Compositions of Fuzzy TΓ-Ideals in Ternary Γ-Semi ring

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Abstract—
The purpose of this paper is to introduce different types of operations on fuzzy TΓ-ideals of ternary Γ-semirings and to prove subsequently these operations give rise to different structures on some classes of fuzzy TΓ-ideals of ternary Γ-semirings. A characterization of a regular ternary Γ-semiring has also been obtained in terms of fuzzy subsets.

Keywords--- **Ternary Γ-Semiring, Regular ternary Γ-semiring, left fizzy TΓ-Ideal, right fizzy TΓ-Ideal, lateral fizzy TΓ-Ideal, fizzy TΓ-Ideal.**

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I. INTRODUCTION

The theory of fuzzy sets was first inspired by Zadeh [9]. Fuzzy set theory has been developed in many directions by many scholars and has evoked great interest among mathematicians working in different fields of mathematics. Fuzzy ideals in rings were introduced by Liu [5] and it has been studied by several authors. Jun [2] and Kim and Park [4] have also studied fuzzy ideals in semirings. In the year 2007, [6] we have introduced the notions of fuzzy ideals and fuzzy quasi-ideals in ternary semirings. In the year 2015, SajaniLavanya and MadhusudhanaRao[6, 7, 8] introduced the notion of ternary Γ-Semirings.

2. PRELIMINARIES

Definition 2.1: Let T and Γ be two additive commutative semigroups. T is said to be a ***Ternary Γ-semiring***if there exist a mappingfromT×Γ×T×Γ×TtoTwhichmaps $(x_1, \alpha, x_2, \beta, x_3) \rightarrow [x_1\alpha x_2\beta x_3]$ satisfying the conditions :
i) $[[a\alpha b\beta c]\gamma d\delta e] = [a\alpha[b\beta c\gamma d]\delta e] = [a\alpha b\beta[c\gamma d\delta e]]$
ii)$[(a + b)\alpha c\beta d] = [a\alpha c\beta d] + [b\alpha c\beta d]$
iii) $[a\alpha (b + c)\beta d] = [a\alpha b\beta d] + [a\alpha c\beta d]$
iv) $[a\alpha b\beta(c + d)] = [a\alpha b\beta c] + [a\alpha b\beta d]$ for all $a, b, c, d \in$ T and $\alpha, \beta, \gamma, \delta \in \Gamma$.

Definition 2.2: An element 0of a ternary Γ-semiring T is said to be an ***absorbing zero*** of T provided $0 + x = x = x + 0$and $0\alpha a\beta b = a\alpha 0\beta b = a\alpha b\beta 0 = 0\ \forall\ a, b, x \in$ T and $\alpha, \beta \in \Gamma$.

Definition 2.3: Let T be ternary Γ-semiring. A non empty subset 'S' is said to be a ***ternary Γ-sub semiring*** of T if S is an additive sub semi group of T and $a\alpha b\beta c \in$ S for all $a,b,c \in$ S and $\alpha, \beta \in \Gamma$.

Definition 2.4: A nonempty subset A of a ternary Γ-semiring T is said to be ***left ternary Γ-ideal*** or simply ***left TΓ-ideal*** of T if (1) $a, b \in$ A implies $a + b \in$ A. (2) $b, c \in$ T, $a \in$ A, $\alpha, \beta \in \Gamma$ implies $b\alpha c\beta a \in$ A.

Definition 2.5: A nonempty subset of a ternary Γ-semiring T is said to be a ***lateral ternary Γ-ideal*** or simply ***lateral TΓ-ideal*** of T if (1) $a, b \in$ A $\Rightarrow a + b \in$ A. (2) $b, c \in$ T, $a \in$ A, $\alpha, \beta \in \Gamma \Rightarrow b\alpha a\beta c \in$ A.

Definition 2.6: A nonempty subset A of a ternary Γ-semiring T is a ***right ternary Γ-ideal*** or simply ***right TΓ-ideal*** of T if (1) $a, b \in$ A $\Rightarrow a + b \in$ A. (2) $b, c \in$ T , $a \in$ A, $\alpha, \beta \in \Gamma \Rightarrow a\alpha b\beta c \in$ A.

Definition2.7:A non empty subset A of a ternary Γ-semiring T is said to be ***ternary Γ-ideal*** or simply ***TΓ-ideal*** of T if

(1) $a, b \in A \Rightarrow a + b \in A$

(2) $b, c \in T$, $a \in A$, $\alpha, \beta \in \Gamma \Rightarrow b\alpha c\beta a \in A$, $b\alpha a\beta c \in A$, $a\alpha b\beta c \in A$.

For more on preliminaries we may refer to the references and their references.

3. *Operations on fuzzy TΓ-ideals*:

Definition 3.1: A non-empty fuzzy subset μ of a ternary Γ-semiring *T*is called a ***fuzzy left*(*lateral, right*)*ternary* Γ-*ideal*** or simply ***fuzzy left T*Γ-*ideal*** of *T* if

(i) $\mu(x + y) \geq \min\{\mu(x), \mu(y)\}$

(ii) $\mu(x\gamma y\delta z) \geq \mu(z)[\mu(x\gamma y\delta z) \geq \mu(y), \mu(x\gamma y\delta z) \geq \mu(x)]\ \forall x, y, z \in T,\ \forall \gamma, \delta \in \Gamma$.

Definition3.2: A non-empty fuzzy subset μ of a ternary Γ-semiring *T* is called a ***fuzzy* Γ-*ideal*** of *T* if

(i) $\mu(x + y) \geq \min\{\mu(x), \mu(y)\}$

(ii) $\mu(x\gamma y\delta z) \geq \mu(x) \vee \mu(y) \vee \mu(z)$ for any $x, y, z \in T$ and $\gamma, \delta \in \Gamma$.

Note3.3:A non-empty fuzzy subset μ of a ternary Γ-semi ring *T* is called a ***fuzzy* Γ-*ideal*** of *T* if it is a fuzzy left TΓ- ideal, a fuzzy right TΓ-ideal and a fuzzy lateral TΓ-ideal of *T.*

Note 3.4: A fuzzy T**Γ**-ideal of a ternary Γ-semi ring T is a non-empty fuzzy subset of T which is a fuzzy left TΓ-ideal, fuzzy lateral TΓ-ideal and fuzzy right TΓ-ideal of T. Throughout this thesis unless otherwise mentioned T denotes a ternary Γ-semiring with unities and FLTΓI(T),FMTΓI(T), FRTΓI(T) and FTΓI(T) denotes respectively the set of all fuzzy left TΓ-ideals, the set of all fuzzy lateral TΓ-ideals, the set of all fuzzy right TΓ-ideals and the set of all fuzzy TΓ-ideals of the ternary Γ-semi ring T. Also we consider that (0) = 1 for a fuzzy left TΓ- ideal(fuzzy lateral TΓ-ideal, fuzzy right TΓ-ideal and fuzzy TΓ-ideal) μ of a ternary Γ-semi ring T.

