∈ T and *p*∈ P such that *p* + *taa* = 1 ⟹ (*t* + P) (*a* + P) (*a* + P) = 1 + P. Then *a* + P is left invertible and it is easy to prove that *a* + P is right and lateral invertible and hence *a* + P is invertible. Conversely assume that T/P is a ternary semi-field and P ⊆ J for some k-ideal J of T. Now we show that J = T. Then there is an element $b$∈ J\P such that $b$ + P is invertible in T/P, implies ($b$ + P)($x$ + P)($x$ + P) = 1 + P⟹$bxx$+ P = 1 + P for some $x$ + P ∈ T/P. Since J is a k-ideal, we have 1∈ J we have P is a maximal k-ideal of T.

(ii)Suppose that P is a maximal k-ideal of T and let L be a k-ideal of T/I such that P/I ⊆ U. There exists a k-ideal J of T such that P/I ⊆ U = J/I by theorem 3.3 (i), we have P ⊆ J hence J = T. Thus L = T/I .

**DEFINITION 3.5 :** If T is a ternary semi-ring, then T is Noetherian (resp.Artinian) if any non-empty set of k-ideals of T has a maximal member (resp.minimal member) with respect to set inclusion. This definition is equivalent to the ascending chain condition (resp.descending chain) on k-ideals. It is easy to see that if I and J are k-ideals of T, then I + J is a k-ideal of T and an intersection of a family of k-ideals of T is a k-ideal.

**THEOREM 3.6 : Let I be a k-ideal of a ternary semi-ring T. T is Noetherian (resp.Artinian) if and only if both I and T/I are Noetherian (resp.Artinian).**

**PROOF :**Given I is a k-ideal of a ternary semi-ring T. Suppose that I and T/I are Noetherian. Now we show that T is Noetherian. Let $J_1 \subseteq J_2 \subseteq J_3 \subseteq \ldots \subseteq J_n \subseteq J_{n+1} \subseteq \ldots$ be an ascending chain of k-ideals of T. Then $J_1 \cap I \subseteq J_2 \cap I \subseteq \ldots (J_n \cap I) \subseteq \ldots$ is an ascending chain of k-ideals of I and by Zorn's lemma, there is a positive integer s such that $J_s \cap I = J_{s+i} \cap I \ \forall\ i \in N$. $\frac{J_1+I}{I} \subseteq \frac{J_2+I}{I} \subseteq \ldots \subseteq \frac{J_n+I}{I} \subseteq \frac{J_{n+1}+I}{I} \subseteq \ldots$ is a chain of k-ideals of T/I. Since T/I is Noetherian, we have there is a positive integer $t$ such that $\frac{J_t+I}{I} = \frac{J_{t+i}+I}{I}$ for all positive integer i. Put $u = \max\{s, t\}$.

Now we show that, for each positive integer, $J_u = J_{u+i}$ . Clearly $J_u \subseteq J_{u+I}$ because $J_1 \subseteq J_2 \subseteq J_3 \subseteq \ldots \subseteq J_n \subseteq J_{n+1} \subseteq \ldots$ is an ascending chain of k-ideals of T. Let $x \in J_{u+I} \Longrightarrow x \in I + J_u \Longrightarrow x = a + b$ for some $a \in$ I and $b \in J_u \subseteq J_{u+i}$. Since $J_{u+I}$ is a k-ideal, we have $a \in J_{u+I} \Longrightarrow a \in I + J_u \Longrightarrow a$ and $b$ are in $J_u$ , because $J_u$ is an ideal. We have $J_{u+i} \subseteq J_u$ . Hence $J_u = J_{u+i}$. Thus T is Noetherian.

Conversely assume that T is Noetherian. Now we show that I and T/I are Noetherian. By 3.3, the chain of T/I must be stationary i.e. the ascending chain $J_1/I \subseteq J_2/I \subseteq \ldots \subseteq J_n/I \subseteq J_{n+1}/I \subseteq \ldots$ Of k-ideals of T/I. Since T is Noetherian, the ascending chain become stationary, so T/I isNoetherian. Similarly I is also Noetherian.

