S^{ℓ} AND S^{r} IDEALS ON d-ALGEBRAS

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ABSTRACT: The notion of d-algebras introduced J.Neggers and H.S.Kim [7] which is generalization of BCK-algebras. They investigated several relations betweend-algebras and BCK-algebras. Ideal theory in d-algebras introduced J.Neggers, Y.B.Jun and H.S.Kim [8] and investigated some relations. In this paper i introduced S^{ℓ} and S^{r}ideals on d-algebras, proved S^{ℓ} is a left d-ideal and proved some theorems regarding to d- ideals.

Key words: BCK-algebra ,d-algebra, Ideal of d-algebra.

1.PRELIMINARIES

1.1 Definition[7]: A d-algebra is a non-empty set X with a constant 0 and a binary operator * satisfying the following axioms

(i) $x * x = 0$

(ii) $0 * x = 0$

(iii) $x * y = 0$ and $y * x = 0 \Rightarrow x = y$.

1.2 Example[7]:

Let X = {0,1,2} be a set with the following cayley table

$*$	0	1	2
0	0	0	0
1	2	0	2
2	1	1	0

Then (x,∗,0) is a d-algebra

1.3 Example[7]: 2 Let X = {0,a,b,c} be a set with the following cayley table

∗	0	a	b	c
0	0	0	0	0
a	a	0	0	b
b	b	b	0	0
c	c	c	a	0

Then (X, ∗,0) is a d-algebra.

1.4 Definition[8]: Let X be a d-algebra and I be a subset of X, then I is called d-ideal of X if it satisfies the following conditions

(i) $0 \in I$

(ii) $x * y \in I$ and $y \in I \Rightarrow x \in I$

(iii) $x \in I$ and $y \in X \Rightarrow x * y \in I$

1.5 Example[8]: Let X = {0,1,2,3} be a d-algebra in which the operation ∗ defined as follows

∗	0	1	2	3
0	0	0	0	0
1	1	0	1	0
2	2	2	0	0
3	3	3	1	0

In X the sets $I_1 = \{0,1\}$ and $I_2 = \{0,2\}$ are d-ideals of X, while $I_3 = \{0,3\}$ and $I_4 = \{0,1.2\}$ are not d-ideals of X = {0,1,2,3}.

1.6 Definition[8]: Let (X, ∗,0) be d-algebra and $x \in X$ .Define $x * x = \{x * a / a \in X\}$. X is said to be edge d-algebra if for any $x \in X$, $x * X = \{x,o\}$.

1.7 Lemma[7]: Let (X, ∗ ,0) be an edge d-algebra ,then $x * 0 = x$ for any $x \in X$.

We assume that (X, ∗ ,0) is a d-algebra with edge property.

1.8 Lemma[7]: Let (X, ∗ ,0) be a d-algebra with edge property ,then $(x * y) * z = (x * z) * y$, for all $x,y,z \in X$

1.9 Proposition[7]: For any d-algebra X ,and for all $x,y,z \in X$, we have

(1) $x * (x * y) = y$,for all $x,y \in X$

(2) $x * (x * (x * y)) = x * y$

(3) $0 * (x * y) = (0 * x) * (0 * y)$

(4) $x * 0 = 0 \Rightarrow x = 0$

(5) $((x * z) * (y * z)) * (x * y) = 0$

(6) $x \leq y \Rightarrow x * z \leq y * z$ and $z * y \leq z * x$

(7) $((x * y) * (x * z)) * (z * y) = 0$

(8) $(x * y) = (x * z) \Rightarrow y = z$ (left cancellation law hold)

(9) $(x * y) * 0 = (x * 0) * (y * 0)$

(10) $x * (y * z) \geq (x * y) * z$

(11) $(x * z) * (y * z) = (x * y)$

(12) $(x * 0) * 0 = x$

2.MAIN PART

2.1 Definition: Suppose S is right ideal of X,define $S^{\ell} = \{ x \in X / x * y = 0 \ \forall \ y \in S \}$ and

Suppose S is a left ideal of X ,define

$S^{r} = \{ x \in X / y * x = 0 \ \forall \ y \in S \}$.

2.2 Theorem: Suppose S is right ideal of X ,then S^{ℓ} is a left ideal.

Proof: (i) Since S^{ℓ} is not empty,then there exist $x \in S^{\ell}$

$\Rightarrow 0 = x * x \in S^{\ell}$

(ii) Suppose $x \in S^{\ell}$ and $y * x \in S^{\ell}$

Since $x \in S^{\ell} \Rightarrow x * z = 0, \forall z \in S$

Since $y * x \in S^{\ell}$

$\Rightarrow (y * x) * z = 0, \forall z \in S$ (By the definition of S^{ℓ})

$\Rightarrow (y * z) * (x * z) = 0$ (By 1.9)

$\Rightarrow (y * z) * 0 = 0$ (since $\Rightarrow x * z = 0$)

$\Rightarrow (y * z) = 0$ (since $x * 0 = 0 \Rightarrow x = 0$)

$\Rightarrow y \in S^{\ell}$ (since by the definition of S^{ℓ})

(iii)Suppose $x \in S^{\ell}$ and $y \in S$

Since $x \in S^{\ell}$ and $x * r = 0 \ \forall \ r \in S$ and $y \in S$

Now $(x * y) * r = (x * r) * y$ (since by above theorem)

$= 0 * y$ (By the definition of S^{ℓ}) $= 0$ (By the definition of d-algebra)

Therefore $(x * y) * r = 0 \Rightarrow x * y \in S^{\ell}$

Hence the theorem.

2.3 Theorem: Let S,T are ideals of d-algebra X .If $S \subseteq T$ then $T^{\ell} \supseteq S^{\ell}$.

Proof: Suppose that $S \subseteq T$

Claim: To show that $T^{\ell} \supseteq S^{\ell}$:

Let $x \in S^{\ell} \Rightarrow x * z = 0, \forall z \in T$

$\Rightarrow x * z = 0, \forall z \in S$ (since $S \subseteq T$

$\Rightarrow x \in S^{\ell}$ (since by the definition)

Hence $T^{\ell} \supseteq S^{\ell}$.

2.4 Theorem: Let S be the ideal of d-algebra X,then $S \subseteq S^{\ell r}$.

Proof: Suppose S be the ideal of d-algebra X.

Claim: $S \subseteq S^{\ell r}$:

Let $x \in S \Rightarrow y * x = 0 \;\; \forall y \in S^{\ell}$

$\Rightarrow x \in S^{\ell r}$.

Therefore $x \in S \subseteq x \in S^{\ell r}$..

Hence : $S \subseteq S^{\ell r}$

2.5 Note: By similar proof of above theorem we have $S \subseteq S^{r\ell}$.

2.6 Theorem: For any ideal S of d-algebra X,

$S^{\ell} = S^{\ell r \ell}$.

Proof: We have $S \subseteq S^{\ell r}$. (Since by 2.4)

$\Rightarrow S^{\ell} \supseteq S^{\ell r \ell}$ (Since by 2.3)..........(i)

Replace S by S^{ℓ}in the above note 2.5,

we have $S^{\ell} \subseteq S^{\ell r \ell}$...........(ii).

Therefore $S^{\ell} = S^{\ell r \ell}$ (since from (i) & (ii)).

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