Definition 3.5: Let T be a ternary Γ-semiring and $\mu_1, \mu_2, \mu_3 \in FLT\Gamma I(T)$ [FMTΓI(T), FRTΓI(T) FTΓI(T)]. Then the $\mu_1 + \mu_2$ sum and the ternary product $\mu_1\Gamma\mu_2\Gamma\mu_3$ and composition $\mu_1 o \mu_2 o \mu_3$ of μ_1 , $\mu_2\, \mu_3$ are defined as follows:

$$(\mu_1 + \mu_2)(x) = \begin{cases} \displaystyle\sup_{x=u+v} \{\min[\mu_1(u), \mu_2(v)] : u, v \in T] \\ 0 \text{ if for any } u,\ v \in T,\ u+v \neq x \end{cases}$$

$$(\mu_1\Gamma\mu_2\Gamma\mu_3)(x) = \begin{cases} \sup_{x=u\gamma v\delta w}[\min\{\mu_1(u), \mu_2(v), \mu_3(w)\} : u, v, w \in T; \gamma, \delta \in \Gamma] \\ 0, \text{ if for any } u, v, w \in T \text{ and for any } \gamma, \delta \in \Gamma, x \neq u\gamma v\delta w \end{cases}$$

$$= \begin{cases} \displaystyle\bigvee_{x = u\gamma v\delta w} [\{\mu_1(u) \wedge \mu_2(v) \wedge \mu_3(w)\} : u, v, w \in T; \gamma, \delta \in \Gamma] \\ 0, \text{ if for any } u, v, w \in T \text{ and for any } \gamma, \delta \in \Gamma, x \neq u\gamma v\delta w \end{cases}$$

$$(\mu_1 o \mu_2 o \mu_3)(x) = \begin{cases} \displaystyle\sup_{x=\sum_{i=1}^{n} u_i\alpha_i v_i\beta_i w_i} [\min_{1\leq i\leq n}\{\mu_1(u_i), \mu_2(v_i), \mu_3(w_i)\} : u_i, v_i, w_i \in T; \alpha_i, \beta_i \in \Gamma] \\ 0, \text{ if for any } u_i, v_i, w_i \in T; \alpha_i, \beta_i \in \Gamma, x \neq \displaystyle\sum_{i=1}^{n} u_i\alpha_i v_i\beta_i w_i \end{cases}$$

$$= \begin{cases} \displaystyle\bigvee_{x = \sum_{i=1}^{n} u_i\alpha_i v_i\beta_i w_i} [\{\mu_1(u_i) \wedge \mu_2(v_i) \wedge \mu_3(w_i)\} : u_i, v_i, w_i \in T; \alpha_i, \beta_i \in \Gamma] \\ 0, \text{ if for any } u_i, v_i, w_i \in T; \alpha_i, \beta_i \in \Gamma, x \neq \displaystyle\sum_{i=1}^{n} u_i\alpha_i v_i\beta_i w_i \end{cases}$$

Since T contains 0, in the above definition the case $x \neq u + v$ for any u, v ∈ T does not arise. Similarly, since T contains left, lateral, right unity, the case $x \neq \sum_{i=1}^{n} u_i \alpha_i v_i \beta_i w_i$ for any $u_i, v_i, w_i \in T; \alpha_i, \beta_i \in \Gamma$ does not arise. In case of product of μ_1, μ_2, μ_3 if T has left, lateral and right unity, then the case $x \neq u\alpha v\beta w$ for any u, v, w ∈ T and $\alpha, \beta \in \Gamma$ does not arise. i.e. in other words there are u, v, w ∈ T and $\alpha, \beta \in \Gamma$ such that $x = u\alpha v\beta w$.

Theorem 3.6: **In a ternaryΓ-semiring T the following are equivalent.**

(i) **μ is a fuzzy left(lateral, right)TΓ-ideals of T**

(ii) **$\chi + \mu \subseteq \mu$ and $\chi\Gamma\chi\Gamma\mu \subseteq \mu$,($\chi\Gamma\mu\Gamma\chi \subseteq \mu$, $\mu\Gamma\chi\Gamma\chi \subseteq \mu$)where χ is the characteristic function of T.**

Proof **:** Let μ be a fuzzy left TΓ-ideal of T. Let $a \in$ T. Suppose there exist $x, y, z \in$ T and $\gamma, \delta \in \Gamma$ such that $a = x\gamma y\delta z$. Then, since μ is a fuzzy left TΓ-ideal of T, we have

$$(\chi+\mu)(x+y) = \sup_{x+y=p+q} [\min[\chi(p),\mu(q)] : p,q \in T]$$
$$\leq \sup_{x=u+v,\, y=s+t} [\min[\chi(u+s),\mu(v+t) : u,v,s,t \in T]$$
$$\leq \sup_{x=u+v,\, y=s+t} [\min[\min[\chi(u),\chi(s)],\min[\mu(v),\mu(t)]] : u,v,s,t \in T]$$
$$= \sup_{x=u+v,\, y=s+t} [\min[\min[\chi(u),\ \mu(v)],\min[\chi(s),\mu(t)]] : u,v,s,t \in T]$$
$$=\min[\sup_{x=u+v}[\min[\chi(u),\ \mu(v)],\sup_{y=s+t}[\min[\chi(s),\mu(t)]] : u,v,s,t \in T]$$
$$=\min[(\chi+\mu)(x),(\chi+\mu)(y)] = \min[\min[\chi(x),\mu(x)],\min[\chi(y),\mu(y)]$$
$$=\min[\min[1,\mu(x)],\min[1,\mu(y)] = \min[\min[\mu(x),\mu(y)]]$$
$$=\min[\mu(x),\mu(y)]$$

Therefore $\chi + \mu \subseteq \mu$. And

$$(\chi\Gamma\chi\Gamma\mu)(a) = \sup_{a=x\gamma y\delta z}[\min\{\chi(x),\chi(y),\mu(z)\}]$$
$$= \sup_{a=x\gamma y\delta z}[\min\{1,1,\mu(z)\}] \quad .$$
$$= \sup_{a=x\gamma y}\{\mu(z)\} = \mu(z)$$

Now since μ is a fuzzy left TΓ-ideal, $\mu(x\gamma y\delta z) \geq \mu(z) \forall x,y,z \in T$ and $\gamma,\delta \in \Gamma$.