**DEFINITION 3.7 :** An ideal I of a ternary semiring T is ***strong irreducible*** if for ideals J and K of J, the inclusion J∩K ⊆ I implies either J ⊆ I or K ⊆ I.

**LEMMA 3.8 : Let I be an ideal of a ternary semiring T. If J, K and L are k-ideals of T containing I, then (J/I) ∩ (K/I) = (L/I) if and only if J∩K = L.**

**PROOF :**Suppose (J/I) ∩ (K/I) = (L/I). Now we show that J∩K = L. Let $x$ ∈ J∩K ⟹$x$ + I ∈ (J/I) ∩ (K/I) = L/I ⟹$x + a_1 = y + a_2$ for some $y$ ∈ L and $a_1, a_2$ ∈ I. Since L is a k-ideal, we have $x$ ∈ L. Thus J∩K ⊆ L. Again suppose that $x$ ∈ L ⟹$x$ + I ∈ L + I = (J/I) ∩ (K/I)⟹$x + b_1 = y + b_2$ and $x + b_1 = z + b_3$ for some $b_1, b_2, b_3$∈ I, $y$ ∈ J and $z$∈ K. Since K and J are k-ideals, we have $x$∈ J ∩K ⟹L ⊆ J∩K. Suppose $x$ ∈ J∩K ⟹$x$ + I ∈ (J/I) ∩ (K/I) = L/I ⟹$x$ ∈ L, we have J ∩K ⊆ L. Hence J∩K = L.

**THEOREM 3.9 : Let T be a ternary semiring, I be an ideal of T and J be a strongly irreducible k-ideal of T with I ⊆J. Then J/I is a strongly irreducible ideal of T/I.**

**PROOF :**Let M and N be k-ideals of T/I such that M∩N ⊆ J/I. Then there are k-ideals K, H of T such that M = K/I and N = H/I. By theorem 3.3 and lemma 3.8, K∩ H ⊆ J. Since J is strongly irreducible, we have K ⊆ J or H ⊆ J. Hence N = H/I or M = K/I ⟹ N ⊆ J/I or M ⊆ J/I. Thus J/I is strongly irreducible.

**DEFINITION 3.10 :**A ternary semiring T is called ***semi-domain like ternary semiring,*** if Z(T) ⊆ nil(T).

**NOTE 3.11 :** A commutative ternary semiring is that an ideal P is prime if and only if T/P is a ternary semi-domain.

**THEOREM 3.12 : Let P be a proper k-ideal of a ternary semiring T. Then P is primary if and only if T/P is a semi-domain like ternary semiring.**

**PROOF :** Given P is a proper k-ideal of a ternary semiring T. Supposet P be a primary ideal of a ternary semiring T. Now we show that T/P is a semi-domainlike ternary semiring. It is enough if we show Z(T/P) ⊆ nil(T). Let $a$ + P ∈Z(T/P) then there exists a non-zero element $b$ + P, $c$ + P of T/P such that ($a$ + P) ($b$ + P) ($c$ + P) = 0 + P ⟹$abc$∈ P and by lemma 3.1 either $b$ + P = P or $c$ + P = P, is a contradiction. Since P is a primary, we have $(a + P)^n = a^n + P = 0 + P$ ⟹$a$ + P ∈ nil(T/P) ⟹ T/P is a semi-domain-like ternary semi-ring.