So in particular, $\mu(z) \leq \mu(a)$ for all $a = x\gamma y\delta z$.

Hence $\sup_{a=x\gamma y\delta z} \mu(z) \leq \mu(a)$. Thus $\mu(a) \geq (\chi\Gamma\mu\Gamma\mu)(a)$.

If there do not exist $x,y,z \in T, \gamma,\delta \in \Gamma$ such that $a = x\gamma y\delta z$ then $(\chi\Gamma\chi\Gamma\mu)(a) = 0 \leq \mu(a)$. Therefore $\chi\Gamma\chi\Gamma\mu \subseteq \mu$.

Conversely, suppose that $\chi\Gamma\chi\Gamma\mu \subseteq \mu$. Let $x, y, z \in$ T and $\gamma, \delta \in \Gamma$ such that $a = x\gamma y\delta z$.

Then $\mu(x\gamma y\delta z) = \mu(a) \geq (\chi\Gamma\chi\Gamma\mu)(a)$. Now

$$(\chi\Gamma\chi\Gamma\mu)(a) = \sup_{a=x\gamma y\delta z}[\min\{\chi(x),\chi(y),\mu(z)\}]$$
$$= \sup_{a=x\gamma y\delta z}[\min\{1,1,\mu(z)\}] \quad .$$
$$= \sup_{a=x\gamma y\delta z}\{\mu(z)\} = \mu(z)$$

Hence $\mu(x\gamma y\delta z) \geq \mu(z) \forall x, y.z \in T$ and $\gamma.\delta \in \Gamma$. Therefore μ is a fuzzy left TΓ-ideal of T.

Similarly, we can prove the remaining parts of the statement.

Theorem 3.7**: In a Γ-semi group S the following are equivalent.**

(i) **μ is a fuzzy TΓ-ideals of S**

(ii) **$\chi + \mu \subseteq \mu$ and $\chi\Gamma\chi\Gamma\mu \subseteq \mu$, $\chi\Gamma\mu\Gamma\chi \subseteq \mu$ and $\mu\Gamma\chi\Gamma\chi \subseteq \mu$, where χ is the characteristic function of T.**

Proof **:** By using the theorems 3.6, we can find the proof of the theorem easily.

Theorem 3.8: Let μ_1, $\mu_2 \in$ FLTΓI(T)[FMTΓI(T), FRTΓI(T), FTΓI(T)]. Then $\mu_1 + \mu_2 \in$ FLTΓI(T)[FMTΓI(T), FRTΓI(T), FTΓI(T)].

Proof: $(\mu_1+\mu_2)(0) = \vee_{0=u+v}[\wedge[\mu_1(u),\mu_2(v)] : u,v\in T] \geq \wedge[\mu_1(0),\mu_2(0)] : u,v\in T] = 1 \neq 0$. Thus $\mu_1+\mu_2$ is nonempty and $(\mu_1+\mu_2)(0)=1$. Let $x, y, z\in$ T and $\alpha, \beta\in\Gamma$. Then

$$(\mu_1+\mu_2)(x+y) = \vee_{x+y=p+q}[\wedge[\mu_1(x),\mu_2(y)] : p,q\in T]$$

$$\geq \vee_{x+y=u+v,\, y=s+t}[\wedge[\mu_1(u+s),\mu_2(v+t)] : u,s,v,t\in T]$$

$$\geq \vee_{x+y=u+v,\, y=s+t}[\wedge[\wedge[\mu_1(u),\mu_1(s)],\wedge[\mu_2(v),\mu_2(t)] : u,s,v,t\in T]$$

$$= \vee_{x+y=u+v,\, y=s+t}[\wedge[\wedge[\mu_1(u),\mu_2(v)],\wedge[\mu_1(s),\mu_2(t)] : u,s,v,t\in T]$$

$$= \wedge[\vee_{x+y=u+v}[\wedge[\mu_1(u),\mu_2(v)], \vee_{y=s+t}\wedge[\mu_1(s),\mu_2(t)]] : u,s,v,t\in T]$$

$$= \wedge[(\mu_1+\mu_2)(x),(\mu_1+\mu_2)(y)]$$

Again $(\mu_1+\mu_2)(x\alpha y\beta z) = \vee_{x\alpha y\beta z=p+q}[\wedge[\mu_1(p),\mu_2(q)]]$

$$\geq \vee_{z=u+v}[\wedge[\mu_1(x\alpha y\beta u),\mu_2(x\alpha y\beta v)]]$$

[Since $x\alpha y\beta z = x\alpha y\beta(u+v) = x\alpha y\beta u + x\alpha y\beta v$]

$$\geq \vee_{z=u+v}[\wedge[\mu_1(u),\mu_2(v)]] = (\mu_1+\mu_2)(z).$$

Hence, $\mu_1+\mu_2 \in FLT\Gamma I(T)$. Similarly, one can prove the remaining parts.

Theorem 3.9: **Let $\mu_1, \mu_2, \mu_3\in$ FLTΓI(T)[FMTΓI(T), FRTΓI(T), FTΓI(T)]. Then**

(i) $\mu_1+\mu_2 = \mu_2+\mu_1$

(ii) $(\mu_1+\mu_2)+\mu_3 = \mu_1+(\mu_2+\mu_3)$.

(iii) $\chi+\mu_1 = \mu_1 = \mu_1+\chi$ **where** χ **is a fuzzy TΓ-ideal of T, defined by** $\chi(x)=\begin{cases}1 & \text{if } x=0\\ 0 & \text{if } x\neq 0\end{cases}$

(iv) $\mu_1+\mu_1=\mu_1$

(v) $\mu_1\subseteq\mu_1+\mu_2$ **and**

(vi) $\mu_1\subseteq\mu_2 \Rightarrow \mu_1+\mu_3\subseteq\mu_2+\mu_3$.

Proof: (i) $(\mu_1+\mu_2)(x) = \vee_{x=u+v}[\wedge[\mu_1(u),\mu_2(v)] : u,v\in T]$

$$= \vee_{x=u+v}[\wedge[\mu_2(v),\mu_1(u)] : u,v\in T] = (\mu_2+\mu_1)(x)$$

Therefore, $\mu_1+\mu_2 = \mu_2+\mu_1$

(ii) Let $x\in$ T

$$[(\mu_1+\mu_2)+\mu_3](x) = \vee_{x=u+v}[\wedge[(\mu_1+\mu_2)(u),\mu_3(v)] : u,v\in T]$$

$$= \vee_{x=u+v}[\wedge[\vee_{u=p+q}[\wedge[\mu_1(p)+\mu_2(q) : p,q\in T],\mu_3(v)] : u,v\in T]]$$

$$= \vee_{x=u+v}[\vee_{u=p+q}[\wedge[\mu_1(p)+\mu_2(q),\mu_3(v)]]$$

$= \vee_{x=p+q+v} [\wedge[\mu_1(p)+\mu_2(q), \mu_3(v)]]$

Similarly, we can deduce that $[\mu_1+(\mu_2+\mu_3)](x) = \vee_{x=p+q+v} [\wedge[\mu_1(p)+\mu_2(q), \mu_3(v)]]$

Therefore, $(\mu_1+\mu_2)+\mu_3 = \mu_1+(\mu_2+\mu_3)$.

(iii) for any $x\in$ T, $(\chi+\mu_1)(x) = \vee_{x=u+v} [\wedge[\chi(u), \mu_1(v)] : u, v \in T] = \wedge[\chi(0), \mu_1(x)] = \mu_1(x)$.

Thus, $\chi+\mu_1 = \mu_1$ and from (i) $\chi+\mu_1 = \mu_1 = \mu_1+\chi$.

(iv) Let $x\in$ T. Then $(\mu_1+\mu_1)(x) = \vee_{x=u+v} [\wedge[\mu_1(u), \mu_1(v)] : u, v \in T]$

$= \vee_{x=u+v} \mu_1(u++v) = \mu_1(x)$. Hence $\mu_1+\mu_1 \subseteq \mu_1$

Again $\mu_1(x) = \wedge[\mu_0(0), \mu_1(x)] \le \vee_{x=u+v} [\wedge[\mu_1(u), \mu_1(v)] : u, v \in T] = (\mu_1+\mu_1)(x)$.

Therefore, $\mu_1 \subseteq \mu_1+\mu_1$. Consequently, $\mu_1+\mu_1 = \mu_1$.

(v) Let $x\in$ T. Then $(\mu_1+\mu_1)(x) = \vee_{x=u+v} [\wedge[\mu_1(u), \mu_1(v)] : u, v \in T]$

$> \wedge[\mu_1(u), \mu_1(0)] = \mu_1(x)$

Therefore, $\mu_1 \subseteq \mu_1+\mu_2$

(vi) $\mu_1 \subseteq \mu_2$, $x\in$ T. Then $(\mu_1+\mu_3)(x) = \vee_{x=u+v} [\wedge[\mu_1(u), \mu_3(v)] : u, v \in T]$

$\le \vee_{x=u+v} [\wedge[\mu_2(u), \mu_3(v)] : u, v \in T] = (\mu_2+\mu_3)(x)$

Hence $\mu_1+\mu_3 \subseteq \mu_2+\mu_3$.

***Theorem 3.10*: Let μ_1, μ_2, $\mu_3\in$ FLTΓI(T)[FMTΓI(T), FRTΓI(T), FTΓI(T)]. Then $\mu_1 o \mu_2 o \mu_3$ ∈FLTΓI(T)[FMTΓI(T), FRTΓI(T), FTΓI(T)].**

Proof: Since $\mu_1 o \mu_2 o \mu_3$ (0)

$$= \bigvee_{0=\sum_{i=1}^{n} u_i\alpha_i v_i\beta_i w_i} [\bigwedge_{1\le i\le n} [\wedge[\mu_1(u_i), \mu_2(v_i), \mu_i(w_i)]] : u_i, v_i, w_i \in T, \alpha_i\beta_i \in \Gamma, n \in Z^+]$$

$$\ge \wedge[\mu_1(0), \mu_2(0), \mu_3(0)] = 1 \ne 0 \text{ (since } \mu_1(0) = \mu_2(0) = \mu_3(0) = 1\text{]}.$$

Therefore, $\mu_1 o \mu_2 o \mu_3$ is non empty and ($\mu_1 o \mu_2 o \mu_3$)(0) = 1.

Now, for $x, y, z\in$ T,

($\mu_1 o \mu_2 o \mu_3$)$(x+y)$

$$= \bigvee_{x+y=\sum_{i=1}^{n} u_i\alpha_i v_i\beta_i w_i} [\bigwedge_{1\le i\le n} [\wedge[\mu_1(u_i), \mu_2(v_i), \mu_i(w_i)]] : u_i, v_i, w_i \in T, \alpha_i\beta_i \in \Gamma, n \in Z^+]$$

$$\geq \vee[\underset{1\leq k\leq l}{\overset{\wedge}{1\leq i\leq m}}[\wedge[\wedge[\mu_1(u_i),\mu_2(v_i),\mu_3(w_i)],\wedge[\mu_1(p_k),\mu_2(q_k),\mu_3(r_k)],\text{for } x=\sum_{i=1}^{m}u_i\alpha_i v_i\beta_i w_i,$$

$$y=\sum_{k=1}^{l}p_k\alpha_k q_k\beta_k r_k,\ u_i,v_i,w_i,p_k,q_k,r_k\in T,\alpha_i,\beta_i,\alpha_k,\beta_k\in\Gamma,\ \ m,l\in Z^+].$$

$$=\wedge[\underset{x=\sum_{i=1}^{m}u_i\alpha_i v_i\beta_i w_i}{\vee}[\underset{1\leq i\leq m}{\wedge}[\wedge[\mu_1(u_i),\mu_2(v_i),\mu_3(w_i)]]:u_i,v_i,w_i\in T,\alpha_i,\beta_i\in\Gamma,\ \ m\in Z^+],$$

$$\underset{y=\sum_{k=1}^{l}p_k\alpha_k q_k\beta_k r_k}{\vee}[\underset{1\leq k\leq l}{\wedge}[\wedge[\mu_1(p_k),\mu_2(q_k),\mu_3(r_k)]]:p_k,q_k,r_k\in T,\alpha_k,\beta_k\in\Gamma,\ \ l\in Z^+]$$

$$=\wedge[(\mu_1 o\mu_2 o\mu_3)(x),(\mu_1 o\mu_2 o\mu_3)(y).$$

Now, $(\mu_1 o\mu_2 o\mu_3)(x\alpha y\beta z)=\underset{x\alpha y\beta z=\sum_{i=1}^{n}u_i\alpha_i v_i\beta_i w_i}{\vee}[\underset{1\leq i\leq n}{\wedge}[\wedge\mu_1(u_i),\mu_2(v_i),\mu_3(w_i)]]:$

$$u_i,v_i,w_i\in T,\alpha_i,\beta_i\in\Gamma,\ \ n\in Z^+]$$

$$\geq\underset{z=\sum_{j=1}^{m}r_j\gamma_j s_j\delta_j t_j}{\vee}[\underset{1\leq j\leq m}{\wedge}[\wedge[\mu_1(x\alpha y\beta r_j),\mu_2(s_j),\mu_3(t_j)]]]$$

$$\geq\underset{z=\sum_{j=1}^{m}r_j\gamma_j s_j\delta_j t_j}{\vee}[\underset{1\leq j\leq m}{\wedge}[\wedge[\mu_1(r_j),\mu_2(s_j),\mu_3(t_j)]]]=(\mu_1 o\mu_2 o\mu_3)(z)$$

Hence, $\mu_1 o\mu_2 o\mu_3\in$ FLTΓI(T). Similarly, we can prove the remaining results.