Conversely assume that T/P is a semi-domain-like ternary semi-ring. Now we show that P is a primary. Let $abc$∈ P, where $a, b, c$ ∈ T. Then ($a$ + P) ($b$ + P) ($c$ + P) = $abc$+ P = 0 + P. By lemma 3.1, $a$ + P = P then $a$∈ P. Similarly $b$ + P = P. We may assume that $a$ + P ≠ P and

**THEOREM 3.13 : Let I be an ideal of a ternary semiring T. Then T/$\sqrt{I}$ is semi-domain-like if and only if T/$\sqrt{I}$ is a ternary semi-domain. In particular, T/nil(T) is semi-domain-like if and only if T/nil(T) is a ternary semi-domain.**

**PROOF :**Given I is an ideal of a ternary semi-ring. Suppose T/$\sqrt{I}$ is a ternary semi-domain-like. Now we show that T/$\sqrt{I}$ is a ternary semi-domain. Suppose $(a + \sqrt{I})(b + \sqrt{I})(c + \sqrt{I}) = (abc + \sqrt{I}) = 0 + \sqrt{I}$ ∈ T/$\sqrt{I}$ with $a + \sqrt{I} \neq 0 + \sqrt{I}$. Then $abc \in \sqrt{I}$ and by lemma 2.1, $a \notin \sqrt{I}$. Since T/$\sqrt{I}$ is ternary semi-domain-like and by lemma 3.9, we have $\sqrt{I}$ is primary. ⟹$b^m \in \sqrt{I}$ for some positive integer m ⟹$b \in \sqrt{I}$ ⟹$b$ + I = 0 + I then T/$\sqrt{I}$ is a semidomain. The other implication is similar.

**DEFINITION 3.14 :**Let T be a ternary semiring. A proper ideal I of T is said to be **weakly primary (resp.weakly prime)** if 0 ≠ $abc$∈ I ⟹$a$∈I or$b^m$∈ I or $c^n$∈ I for some odd positive integers $m$ and $n$ (resp. $a$ ∈ I or $b$ ∈ I or $c$ ∈ I)

**NOTE 3.15 :**(i) A primary ideal (resp.prime ideal) is a weakly primary (resp.prime)ideal.

(ii)A weakly primary ideal (resp.a weakly prime ideal) need not be primary (resp.prime). Clearly, every weakly prime is weakly primary.

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**THEOREM 3.16 : Let T be a ternary semiring, I be an ideal of T and P be a k-ideal of T with I ⊆ P. Then the following hold:**

**(i)If P is a weakly primary ideal (resp.weakly prime ideal) of T, then P/I is a weakly primary ideal (resp.weakly prime ideal) of T/I .**

**(ii) If both I and P are weakly primary (resp.weakly prime ideal) ideal, then P is a weakly primary (resp. weakly prime) ideal.**

**PROOF :**(i) Assume that P is weakly primary. Now we show that P/I is a weakly primary ideal of T/I. Let $a$ + I, $b$ + I ∈ T/I . Suppose 0 + I ≠ ($a$ + I) ($b$ + I) ($c$ + I) = $abc$+ I ∈ P/I ⟹ by lemma 3.1, 0 ≠ $abc$∈ P. Since P is a primary, we have $a$∈P or$b^m$∈ P or $c^n$∈ P for some odd positive integers $m$ and $n$. If $a$∈ P then by lemma 3.2 $a$ + I ∈ P/I. Suppose $b^m$∈ P ⟹$(b + I)^m = b^m$ + I ∈ P/I. Similarly $c^n$ + I ∈ P/I. Thus P/I is a weakly primary ideal of T/I . The other implication is similar.

*(ii)Let I and P/I be weakly primary ideal. Now we show that P is a weakly primary ideal.*

Suppose 0 ≠ $abc$∈ P, where $a, b, c$∈ T. If $abc$∈I and since I is a weakly primary ideal, we have $a$∈ I ⊆ P or $b^m$∈ I ⊆ P or $c^n$∈I ⊆ P for some odd positive integers m and n. Suppose $abc$∈ I, by lemma

3.1, $0 + I \neq (abc + I) = (a + I)(b + I)(c + I) \in P/I \Longrightarrow (a + I) \in P/I$ or $(b + I)^m = b^m + I \in P/I$ for some odd positive integer m or $(c + I)^n = c^n + I \in P/I$ for some odd positive integer n, because P/I is a weakly primary. If $a + I \in P/I$ then $a \in P$ by lemma 3.1. If $b^m + I \in P/I$ then $b^m \in P$ and $c^n + I \in P/I$ then $c^n \in P$. Hence P is weakly primary.