***Theorem 3.11*: Let μ_1, μ_2, μ_3∈ FLTΓI(T)[FMTΓI(T), FRTΓI(T), FTΓI(T)]. Then** $\mu_1\Gamma\mu_2\Gamma\mu_3\subseteq\mu_1 o\mu_2 o\mu_3$.

***Proof*:** If for any *u, v, w* ∈ T and for any *α,* ∈Γ, *uαvβw≠x* then $\mu_1\Gamma\mu_2\Gamma\mu_3\subseteq\mu_1 o\mu_2 o\mu_3$.

For any *x*∈ T, $(\mu_1 o\mu_2 o\mu_3)(x)=\underset{x=\sum_{i=1}^{n}u_i\alpha_i v_i\beta_i w_i}{\vee}[\underset{1\leq i\leq n}{\wedge}[\wedge\mu_1(u_i),\mu_2(v_i),\mu_3(w_i)]]:$

$$u_i,v_i,w_i\in T,\alpha_i,\beta_i\in\Gamma,\ \ n\in Z^+]$$

$$\geq\underset{x=u\alpha v\beta w}{\vee}[\wedge\mu_1(u),\mu_2(v),\mu_3(w)]=(\mu_1\Gamma\mu_2\Gamma\mu_3)(x)$$

Thus, $\mu_1\Gamma\mu_2\Gamma\mu_3\subseteq\mu_1 o\mu_2 o\mu_3$.

***Theorem 3.12*: Let μ_1 be a fuzzy left TΓ-ideal, μ_2 be a fuzzy lateral TΓ-ideal and $\mu_3$be a fuzzy right TΓ-ideal of a ternaryΓ-semiring T. Then** $\mu_1\Gamma\mu_2\Gamma\mu_3\subseteq\mu_1\cap\mu_2\cap\mu_3$.

***Proof*:** Let μ_1 be a fuzzy left TΓ-ideal and μ_2 be a fuzzy lateral TΓ-ideal and μ_3 be a fuzzy right TΓ-ideal of a ternary**Γ**-semiring T.

Let *x* ∈ T. Suppose there exist $u,v,w\in T$ and $\alpha,\beta\in\Gamma$ such that $x=u\alpha v\beta w$.

Then $(\mu_1\Gamma\mu_2\Gamma\mu_3)(x) = Sup_{x=u\alpha v\beta w} \min\{\mu_1(u), \mu_2(v), \mu_3(w)\}$

$\leq Sup_{x=u\alpha v\beta w}[\min\{\mu_1(u\alpha v\beta w), \mu_2(u\alpha v\beta w), \mu_3(u\alpha v\beta w)\}]$

$= \min\{\mu_1(x), \mu_2(x), \mu_3(x)\} = (\mu_1 \cap \mu_2 \cap \mu_3)(x)$.

Suppose there do not exist $u, v, w \in T$ such that $x = u\alpha v\beta w$.

Then $(\mu_1\Gamma\mu_2\Gamma\mu_3)(x) = 0 \leq (\mu_1 \cap \mu_2 \cap \mu_3)(x)$. Therefore, $\mu_1\Gamma\mu_2\Gamma\mu_3 \subseteq \mu_1 \cap \mu_2 \cap \mu_3$.

***Theorem 3.13*: Let T be a multiplicatively regular ternary Γ-semiring and μ_1, and μ_3 be three fuzzy subsets of T. Then** $\mu_1 \cap \mu_2 \cap \mu_3 \subseteq \mu_1\Gamma\mu_2\Gamma\mu_3$.

***Proof* :** Let $c \in$ T. Since T is multiplicatively regular, then there exists two elements $x, y \in$ T and $\gamma_1, \gamma_2, \gamma_3, \gamma_4 \in \Gamma$ such that $c = c\gamma_1 x\gamma_2 c\gamma_3 y\gamma_4 c$.

Now, $(\mu_1\Gamma\mu_2\Gamma\mu_3)(c) = Sup_{c=u\alpha v\beta w}[\min\{\mu_1(u), \mu_2(v), \mu_3(w)\} : u, v, w \in T, \alpha, \beta \in \Gamma]$

$\geq \min\{\mu_1(c\gamma_1 x\gamma_2 c), \mu_2(c), \mu_3(c)\}$

[since $c = c\gamma_1 x\gamma_2 c\gamma_3 y\gamma_4 c = c\gamma_1 x\gamma_2 c\gamma_3 y\gamma_4 c\gamma_1 x\gamma_2 c\gamma_3 y\gamma_4 c$]

$= (\mu_1 \cap \mu_2 \cap \mu_3)(c)$. Therefore, $\mu_1 \cap \mu_2 \cap \mu_3 \subseteq \mu_1\Gamma\mu_2\Gamma\mu_3$.

Definition 3.14: An element a of a ternary Γ-semiring T is said to be ***ternary multiplicatively regular*** if there exist $x, y \in$ T and $\alpha, \beta, \gamma, \delta \in \Gamma$ such that $a\alpha x\beta a\gamma y\delta a = a$.

***Theorem 3.15*: A ternary Γ-semiring T is multiplicatively regular if and only if AΓBΓC = A∩B∩C for all left TΓ-ideals A, for all lateral TΓ-ideals B and for all right TΓ-ideals C of T.**

Theorem 3.16: In a ternary Γ-semiring T the following are equivalent.

(i) T is multiplicatively regular

(ii) $\mu_1\Gamma\mu_2\Gamma\mu_3 = \mu_1 \cap \mu_2 \cap \mu_3$ **for every fuzzy left TΓ-ideal** μ_1 **and every fuzzy lateral TΓ-ideal** μ_2 **and every fuzzy right TΓ-ideal** μ_3 **of T.**

***Proof*:** Let T be a regular ternary Γ-semiring. Then by theorem 3.13, $\mu_1 \cap \mu_2 \cap \mu_3 \subseteq \mu_1\Gamma\mu_2\Gamma\mu_3$. Again by theorem 3.12, $\mu_1\Gamma\mu_2\Gamma\mu_3 \subseteq \mu_1 \cap \mu_2 \cap \mu_3$. Hence $\mu_1\Gamma\mu_2\Gamma\mu_3 = \mu_1 \cap \mu_2 \cap \mu_3$.

Conversely, let T be a ternary Γ-semiring and for every fuzzy left TΓ-ideal μ_1, every fuzzy lateral TΓ-ideal μ_2 and every fuzzy right TΓ-ideal of T, $\mu_1\Gamma\mu_2\Gamma\mu_3 = \mu_1 \cap \mu_2 \cap \mu_3$. Let L, M and R be respectively a left TΓ-ideal, a lateral TΓ-ideal and a right TΓ-ideal of T and $x \in$ L ∩ M ∩ R. Then $x \in$ L, $x \in$ M and $x \in$ R. Hence $\chi_L(x) = \chi_M(x) = \chi_R(x) = 1$ (where $\chi_L(x)$, $\chi_M(x)$ and $\chi_R(x)$ are respectively the characteristic function of L, M and R). Thus $(\chi_L \cap \chi_M \cap \chi_R)(x) = \min\{\chi_L(x), \chi_M(x), \chi_R(x)\} = 1$. Since χ_L is a fuzzy left TΓ-ideal of T, χ_M is a fuzzy lateral TΓ-ideal of T and χ_R is a fuzzy right TΓ-ideal of T. Therefore by hypothesis, $\chi_L\Gamma\chi_M\Gamma\chi_R = \chi_L \cap \chi_M \cap \chi_R$. Hence $(\chi_L\Gamma\chi_M\Gamma\chi_R)(x) = 1$.

i.e., $\sup_{x=u\gamma v\delta w}[\min\{\chi_L(u), \chi_M(v), \chi_R(w)\} : u, v, w \in T; \gamma, \delta \in \Gamma] = 1$. This implies that there exist some r, $s, t \in$ T and $\gamma_1, \gamma_2 \in \Gamma$ such that $x = r\gamma_1 s\gamma_2 t$ and $\chi_L(r) = \chi_M(s) = \chi_R(t) = 1$. Hence $r \in$ L, $s \in$ M and $t \in$ R. Therefore $x \in$ LΓMΓR. Thus L ∩ M ∩ R ⊆ LΓMΓR. Also LΓMΓR ⊆ L ∩ M ∩ R. Hence LΓMΓR = L ∩ M ∩ R. Consequently, by theorem 3.15, the ternary Γ-semiring T is multiplicatively regular.

***Theorem 3.17*: Let** $\mu_1, \mu_2, \mu_3 \in FT\Gamma I(T)$. **Then** $\mu_1\Gamma\mu_2\Gamma\mu_3 \subseteq \mu_1 \cap \mu_2 \cap \mu_3 \subseteq \mu_1, \mu_2, \mu_3$.

***Proof* :** By the theorem 3.11, $\mu_1\Gamma\mu_2\Gamma\mu_3 \subseteq \mu_1 o \mu_2 o \mu_3$. For any $x \in$ T, if $(\mu_1 o \mu_2 o \mu_3)(x) = 0$, then obviously $\mu_1 o \mu_2 o \mu_3 \subseteq \mu_1 \cap \mu_2 \cap \mu_3$. Now for any $x \in$ T,

$$(\mu_1 o \mu_2 o \mu_3)(x) = \bigvee_{x = \sum_{i=1}^{n} u_i\alpha_i v_i\beta_i w_i} \Big[\bigwedge_{1 \leq i \leq n} [\wedge \mu_1(u_i), \mu_2(v_i), \mu_3(w_i)]\Big] : u_i, v_i, w_i \in T, \alpha_i, \beta_i \in \Gamma, \ n \in Z^+]$$

$$\leq \bigvee_{x=\sum_{i=1}^{n} u_i\alpha_i v_i\beta_i w_i} [\bigwedge_{1\leq i\leq n} [\wedge \mu_1(u_i\alpha_i v_i\beta_i w_i), \mu_2(u_i\alpha_i v_i\beta_i w_i), \mu_3(u_i\alpha_i v_i\beta_i w_i)]] : u_i, v_i, w_i \in T, \alpha_i, \beta_i \in \Gamma, \quad n \in Z^+]$$

$\leq \Lambda[\mu_1(x),\mu_2(x),\mu_3(x)] = (\mu_1\cap\mu_2\cap\mu_3)(x)$. Therefore, $\mu_1 o \mu_2 o \mu_3 \subseteq \mu_1 \cap \mu_2 \cap \mu_3$.

Again $(\mu_1\cap\mu_2\cap\mu_3)(x) = \Lambda[\mu_1(x),\mu_2(x),\mu_3(x)] \leq \mu_1(x)$. Thus $\mu_1\cap\mu_2\cap\mu_3\subseteq\mu_1$. Similarly, it can be shown that $\mu_1\cap\mu_2\cap\mu_3\subseteq\mu_2$ and $\mu_1\cap\mu_2\cap\mu_3\subseteq\mu_3$. Hence the theorem.

***Theorem 3.18*: Let $\mu_1, \mu_2, \mu_3, \mu_4 \in$ FLTΓI(T)[FMTΓI(T), FRTΓI(T), FTΓI(T)]. Then $\mu_1\Gamma\mu_2\Gamma\mu_3\subseteq\mu_4$ if and only if $\mu_1 o \mu_2 o \mu_3\subseteq\mu_4$.**

***Proof*:** Since $\mu_1\Gamma\mu_2\Gamma\mu_3\subseteq\mu_1 o\mu_2 o\mu_3$ it follows that $\mu_1 o\mu_2 o\mu_3\subseteq\mu_4$ implies that $\mu_1\Gamma\mu_2\Gamma\mu_3\subseteq\mu_4$. Assume that $\mu_1\Gamma\mu_2\Gamma\mu_3\subseteq\mu_4$.