**DEFINITION 3.17 :** Let I be an ideal of a ternary semi-ring T. An element $a \in T$ is called **weakly prime** toif $0 \neq raa \in I$ ( $r \in T$) $\Longrightarrow r \in I$. A proper ideal I of a ternary semi-ring T is called **weakly primal** if the set $P = P(I) \cup \{0\}$ form an ideal, where P(I) is the set of elements of T that are not weakly prime to I. This ideal is called the **weakly adjoint ideal** P of I

**THEOREM 3.18 : Let J be a weakly prime k-ideal of a ternary semi-ring T and I be a proper k-ideal of T with J ⊆ I. Then I is a weakly primal ideal of T if and only if I/J is a weakly primal ideal of T/J. In particular, there is a bijective correspondence between the weakly primal ideals of T containing J and the weakly primal ideals of T/J.**

**PROOF :**Give J is a weakly prime k-ideal of a ternary semi-ring T and I is a proper k-ideal of T with $J \subseteq I$. Suppose I is a weakly primal ideal of T. Now we show that I/J is a weakly primal ideal of T/J. Since J is a weakly prime k-ideal of a ternary semi-ring T and I is a proper k-ideal of T with $J \subseteq I$ then by [3, Remark 3.2 and theorem 3.4], $J \subseteq P$ and P is a weakly prime ideal of T. By theorem 3.17 P/J is a weakly prime ideal of T/J. It is enough if we show $(P/J)^*$ satisfies (*). Let $a + J \in (P/J)^*$ where $a \in P$. Since $0 + J \neq a + J$ and by lemma 3.1, $a \neq 0$ and $a$ is not weakly prime to I. Then there exists $r \in T - I$ such that $0 \neq raa \in I$. If $0 \neq raa \in J$, then J is a weakly prime gives $r \in J$, is a contradiction to the fact $r \notin I$ and $J \subseteq I$. Then we may assume that $0 \neq raa \notin J \Longrightarrow 0 \neq (r + J)(a + J)(a + J) \in I/J$ with $(r + J) \notin I/J$ □$a + J$ is not a weakly prime to I/J, where $b + J \neq 0 + J$ is not weakly prime to I/J, where $b$ □ I. Then there exists $c + J$ □ $T/J - I/J$ such that $0 \neq (c + J)(b + J)(d + J) = cbd + J$ □ I/J □ $cbd$□ I with $c$ □ I by lemma 2.1. Thus $b \neq 0$ is not a weakly prime to I. Hence $b + J$ □ $(P/J)^*$. Therefore I/J is a weakly primal ideals of T.

Conversely suppose that I/J is a P/J – weakly primal ideal of T/J. We show that I is a P-weakly primal ideal of T. By [3, Theorem 3.4] and Theorem 3.17, P is a weakly prime ideal of T. It is enough is we show $P^*$ satisfies (*). Let $a$ □ $P^*$. by theorem [3, remarks 3.2], we can assume that $a$ □ J. As J is a weakly prime ideal of $0 \neq a + J$ □ P/J, there exists $r + J$ □ $T/J - I/J$ such that $0 \neq (a + J)(a + J)(r + J) = aar + J$ □ I/J □ $0 \neq aar$□ I with r □ J. Thus $a$ is not a weakly prime to I. Now we assume that $a$ is not a weakly prime to I ( so$a \neq 0$). We show that $a$ □P . Assume that $a$ □ I. Then there is an element $r$ is in T\I such that $0 \neq aar$□ I implies $0 \neq (r + J)(a + J)(a + J) = raa + J$ □ I/J with $r + J$ □ I/J. Hence $a + J$ □ $(P/J)^*$ since I/J is a P/J- weakly primal. Thus $a$ □ P. Hence I is a P-weakly primal ideal of T.

**CONCLUSION :**In this paper mainly we studied about ideals in quotient ternary semiring.

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