Let $x\in$ T and $x = \sum_{i=1}^{n} u_i\alpha_i v_i\beta_i w_i$, $u_i, v_i, w_i \in T, \alpha_i, \beta_i \in \Gamma, \quad n \in Z^+$. Then

$$\mu_4(x) = \mu_4(\sum_{i=1}^{n} u_i\alpha_i v_i\beta_i w_i) \geq \wedge[\mu_4(u_1\alpha_1 v_1\beta_1 w_1), \mu_4(u_2\alpha_2 v_2\beta_2 w_2), \ldots.., \mu_4(u_n\alpha_n v_n\beta_n w_n)$$

$$\geq \wedge[(\mu_1\Gamma\mu_2\Gamma\mu_3)(u_1\alpha_1 v_1\beta_1 w_1), (\mu_1\Gamma\mu_2\Gamma\mu_3)(u_2\alpha_2 v_2\beta_2 w_2), \ldots..(\mu_1\Gamma\mu_2\Gamma\mu_3)(u_n\alpha_n v_n\beta_n w_n)]$$

$$\geq \wedge[\wedge(\mu_1(u_1), \mu_2(v_1), \mu_3(w_1)], \wedge(\mu_1(u_2), \mu_2(v_2), \mu_3(w_2)], \ldots.. \wedge(\mu_1(u_n), \mu_2(v_n), \mu_3(w_n)]].$$

$$\mu_4(x) \geq \bigvee_{x=\sum_{i=1}^{n} u_i\alpha_i v_i\beta_i w_i} [\bigwedge_{1\leq i\leq n} [\wedge[\mu_1(u_i), \mu_2(v_i), \mu_3(w_i)]]]$$

$= (\mu_1 o\mu_2 o\mu_3)(x)$. Thus $\mu_1 o\mu_2 o\mu_3\subseteq\mu_4$.

***Theorem 3.19*: Let $\mu_1, \mu_2, \mu_3, \mu_4 \in$ FLTΓI(T)[FMTΓI(T), FRTΓI(T), FTΓI(T)]. Then**

$(i)\quad (\mu_1 o \mu_2) o \mu_3 = \mu_1 o (\mu_2 o \mu_3)$

$(ii)\quad \mu_1 \subseteq \mu_2 \Rightarrow \mu_1 o \mu_3 o \mu_4 \subseteq \mu_2 o \mu_3 o \mu_4$

$(iii)\quad \mu_1 o \mu_2 o \mu_3 = \mu_2 o \mu_3 o \mu_1 = \mu_3 o \mu_1 o \mu_2$

If T is a commutative ternary Γ-Semiring.

$(iv)\quad eoeo\mu_1 = \mu_1$ where $e \in FLT\Gamma I(T)$ is definied by $e(x) = 1$ for all $x \in T$

[respectively, $eo\mu_1 oe = \mu_1$, $\mu_1 oeoe = \mu_1$, $eoeo\mu_1 = eo\mu_1 oe = \mu_1 oeoe = \mu_1$].

***Proof*:** (i): Proof of (i) follows from the definition.

(ii) : $\mu_1\subseteq\mu_2$ Now

$$(\mu_1 o \mu_3 o \mu_4)(x) = \bigvee_{x=\sum_{i=1}^{n} u_i\alpha_i v_i\beta_i w_i} [\bigwedge_{1\leq i\leq n} [\wedge \mu_1(u_i), \mu_3(v_i), \mu_4(w_i)]] : u_i, v_i, w_i \in T, \alpha_i, \beta_i \in \Gamma, \quad n \in Z^+]$$

$$\leq \vee[\bigwedge_{1\leq i\leq n} [\wedge[\mu_2(u_1), \mu_3(v_1), \mu_4(w_i)]]] = (\mu_2 o \mu_3 o \mu_4)(x)$$

Thus, $\mu_1 o \mu_3 o \mu_4 \subseteq \mu_2 o \mu_3 o \mu_4$.

(iii): $$(\mu_1 o \mu_2 o \mu_3)(x) = \bigvee_{x=\sum_{i=1}^{n} u_i\alpha_i v_i\beta_i w_i} [\bigwedge_{1\leq i\leq n} [\wedge \mu_1(u_i), \mu_2(v_i), \mu_3(w_i)]] : u_i, v_i, w_i \in T, \alpha_i, \beta_i \in \Gamma, \quad n \in Z^+]$$

$$= \bigvee_{x=\sum_{i=1}^{n} v_i\beta_i w_i\alpha_i u_i} [\bigwedge_{1\le i\le n} [\wedge \mu_2(v_i), \mu_3(w_i), \mu_1(u_i)]]$$

$$= (\mu_2 o\mu_3 o\mu_1)(x)$$

Therefore, $(\mu_1 o\mu_2 o\mu_3)(x) = (\mu_2 o\mu_3 o\mu_1)(x) \Rightarrow \mu_1 o\mu_2 o\mu_3 = \mu_2 o\mu_3 o\mu_1$

Similarly, we can show that $\mu_2 o\mu_3 o\mu_1 = \mu_3 o\mu_1 o\mu_2$

$\therefore \mu_1 o\mu_2 o\mu_3 = \mu_2 o\mu_3 o\mu_1 = \mu_3 o\mu_1 o\mu_2$

(iv): As T is with left unity $e_i \in$ L, which is defined by $\sum_{i=1}^{n} e_i\alpha_i e_i\beta_i x = x$.

For every $x \in$ T we have,

$$(eoeo\mu_1)(x) = \bigvee_{x=\sum_{i=1}^{n} u_i\alpha_i v_i\beta_i w_i} [\bigwedge_{1\le i\le n} [\wedge e(u_i), e(v_i), \mu_1(w_i)]] : u_i, v_i, w_i \in T, \alpha_i, \beta_i \in \Gamma, \ n \in Z^+]$$

$$= \vee[\bigwedge_{1\le i\le n} [\wedge[e, e, \mu_1(w_i)]]] = \vee[\bigwedge_{1\le i\le n} [\mu_1(w_i)] \le \vee[\bigwedge_{1\le i\le n} [\mu_1(u_i\alpha_i v_i\beta_i w_i)]]$$

$$\le \mu_1(\sum_{i=1}^{n} u_i\alpha_i v_i\beta_i w_i) = \mu_1(x)$$ Therefore, $(eoeo\mu_1) \le \mu_1$

Again $$(eoeo\mu_1)(x) = \bigvee_{x=\sum_{i=1}^{n} u_i\alpha_i v_i\beta_i w_i} [\bigwedge_{1\le i\le n} [\wedge e(u_i), e(v_i), \mu_1(w_i)]] : u_i, v_i, w_i \in T, \alpha_i, \beta_i \in \Gamma, \ n \in Z^+]$$

$$= \vee[\bigwedge_{1\le i\le n} [\wedge[e, e, \mu_1(w_i)]]] = \mu_1(x)$$

So $\mu_1 \subseteq eoeo\mu_1$ and hence $\mu_1 = eoeo\mu_1$

The following theorem shows that ternary multiplication is distributive over addition from three sides.

***Theorem 3.20*: Let $\mu_1, \mu_2, \mu_3 \in$ FLTΓI(T)[respectively, FMTΓI(T), FRTΓI(T), FTΓI(T]. Then**

(i) $\mu_1 o(\mu_2 + \mu_3)o\mu_4 = \mu_1 o\mu_2 o\mu_4 + \mu_1 o\mu_3 o\mu_4$

(ii) $(\mu_2 + \mu_3)o\mu_1 o\mu_4 = \mu_2 o\mu_1 o\mu_4 + \mu_3 o\mu_1 o\mu_4$

(iii) $\mu_1 o\mu_4 o(\mu_2 + \mu_3) = \mu_1 o\mu_4 o\mu_3 + \mu_1 o\mu_4 o\mu_3$

***Proof*:** Let $x \in$ T be arbitrary. Then

$$(\mu_1 o(\mu_2 + \mu_3)o\mu_4)(x) = \bigvee_{x=\sum_{i=1}^{n} u_i\alpha_i v_i\beta_i w_i} [\bigwedge_{1\le i\le n} [\wedge[\mu_1(u_i), (\mu_2 + \mu_3)(v_i), \mu_4(w_i)]]$$

$$: u_i, v_i, w_i \in T, \alpha_i, \beta_i \in \Gamma, n \in Z^+]$$

$$= \vee[\bigwedge_{1\le i\le n} [\wedge[\mu_1(u_i), \vee[\wedge[\mu_2(r_i), \mu_3(s_i)], \mu_4(w_i)]]]$$

$$= \bigvee_{x=\sum_{i=1}^{n}(u_i\alpha_i r_i\beta_i w_i + u_i\alpha_i s_i\beta_i w_i)} [\bigwedge_{1\le i\le n} [\wedge[\mu_1(v_i), \mu_2(v_i), \mu_3(s_i), \mu_4(w_i)]]]$$

$$\le \bigvee_{x=\sum_{j=1}^{n} p_j\delta_j q_j\varepsilon_j t_i + \sum_{k=1}^{m} p_k'\delta_k' q_k'\varepsilon_k' t_k'} [\wedge[\wedge[\bigwedge_{1\le j\le n}[\mu_1(p_j), \mu_2(q_j), \mu_4(t_j)],$$

$$\wedge[\bigwedge_{1\le k\le m}[\mu_1(p_k'), \mu_3(q_k'), \mu_4(t_k')]]]]]$$

$$= \vee[\wedge[(\mu_1 o\mu_2 o\mu_4)(u), (\mu_1 o\mu_3 o\mu_4)(v)] : u = \sum_{j=1}^{n} p_j\delta_j q_j\varepsilon_j t_i, v = \sum_{k=1}^{m} p_k'\delta_k' q_k'\varepsilon_k' t_k']$$

$$= ((\mu_1 o\mu_2 o\mu_4) + (\mu_1 o\mu_3 o\mu_4))(x)$$

Thus $\mu_1 o(\mu_2+\mu_3)o\mu_4 \subseteq \mu_1 o\mu_2 o\mu_4 + \mu_1 o\mu_3 o\mu_4$

Since $\mu_2 \subseteq \mu_2 + \mu_3$, therefore, $\mu_1 o\mu_2 o\mu_4 \subseteq \mu_1 o(\mu_2+\mu_3)o\mu_4$.
Similarly, $\mu_1 o\mu_3 o\mu_4 \subseteq \mu_1 o(\mu_2+\mu_3)o\mu_4$.
Thus $(\mu_1 o\mu_2 o\mu_4) + (\mu_1 o\mu_3 o\mu_4) \subseteq \mu_1 o(\mu_2+\mu_3)o\mu_4 + \mu_1 o(\mu_2+\mu_3)o\mu_4 = \mu_1 o(\mu_2+\mu_3)o\mu_4$.
Hence we conclude that $\mu_1 o(\mu_2+\mu_3)o\mu_4 = \mu_1 o\mu_2 o\mu_4 + \mu_1 o\mu_3 o\mu_4$. Proof of (ii) and (iii) follows similarly.

***Theorem 3.21*: Let μ_1, μ_2, μ_3 be three fuzzy left TΓ-Ideals (fuzzy lateral TΓ-ideals, fuzzy right TΓ-ideals, fuzzy TΓ-ideals) of a ternary Γ-semiring T. Then $\mu_1 + \mu_2$ is the unique minimal element of te family of all fuzzy left TΓ-Ideals (fuzzy lateral TΓ-ideals, fuzzy right TΓ-ideals, fuzzy TΓ-ideals) of a ternary Γ-semiring T containing μ_1, $\mu_2$and $\mu_1 \cap \mu_2 \cap \mu_3$is the unique maximal element of the family of all fuzzy left TΓ-Ideals (fuzzy lateral TΓ-ideals, fuzzy right TΓ-ideals, fuzzy TΓ-ideals) of a ternary Γ-semiring T containing μ_1, μ_2, μ_3.**
Proof: Let μ_1, μ_2, $\mu_3 \in$ FLTΓI(T). Then by theorem 2.2.10(v), μ_1, $\mu_2 \subseteq \mu_1 + \mu_2$. Suppose $\mu_1 \subseteq \psi$ and $\mu_2 \subseteq \psi$ where, $\psi \in$ FLTΓI(T). Now for any $x \in$ T,

$$(\mu_1+\mu_2)(x) = \bigvee_{x=y+z} [\wedge[\mu_1(y), \mu_2(z)] : y, z \in T]$$
$$\le \vee[\wedge[\psi(y), \psi(z)] \le \vee[\psi(x+y)] = \psi(x)$$

Thus $\mu_1 + \mu_2 \subseteq \psi$. Again $\mu_1 \cap \mu_2 \cap \mu_3 \subseteq \mu_1, \mu_2, \mu_3$.

Let us suppose that $\phi \in$ FLTΓI(T) be such that $\phi \subseteq \mu_1$, $\phi \subseteq \mu_2$ and $\phi \subseteq \mu_3$.
Then for any $x \in$ T, $(\mu_1 \cap \mu_2 \cap \mu_3)(x) = \wedge[\mu_1(x), \mu_2(x), \mu_3(x)] \ge \wedge[\phi(x), \phi(x), \phi(x)] = \phi(x)$.
Thus $\phi \subseteq \mu_1 \cap \mu_2 \cap \mu_3$. Uniqueness of $\mu_1 + \mu_2$and $\mu_1 \cap \mu_2 \cap \mu_3$ with the stated properties are obvious. Proofs of other cases follow similarly.

CONCLUSION

Our main purpose in this paper is to introduce the operations on fuzzy TΓ-ideal in ternary Γ-semirings. We give some characterizations of fuzzy TΓ-ideals.

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